Proceedings of the 2002 IEEE/RSJ
Intl. Conference on Intelligent Robots and Systems
EPFL, Lausanne, Switzerland • October 2002

Composite-H_∞ Controller Synthesis for Flexible Joint Robots

H. D. Taghirad[1], Gh. Bakhshi

Advanced Robotics and Automated Systems,
Department of Electrical Engineering,
K.N. Toosi University of Technology,
Tehran, Iran, 16314
[1]*Taghirad@novindana.com*

Abstract

In this paper a robust composite control algorithm is proposed for flexible joint manipulators, with the emphasis on satisfying control effort limitations. An H_∞ framework is used for the slow subsystem controller design, instead of robust PID synthesis introduced in the literature. Linear identification techniques are used to represent the nonlinear dynamics of the system into a linear model plus multiplicative uncertainty. An H_∞ controller is designed in the framework of composite control, in order to optimize the required control effort, along with satisfying robust stability and desirable performance. The effectiveness of the proposed control law is compared with other methods through a simulation study. The comparison results show a significant improvement in control effort, while satisfying both stability and performance requirements.

1. Introduction

The importance of flexibility in modeling and control synthesis of industrial robots has been experimentally shown in [6]. On the other hand, harmonic drives and torque transducers are extensively used in industrial and space robots. The joint flexibility produced by these elements, is one of the main reasons of widespread international attention to the topic of control of flexible joint manipulators in recent years. Spong et al observed that, by neglecting the flexibility of such systems, the designed controller, may even cause instability [5]. Singular perturbation is the main idea for modeling of flexible joint manipulators, and to separate the slow and fast dynamics. In most representations the elastic deformations are considered small, and the elasticity of the joint is modeled with linear torsional spring. The controller synthesis is, hence, separated into two parts in order to control the fast and slow dynamics, respectively. This type of control design is named *composite control* in the literature [4, 5]. Spong showed that by neglecting the rotor motion from kinetic energy derivation of the system, FJR becomes feedback linearizable, and hence, introduced a feedback linearization control algorithm for such systems. The main drawback of this method is its dependence to the acceleration and jerk measurements [4]. To avoid these costly measurements, a novel integral manifold control is introduced, by Spong et al in [5]. In this method the first order approximation of the flexible reduced order model is used, and hence, a correction term is added to the composite control law. In order to design the rigid control term many algorithms are proposed in the literature. Adaptive schemes are proposed by Khorasani [3], and followed by Ghorbel et al [2], while Taghirad and Khosravi proposed a simple form of PID controller [8]. The PID controller is designed with an emphasis on robust stability of the system, imposed to parameter and input uncertainties. One of the drawbacks of the aforementioned algorithms is their relatively high control efforts needed to accomplish a good performance. In this paper the control synthesis of FJR's are reconsidered with the emphasis on robust performance and especially to optimize the required control effort, while obtaining robust stability and desired performance. In continuation of the previous results, the main significance of this method is providing a systematic approach to use the control effort limitations directly into controller synthesis. In order to introduce the proposed method, previous results on robust composite control of flexible joint robots are overviewed in next section. The representation of the nonlinear dynamics of the system in terms of a linear model and multiplicative uncertainty is elaborated next, and an H_∞-based robust controller is designed for the system. By analyzing the performance of the closed-loop system, it is observed that, the low frequency content reference inputs are well tracked, with a limited control effort, while the controller synthesis enables the designer to make a suitable compromise between the required bandwith and the control effort limitation. Moreover, in order to have the benefits of composite control law in addition to that of H_∞ synthesis, these methods are combined and a composite H_∞ controller is designed for the system. Through a simulation study it is shown that the composite H_∞ controller provides significant improvement in control effort, while satisfying both stability and performance requirements.

2. Composite Control of FJR

The singularly perturbed model of an n-link flexible joint robot with revolute joints, can be represented as [8]:

$$\begin{cases} \ddot{q} = a_1(q,\dot{q}) + A_1(q)z \\ \varepsilon\ddot{z} = a_2(q,\dot{q}) + A_2(q)z + B_2 u \end{cases} \tag{1}$$

In which, q is the link position vector, z is the spring elastic force vector, and ε is the inverse of spring stiffness $1/k$, and the other parameters are as following:

$$\begin{aligned} A_1 &= -M^{-1}(q), a_1 = -M^{-1}(q)N(q,\dot{q}) \\ a_2 &= -\varepsilon J^{-1}D\dot{z} + J^{-1}D\dot{q} - J^{-1}T_F - M^{-1}(q)N(q,\dot{q}) \\ A_2 &= -\left(M^{-1}(q) + J^{-1}\right), \ B_2 = -J^{-1} \end{aligned} \quad (2)$$

Where, $M(q)$ is the mass matrix, $N(q,\dot{q})$ is the inertial centrifugal and Coriolis torque vector, J is the Matrix of actuator moments of inertia, D is the vector of damping coefficients, and T_F is the external disturbance torque vector, and the effect of unmodeled dynamics of the system [8].

Defining the integral manifold as M_ε in form of $M_\varepsilon : z = H(q,\dot{q},u,\varepsilon)$ and limiting the system dynamics to remain within this manifold, the reduced order flexible model of the system, satisfies the following equation:

$$\ddot{q} = a_1(q,\dot{q}) + A_1(q)H(q,\dot{q},u,\varepsilon) \quad (3)$$

The integral manifold remains invariant, provided that the fast dynamics of the system is stabilized with the fast control term of a composite control law as following [8]:

$$u = u_s(q,\dot{q},\varepsilon) + u_f(\eta,\dot{\eta}) \quad (4)$$

In which u_f is designed based on asymptotic stability of fast subsystem, and u_s stabilizes the slow subsystem robustly with a PID design scheme elaborated in [8]. η is the variable indicating the variations of fast dynamics variables from integral manifold:

$$\eta = z - H(q,\dot{q},u_s,\varepsilon) \quad (5)$$

In which it is assumed that $u_f(0,0) = 0$. Hence, the fast subsystem dynamics satisfies the following equation:

$$\varepsilon\ddot{\eta} = A_2(q)\cdot\eta + B_2 u_f \quad (6)$$

This is a dynamic relation with respect to q and its stability can be obtained with pole placement method. The slow subsystem control effort u_s, has a correction term, which is obtained by first order approximation of u_s, and H about ε=0 as following:

$$\begin{aligned} u_s(q,\dot{q},\varepsilon) &= u_o(q,\dot{q}) + \varepsilon\, u_1(q,\dot{q}) + \dots \\ H(q,\dot{q},u_s,\varepsilon) &= H_o(q,\dot{q},u_s) + \varepsilon\, H_1(q,\dot{q},u_s) + \dots \end{aligned} \quad (7)$$

These approximations must satisfy the manifold invariance condition; hence:

$$\varepsilon\,\ddot{H} = a_2(q,\dot{q},\varepsilon\dot{H}(q,\dot{q},u,\varepsilon)) + A_2(q)\ H(q,\dot{q},u,\varepsilon) + B_2 u \quad (8)$$

By equating the similar order terms in the above relation, first u_o is designed based on the robust stability of the rigid dynamics, and then the correction terms u_i: $i=1,2,\dots$ can be designed up to desired order in terms of u_o. The details of stability theorems of this design procedure are given in [8], and a simulation-based comparison study is forwarded in [9]. It is observed that in all compared methods in this simulation study, the required control efforts show some peaks in small time intervals, which is hardly implementable. In order to remedy this shortcoming an improvement in the design procedure is proposed here within a robust H_∞ design framework.

3. H_∞ Controller design for FJR

Since the required objectives of robust stability, fast and suitable tracking response and disturbance attenuation despite the limited control effort, are well suited into an H_∞ design framework, in this section the FJR controller design is reformulated such that this methodology can be applied. In order to apply H_∞ synthesis to this problem, the nonlinear model of FJR is represented by a linear model and multiplicative uncertainty, using a systematic linear identification scheme. In this representation, the nominal model replicates the dynamic behavior of the system, only at nominal conditions, and all nonlinear interactions, unmodeled dynamics and the disturbances are encapsulated via an unstructured uncertainty representation. This idea is used extensively in many applications, where linear H_∞ schemes are used in controller design of some nonlinear systems [7].

In order to represent a system into this form, suppose the true system belongs to a family of plants ∏, which is defined by using the following perturbation to the nominal plant P_o:

$$\forall P(s) \in \Pi \qquad P(s) = \left(1 + \Delta(s)W(s)\right)P_o(s) \quad (9)$$

In this equation $W(s)$ is a stable transfer function indicating the upper bound of uncertainty and $\Delta(s)$ indicates the admissible uncertainty block, which is a stable but unknown transfer function with $\|\Delta\|_\infty < 1$. In this general representation $\Delta(s)W(s)$ describes the normalized perturbation of the true plant from nominal plant, and is quantitatively determined through identification in each frequency:

$$\frac{P(j\omega)}{P_0(j\omega)} - 1 = \Delta(j\omega)W(j\omega) \quad (10)$$

In which $\|\Delta\|_\infty < 1$; hence,

$$\left|\frac{P(j\omega)}{P_0(j\omega)} - 1\right| \le |W(j\omega)|, \forall\omega \quad (11)$$

Where, $|W(j\omega)|$ represents the uncertainty profile with respect to frequency. Nominal plant P_o, can be evaluated experimentally, through a series of frequency response estimates of the system in the operating regime [7]. Linear identification for the system can be applied with different input amplitudes, while their outputs are measured and logged. By minimizing the least squares of the prediction error, from the set of input-output information, a set of linear models are estimated for the system, which can be considered as Π. The uncertainty upper bound $W(s)$, is then obtained using Eq. (11), while the nominal plant P_o is selected from the average fit over all the individual identified plants. By this means, not only the nominal plant of the system is obtained, but also a measure of its perturbations, will be represented by the multiplicative uncertainty. This representation is highly effective, if the system variations from its nominal conditions are not large, especially within the desired closed-loop bandwidth of the system. In order to illustrate the details and effectiveness of this method, it is implemented for the one-link flexible joint manipulator introduced by Spong [4].

Table 1: *One-link flexible joint robot parameters*

Parameter	Nominal Values
Mass	M=1
Joint Stiffness	K=100
Length (2L)	L=1
Moment of Inertia	I=1
Rotor Inertia	J=1

The system dynamics are represented as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{-MgL}{I}\sin(x_1) - \dfrac{k}{I}(x_1 - x_3) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \dfrac{k}{J}(x_1 - x_3) + \dfrac{1}{J}u \end{cases} \quad (12)$$

In which x_1 and x_3 are the link and rotor angles, respectively and the system parameters are given in Table 1.

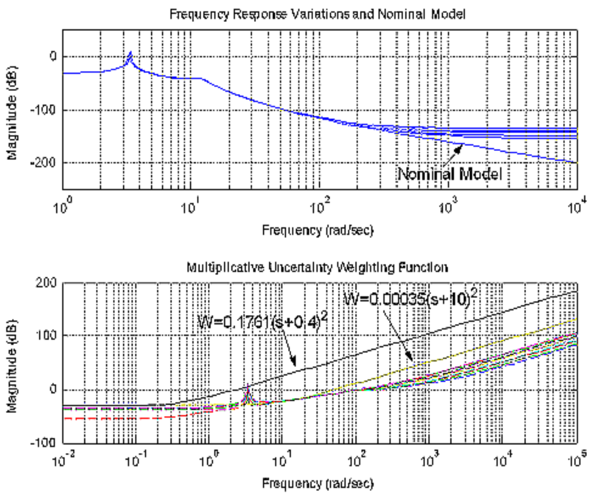

Figure 1: *Frequency response estimates of open-loop one-link flexible joint robot; Nominal plant and uncertainty profile.*

Figure 1, illustrates some frequency response estimates of the system and its relating uncertainty profile. Multi-sine and chirp functions with different amplitudes and frequencies are used as inputs, and least-square estimation methods[1], are used to find the estimates. The nominal model of the system P_o, is determined from average of the estimated models, whose transfer function is as following:

$$P_o = \frac{0.0079\ s^2 + 1.407\ s + 44.48}{s^4 + 3.3\ s^3 + 159\ s^2 + 42.8\ s + 1696} \quad (13)$$

The nominal plant has four poles at s = -0.018±3.39j and s=-1.63±12.03j, and two zeros at s=-41, -137, with a DC gain of –31 *dB*. The uncertainty weighting function is estimated as $W(s)$=0.176(s+0.4)^2. Relatively small uncertainty at low frequencies is promising a suitable H_∞ controller design, but sharp increase of the uncertainty at ω=0.4 *rad/sec* warns about the limitations on achievable closed-loop bandwidth for this system. The peak of uncertainty estimation at frequency 33.5 rad/sec exposes this bandwidth limitation. Hence, a second choice of the uncertainty profile is also illustrated in Figure 1 and in Equation 14, in which the sharp peaks are neglected. In practice due to the conservativeness of H_∞ design, this assumption will reduce the bandwidth limitation without any practical stability concerns.

$$W(s) = 3.5\times10^{-4}(s+10)^2. \quad (14)$$

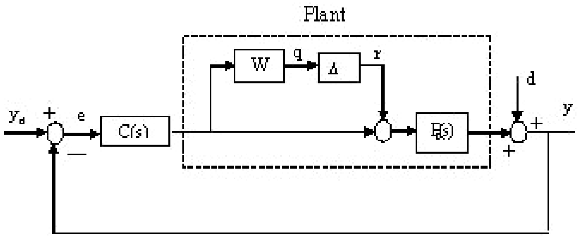

Figure 2: *The closed-loop block diagram of system with input multiplicative uncertainty.*

Figure 2, illustrates the block diagram representation of the system with multiplicative uncertainty. The objectives of controller design are robust stability and good tracking performance in presence of torque disturbances, despite the *limited control effort.* All these objectives can be simultaneously optimized by the solution of a mixed-sensitivity problem formulated on the generalized plant illustrated in Figure 3. The robust stability is guaranteed by minimizing the infinity norm of weighted transfer function from y_d to z_1, which is equivalent to the weighted complementary sensitivity function: $\|WT\|_\infty<1$ (Small-gain theorem). The tracking performance and disturbance attenuation is obtained by minimizing the infinity norm of y_d to z_2, or the weighted sensitivity function $\|W_sS\|_\infty<1$.

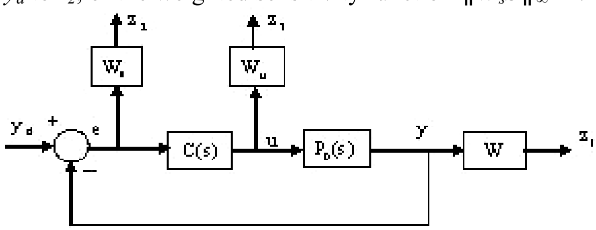

Figure 3: *Block diagram representation of mixed-sensitivity solution for system.*

The sensitivity weighting function W_s, must be selected such that the performance requirements like response speed and steady state errors are satisfied. Finally, the infinity norm of y_d to z_3, or weighted control effort transfer function penalizes controllers with high control effort, and provides a media in our optimization to include directly the control effort limitations into the controller synthesis. Hence, by simultaneously optimization of the infinity norm of the transfer matrix $\|T_{yz}\|_\infty<1$, all the objectives are satisfied, provided that a solution exists to the mixed-sensitivity problem. Many tractable numerical solutions exist for this problem[2]. The problem has been solved for

[1] "PEM" function in Matlab is used

[2] "Hinfsyn" function in μ-synthesis is used.

the represented FJR, with an upper bound for control effort corresponding to W_u=0.001. This selection indicates that the maximum amplitude of the control effort for all reference inputs can only be smaller than W_u^{-1}. The sensitivity weighting function is determined in order to have solution for the mixed sensitivity problem as well as to have maximum reachable bandwidth as following:

$$W_s(s) = \frac{s+160}{5(s+1.6)} \tag{15}$$

This selection indicates that the disturbance attenuation gain must be at least 20:1 or the steady state error to a unit step is smaller than 5%, while the designed bandwidth is about 1.6 *rad/sec.* The H_∞ solution for this problem is calculated from numerical solutions as following with a DC gain of *57 dB*.

$$C(s) = \frac{4.14\times10^{11}(s+0.018\pm3.38j)(s+1.63\pm12.03j)}{(s+1.6)(s+42.04)(s+94.3\pm4.1j)(s+1.5\times10^{6})} \tag{16}$$

To analyze the performance of the closed loop system, the nonlinear model of the system is used in simulations. First a sinusoid reference trajectory with frequency *2 rad/sec* is considered, and the closed-loop response is illustrated in Figure 4. The tracking error is quite small, despite the limited control effort of order 10^3 (as assigned by W_u). Figure 5 illustrates the closed-loop tracking performance of the system with the proposed controller, in which the settling time is desirably short, and the tracking performance is quite suitable. Comparing these results to the composite controls introduced in [4] and [8], comparable tracking performance are obtained in spite of much smaller control effort. In addition to that, the H_∞ controller is much simpler in structure and easily tunable and implementable. These desirable performances cannot be obtained if in the uncertainty representation, the sharp peaks were not neglected. Despite the practical importance of this method, since the uncertainty representations are not accurately concurrent with the identification results, robust stability of the closed-loop system cannot be claimed rigorously. In order to remedy this theoretical draw back, the composite controller design is combined with the proposed H_∞, replacing the robust PID design in the composite control law with an H_∞ controller, designed to reduce the control effort. The details of the proposed composite H_∞ are elaborated in the next section.

4. Composite H_∞ Control law

Consider the composite control structure introduced in section 2 and in [8]. The control effort is composed of three terms, in which according to Equation 4, u_f is designed to stabilizes the fast subsystem, and u_s has two main components: u_o, and u_1, corresponding to the robust PID and the correction term from the integral manifold contribution in the slow subsystem control law.

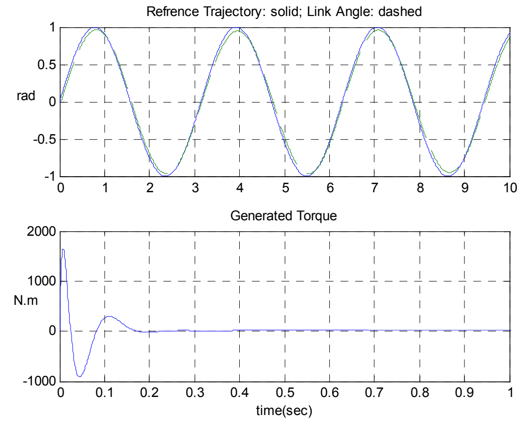

Figure 4: *Closed-loop tracking performance of* H_∞ *controller to sinusoid reference command.*

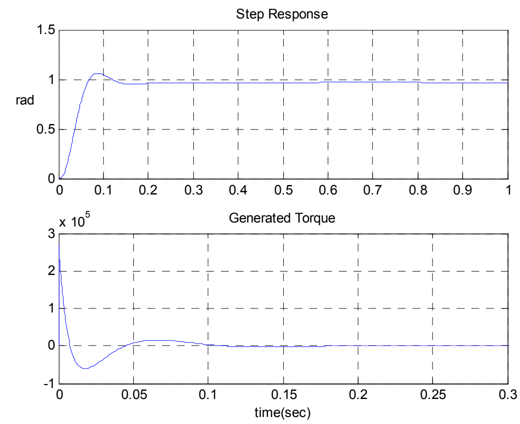

Figure 5: *Closed-loop tracking performance of* H_∞ *controller to a unit step reference command.*

Rigorous Lyapunov-based stability theorems are introduced in [8], which guarantees the robust stability of the closed-loop system with this control configuration. We propose, replacing the robust PID controller which is designed with only robust stability concern, with an H_∞ controller designed for robust stability and tracking, especially to reduce the undesirable control effort. In order to accomplish that, similar identification schemes are applied to the closed loop system with the above composite control configuration, and different input-output data is collected from the output of the PID and the plant blocks, respectively. By this means, the overall closed-loop system with fast control u_f, and u_1, is identified, excluding only u_o term generated by the PID term. Hence, similar H_∞ synthesis approach can be forwarded for this system to replace the PID with an H_∞-based controller. By this means not only the benefits of robust tracking synthesis is obtained, despite the control effort saturation, but also the robust stability theorems developed for the composite control law can be applied to the overall closed-loop system.

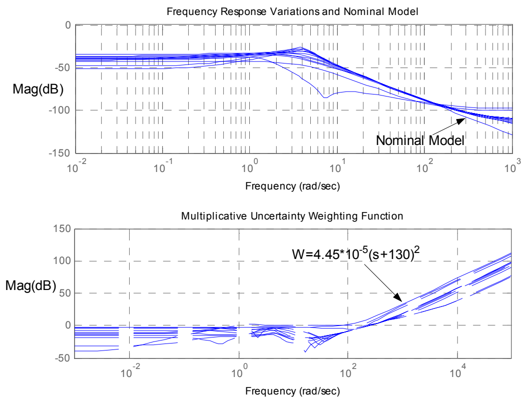

Figure 6: *Frequency response estimates of composite controlled one-link flexible joint robot; Nominal plant and uncertainty profile.*

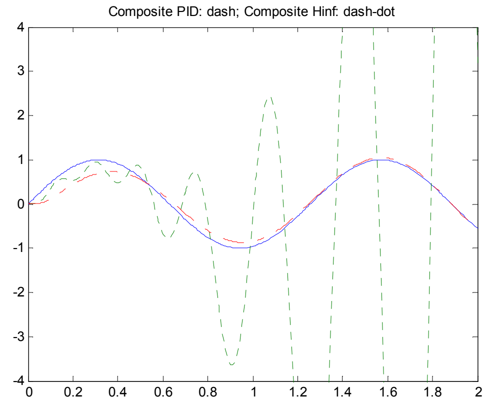

Figure 7: *Closed-loop tracking performance of composite H_∞ compared to composite PID, to a sinusoid reference command.*

Let us apply the combined composite and H_∞ algorithm for the one-link flexible robot of [4], and compare the results to the previous algorithms proposed in this paper. Similar identification simulations are forwarded for this configuration, and the estimated frequency response estimates and its uncertainty profile is given in Figure 6. The nominal plant of the identified model and the uncertainty profile upper bound are determined as following:

$$P_0(s) = \frac{0.369\ (s+0.396)}{(s+0.69)(s+1.46\pm 4.16j)} \qquad (17)$$

$$W(s) = 4.45\times 10^{-5}(s+130)^2 \qquad (18)$$

The uncertainty profile obtained for this system has DC gain of *–2.5 dB*, but provides much wider frequency band-width. The reason for this improvement in uncertainty description is the effect of feedback, which makes the system more linear. Hence, much suitable tracking performance is expected for this representation. Using the control effort weighting function as $W_u = 3\times 10^{-4}$, the mixed-sensitivity problem is solvable for the following performance weighting function:

$$W_s(s) = \frac{s+80}{5(s+0.8)} \qquad (19)$$

The selected performance weighting function indicates a bandwidth of *0.8 rad/sec*, with a disturbance attenuation ratio of *20:1*. The H_∞ solution to the mixed sensitivity problem similar to previous section is calculated from numerical software. The designed controller has the following transfer function:

$$C(s) = \frac{9.4\times 10^9 (s+0.69)(s+1.46\pm 4.16j)}{(s+0.4)(s+0.8)(s+53.3)(s+2.89\times 10^6)} \qquad (20)$$

with a DC gain of 68 dB. As it is observed from the poles and zeros of the controller transfer function, it can be reduced to a lower order form. The controller is reduced into a second order controller, which is quite close to the robust PID form designed in [8]. The reduced order controller has the following transfer function:

$$C(s) = \frac{3240(s+1.81\pm 4.03j)}{(s+0.4)(s+56.6)} \qquad (21)$$

In which the DC gain of controller remains as before. The tracking performance of the system is illustrated in Figure 7. The reference signal is a sinusoid with *5 rad/sec* frequency and the simulation is executed with a saturation block for a maximum control effort of *4000 N.m* for composite PID controller, and *1000 N.m* for composite H_∞. This result illustrates the effectiveness of the proposed composite H_∞ to provide larger bandwidth and suitable tracking error, despite a limited control effort. The robust PID controller results into instability in presence of control effort limitations, even with four times larger bounds on the control effort of composite H_∞.

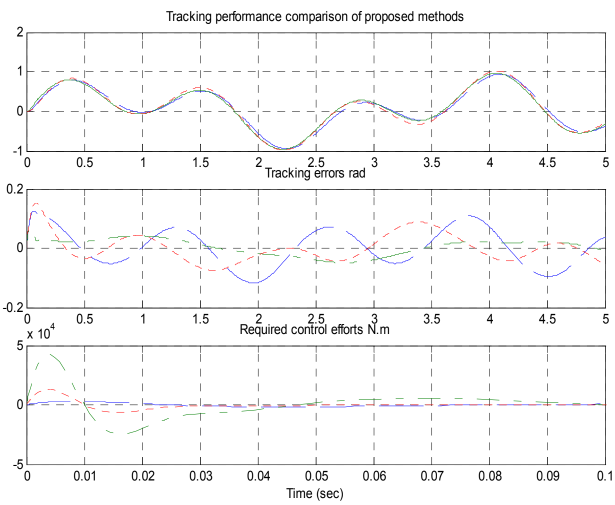

Figure 8: *Tracking performance and control effort comparison of composite PID, H_∞ and Composite H_∞.*

To have a comparison between the results obtained with the two proposed methods and composite PID method [7], the tracking performance and control effort of them for a multi-sine reference command is illustrated in Figure 8. The closed-loop response of the system with H_∞ controller is plotted with dashed line, composite PID with dash-dotted line, and the proposed composite H_∞ with dotted line. In the first figure the overall tracking performance of all methods are observed to be quite comparable, while in the second graph the tracking error is shown more accurately. In the last graph the control effort required by each method is given. Table 2 gives the two- and infinity-norms of the tracking error and corresponding control effort. The tracking error of the composite H_∞ has similar error compared to the composite PID, but requiring only about a quarter of corresponding control effort. H_∞ controller results have the least control effort, on the other hand, but with relatively larger tracking error. This comparison clearly illustrates the effectiveness of combining composite control law with H_∞ synthesis, in order to obtain a suitable tracking performance, with much smaller required control effort. Practical control effort limitations may cause even instability in the composite PID algorithm, while this limitation is fully acknowledged and compensated for, by the proposed composite H_∞ synthesis.

Table 2: *Tracking error and control effort measures for different control algorithms.*

Control Method	$\|e\|_2$	$\|e\|_\infty$	$\|u\|_\infty$
H_∞ control	2.52	0.12	0.03×10^5
Composite PID	1.08	0.077	0.42×10^5
Composite H_∞	1.71	0.151	0.13×10^5

5. Conclusions

In this paper, the robust control of flexible joint manipulators, with the emphasis on the performance and control effort limitations, is analyzed in detail. First, previous research on composite PID control law is elaborated, and the main draw back of it to provide solutions for limited control effort is described. Then with a new approach to the control synthesis of the system, an H_∞ framework is proposed. In this framework, linear identification techniques are used to represent the open-loop system nonlinear dynamics as a linear model with multiplicative uncertainty. Then, an H_∞ controller is designed for the system with the emphasis on robust performance and especially limited control effort. By this means relatively suitable tracking performance is obtained with much smaller control effort, compared to that of composite PID controller. In order to have the benefits of composite control in addition to H_∞ controller, it is proposed to combine these methods, in which the PID controller of the composite algorithm is replaced with an H_∞ controller, designed for performance. This has been accomplished by the proposed algorithm and the tracking performance and control effort limitations are compared in a simulation study. It is observed that on the contrary to composite PID control, the composite H_∞ control is robustly stable, despite control effort limitations. Moreover, it is shown in the simulation study, that the proposed composite H_∞ control law can provide similar tracking performance to composite PID, with much smaller control effort. This yields to a significant improvement in control effort, while satisfying the robustness and performance requirements.

References

[1] R.A. Al-Ashoor, R.V. Patel. And K. Khorasani, "Robust adaptive controller design and stability analysis for flexible joint manipulators", IEEE Trans. Systems, Man and Cybernetics, 23(2): 598-602, 1993.

[2] F. Ghorbel, J.Y. Hung, and M.W. Spong, "Adaptive control of flexible joint manipulators", Proceedings of 29th Conf. Decision and Control, 2:1188-1193, 1990.

[3] K. Khorasani, "Adaptive control of flexible joint robots", IEEE Trans. Robotics and Automation, 8(2): 250-257, 1992.

[4] M.W. Spong, "Modeling and control of elastic joint robots" Journal of Dynamic Systems, Measurement and Control, 109:310-319, 1987.

[5] M.W. Spong, K. Khorasani, and P.V. Kokotovich, "An integral manifold approach to the feedback control of elastic joint robots", IEEE Journal of robotics and Automation, RA-3(4):291-300. Aug 1987.

[6] L.M. Sweet and M.C. Good, "Redefinition of the robot motion control problem: Effects of plant dynamics, drive system constraints, and user requirements", Proceedings of 23rd IEEE Conf. On Decision and Control, 1984.

[7] H.D. Taghirad and P.R. Belanger, "H_∞-Based robust torque control of harmonic drive systems", Journal of Dynamic Systems, Measurements and Control, 123(3): 338-345, 2001.

[8] H.D. Taghirad and M.A. Khosravi, "Stability analysis, and robust PID design for flexible joint robots", Proceedings of the International Symposium on Robotics, 1:144-149, 2000.

[9] H.D. Taghirad and M.A. Khosravi, "Stability analysis, and robust composite controller design for flexible joint manipulators", In Proceedings of IROS'02, 2002.