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Design and Simulation of Robust Composite Controllers for Flexible Joint Robots

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Abstract

In this paper the control of flexible joint manipulators is studied in detail. A composite control algorithm is proposed for the flexible joint robots, which consists of two main parts. Fast control, u_f, which guarantees that the fast dynamics remains asymptotically stable, and the corresponding integral manifold remains invariant. Slow control ,u_s, itself consists of a robust PID designed based on the rigid model, and a corrective term designed based on the reduced flexible model. The stability of the overall closed loop system is proved to be UUB stable, by Lyapunov stability analysis. Finally, the effectiveness of the proposed control law is verified through simulations. It is shown that the proposed control law ensure the robust stability and performance, despite the modeling uncertainties.

I. Introduction

After the inception of harmonic drive, multiple–axis flexible robot manipulators are widely used in industrial and space applications. In early eighties researchers showed that the use of control algorithms developed based on rigid robot dynamics on real non–rigid robots is very limited and may even cause instability [15]. To avoid this problem, many researchers have proposed control algorithms based on slow and fast dynamics of the system. Among them, in adaptive methods many algorithms are developed for FJR's, in most of which a term due to the fast subsystem is added to the adaptive algorithm based on rigid models [3, 4]. In robust methods by considering model uncertainties the stability of the fast subsystem is first analyzed and by the use of robust control synthesis, a robust controller is designed for the slow subsystem [1, 7]. Hence, most of the research on FJR's are concentrated on nonlinear control schemes. In this paper we propose a new method based on the simple form of PID, and analyze the robust stability of the uncertain closed–loop system in the presence of structured and unstructured uncertainties. In this analysis we introduce an integral manifold plus a composite control law in order to restrain the integral manifold invariant and to satisfy asymptotic stability requirement. The control effort consists of three elements, the first element is designed for the fast subsystem, the second term is a robust PID control designed for the rigid subsystem and the third term is a corrective term designed based on the first order approximation of the reduced flexible system. Based on the Lyapunov stability theory the complete closed–loop system is proven to be UUB stable. In order to verify the effectiveness of the proposed design method, and to compare its results to that presented in the literature, simulation of single and two link flexible joint manipulators are examined. It is shown in this study that the proposed control law ensure the robust stability and performance, despite the modeling uncertainties.

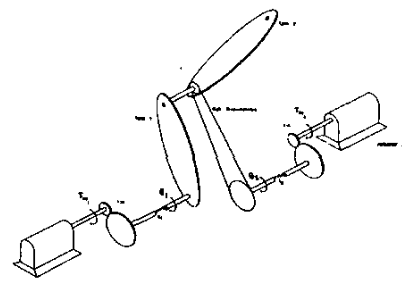

Fig. 1. Two-link Flexible Joint Manipulator.

II. Flexible Joint Robot Modeling

Spong [13], has derived a nonlinear dynamical model for FJR using singular perturbation, in which the slow states are the position and velocities of the joints and the fast states are the forces and their derivatives. In order to model an N–axis robot manipulator with n revolute joints assume that: $\hat{q}_i : i = 1, 2, ..., n$ denote the position of i'th link and $\hat{q}_i : i = n+1, n+2, ..., 2n$ denote the position of the i'th actuator scaled by the actuator gear ratio. If the joint is rigid $\hat{q}_i = \hat{q}_{n+i} \forall i$. For flexible joint, if the flexibility is modeled with a linear torsional spring with constant k_i, the elastic force z_i is derived from:

$$z_i = k_i(\hat{q}_i - \hat{q}_{n+i}) \quad (1)$$

The spring constants k_i's are relatively large and rigidity is modeled by the limit $k_i \to \infty$. Let u_i denotes the generalized force applied by the i'th actuator and use the notation:

$$q = (\hat{q}_1, ..., \hat{q}_n, \hat{q}_{n+1}, ..., \hat{q}_{2n})^T = (q_1^T | q_2^T)^T \quad (2)$$

The equation of motion of the system can be written in the following form using Euler–Lagrange formulation.

$$\begin{cases} M(q_1)\ddot{q}_1 + N(q_1, \dot{q}_1) = K(q_2 - q_1) \\ J\ddot{q}_2 = K(q_1 - q_2) - D\dot{q}_2 + T_F + u \end{cases} \quad (3)$$

in which,

$$N(q_1, \dot{q}_1) = V_m(q_1\dot{q}_1)\dot{q}_1 + G(q_1) + F_d\dot{q}_1 + F_s(\dot{q}_1) + T_d \quad (4)$$

and K is the joint stiffness matrix, $M(q_1)$ is the mass matrix, $V_m(q_1\dot{q}_1)$ is the matrix of Coriolis and centrifugal terms, $G(q_1)$ is the vector of gravity terms, F_d is the viscous friction matrix, $F_s(\dot{q}_1)$ is the Coulomb friction vector, T_d is the vector of the joint bounded unmodeled dynamics, J is the actuator moments of inertia matrix, D is the actuator viscous friction matrix, and T_F is the actuator bounded unmodeled dynamics. For all revolute manipulators, it is shown in [2, 10], that

$$m_1 I \leq M(q_1) \leq m_2 I \quad ; \quad ||V_m(q_1\ \dot{q}_1)|| \leq \zeta_c ||\dot{q}_1|| \tag{5}$$

$$||G(q_1)|| \leq \zeta_g \quad ; \quad ||F_d \dot{q}_1 + F_s(\dot{q}_1)|| = \zeta_{f0} + \zeta_{f1}||q_1|| \tag{6}$$

$$j_1 I \leq J \leq j_2 I \quad ; \quad d_1 I \leq D \leq d_2 I \tag{7}$$

Moreover, if the perturbations are bounded:

$$||T_d|| \leq \zeta_e \quad ; \quad ||T_F|| \leq \zeta_{f2} \tag{8}$$

in which $\zeta_{f2}, \zeta_e, d_2, d_1, j_2, j_1, \zeta_{f1}, \zeta_{f0}, \zeta_g, \zeta_c, m_2, m_1$ are positive real constants. If the joints are all rigid:

$$M_t(q)\ddot{q} + N_t(q, q) = u_0 \tag{9}$$

in which $q = q_1$ and M_t is a positive definite matrix. This model is the model of FJR where $k \to \infty$ verifying that the FJR model is a singularly perturbed model of rigid system. Assume that all spring constants are equal the elastic forces of the springs can be calculated by:

$$z = k(q_1 - q_2)\,,\; K = kI \tag{10}$$

in order to use a small quantity for singular perturbation define $\epsilon = \frac{1}{k}$ by which for rigid system ($k \to \infty$) in this form we have $\epsilon \to 0$. Multiplying M^{-1} to the both side of 3 and taking $z = k(q_1 - q_2)$, $q = q_1$, and using $\dot{q}_2 = \dot{q}_1 - \epsilon\dot{z}$:

$$\begin{cases} \ddot{q} = a_1(q, \dot{q}) + A_1(q)z \\ \epsilon\ddot{z} = a_2(q, \dot{q}, \epsilon\dot{z}) + A_2(q)z + B_2 u \end{cases} \tag{11}$$

in which,

$$A_1 = -M^{-1}(q) \quad ; \quad a_1 = -M^{-1}(q)N(q, \dot{q}) \tag{12}$$

$$a_2 = -\epsilon J^{-1}D\dot{z} + J^{-1}D\dot{q} - J^{-1}T_F - M^{-1}(q)N(q, \dot{q}) \tag{13}$$

$$A_2 = -(M^{-1}(q) + J^{-1})\,,\; B_2 = -J^{-1} \tag{14}$$

Equation 11 represents FJR as a nonlinear and coupled system. This representation includes both rigid and flexible subsystems in form of a singular perturbation model.

III. Reduced Flexible Model

The singular perturbation model of the FJR is given in Equation 11, This model represents the flexibility in the joints, however, the reduced order model is the model of rigid system, which can be easily derived from Equation 11 by setting $\epsilon = 0$. With some matrix manipulation it can be shown that:

$$(M + J)\ddot{q} + N - T_F + D\dot{q} = u_0$$

Rewrite this equation in this form:

$$M_t(q)\ddot{q} + N_t(q, \dot{q}) = u_0 \tag{15}$$

in which

$$M_t(q) = M(q) + J \tag{16}$$

$$N_t(q, \dot{q}) = N(q, \dot{q}) - T_F + D\dot{q} =$$
$$V_m(q, \dot{q})\dot{q} + G(q) + (F_d + D)\dot{q} + F_s(\dot{q}) + T_d - T_F \tag{17}$$

This representation introduces a 2n dimension manifold, M_o, which is called the rigid manifold. If $\epsilon \neq 0$ the produced manifold M_ϵ, which is a function of ϵ represents the flexible system. To define flexible manifold M_ϵ assume:

$$z = H(q, \dot{q}, u, \epsilon) \qquad q\epsilon R^n, u\epsilon R^n, z\epsilon R^n \tag{18}$$

$$\dot{z} = \dot{H}(q, \dot{q}, u, \epsilon) \qquad q\epsilon R^n, u\epsilon R^n, z\epsilon R^n \tag{19}$$

M_ϵ is an integral manifold for the flexible system if for each initial condition

$$\begin{cases} z(t) = \Delta \\ \dot{z}(t) = \Delta' \end{cases} \text{ and } \begin{cases} q(t) = \zeta \\ \dot{q}(t) = \zeta' \end{cases}$$

in M_ϵ all trajectories of $q(t)$ and $z(t)$ for $t > t_o$ remain in the manifold M_ϵ. In other words $\forall t > t_o$:

$$z(t) = H(q(t), \dot{q}(t), u(t), \epsilon) \tag{20}$$

$$\dot{z}(t) = \dot{H}(q(t), \dot{q}(t), u(t), \epsilon) \tag{21}$$

Now, the reduced flexible model can be derived by replacing $z, \dot{z}$ with $H, \dot{H}$ in Equation 11.

$$\ddot{q} = a_1(q, \dot{q}) + A_1(q)H(q, \dot{q}, u, \epsilon) \tag{22}$$

The order of this equation is equal to the rigid system, however, this model includes the effects of flexibility in form of an invariant integral manifold embedded in itself. Hence, this reduced order model is not an approximation of the FJR model, but it represents its projection on the integral manifold.

IV. Composite Control

In order to have a valid reduced flexible model for the system, it is essential that the M_ϵ be an invariant manifold, or the fast dynamics be asymptotically stable. This can be satisfied using a composite control scheme [6]. In this framework the control effort u consists of two main parts, u_s the control effort for slow subsystem, and u_f the control effort for fast subsystem, as:

$$u = u_s(q, \dot{q}, \epsilon) + u_f(\eta, \dot{\eta}) \tag{23}$$

in which $u_f(\eta, \dot{\eta})$ is designed such that the fast dynamics becomes asymptotically stable. η denotes the deviations of fast state variables from the integral manifold.

$$\eta = z - H(q, \dot{q}, u_s, \epsilon) \tag{24}$$

$$\dot{\eta} = \dot{z} - \dot{H}(q, \dot{q}, u_s, \epsilon) \tag{25}$$

The slow component of the control effort, $u_s(q, \dot{q}, \epsilon)$, is also designed based on the reduced flexible model. We describe the design technique for u_f and u_s in the next subsections, respectively.

A. Fast Subsystem Dynamics and Control

Recall Equation 24; hence,

$$\epsilon\ddot{\eta} = [a_2(q, \dot{q}, \epsilon\dot{z}) - a_2(q, \dot{q}, \epsilon\dot{H})] + A_2(q)\eta + B_2 u_f \tag{26}$$

Substitute the value of a_2 and use fast time scale $\tau = \frac{t}{\sqrt{\epsilon}}$ with some manipulations we reach to [5]:

$$\epsilon\ddot{\eta} = A_2(q)\eta + B_2 u_f \tag{27}$$

and in state space form:

$$\epsilon \begin{bmatrix} \dot{\eta} \\ \ddot{\eta} \end{bmatrix} = \begin{bmatrix} \emptyset & \epsilon I \\ A_2(q) & \emptyset \end{bmatrix} \begin{bmatrix} \eta \\ \dot{\eta} \end{bmatrix} + \begin{bmatrix} \emptyset \\ B_2 \end{bmatrix} u_f \tag{28}$$

The flexible modes are not stable since the eigenvalues are on the imaginary axis. Hence, u_f must be designed such that the eigenvalues are shifted to the open left half plane in order to guarantee stability.

Theorem 1: The diagonal and positive definite matrices K_{pf} and K_{vf} exist such that the closed loop system including the subsystem 27 with the control effort $u_f = K_{pf}\eta + K_{vf}\dot{\eta}$ becomes globally asymptotically stable. (Proof in [17])

B. Control of Reduced Flexible Model

The reduced flexible model represents the effect of flexibility in the form of the flexible integral manifold. In this section a robust control algorithm is proposed for the system based on this model. In order to accurately derive a robust control law $u_s(q,\dot{q},\epsilon)$ for the system, manipulation of partial differential equation is necessary. To avoid complex manipulations, we propose deriving the robust control law $u_s(q,\dot{q},\epsilon)$ to any order of ϵ from the series expansion of the integral manifold to the same order of ϵ.

$$H(q,\dot{q},u_s,\epsilon) = H_0(q,\dot{q},u_s) + \epsilon H_1(q,\dot{q},u_s) + \cdots \qquad (29)$$

and implement the controller $u_s(q,\dot{q},\epsilon)$ in the same form as:

$$u_s(q,\dot{q},\epsilon) = u_0(q,\dot{q}) + \epsilon u_1(q,\dot{q}) + \cdots \qquad (30)$$

in which the functions $H_i(q,\dot{q},u_s), u_i(q,\dot{q}),\ i = 0,1,\cdots$ are calculated iteratively without need to solve the partial differential equations. It is important to note that as $\epsilon \to 0$, u_s tends to rigid control, and H tends to rigid integral manifold. u_0 is designed using a robust design technique based on the rigid reduced order model ($\epsilon = 0$), and H_0 is calculated from:

$$H_0 = -A_2^{-1}(a_{20} + B_2 u_0) \qquad (31)$$

in which:

$$a_{20} = a_2(q,\dot{q},0) = J^{-1}D\dot{q} - J^{-1}T_F(q,\dot{q}) - M^{-1}(q)N(q,\dot{q}) \qquad (32)$$

Let:

$$a_2(q,\dot{q},\epsilon\dot{H}) = a_{20} + \epsilon\Delta a_2 + O(\epsilon^2)$$

in which a_{20} is given in Equation 32, and compare to Equation 13 we reach to:

$$\begin{cases} \Delta a_2 = -J^{-1}D\dot{H} \\ \Delta a_{20} = -J^{-1}D\dot{H}_0 \end{cases}$$

Hence,

$$\epsilon\ddot{H}_0 = a_{20} + A_2H_0 + B_2u_0 + \epsilon(\Delta a_{20} + A_2H_1 + B_2u_1) + O(\epsilon^2) \qquad (33)$$

and,

$$\ddot{H}_0 = \Delta a_{20} + A_2H_1 + B_2u_1 \qquad (34)$$

Therefore,

$$H_1 = A_2^{-1}(\ddot{H}_0 - \Delta a_{20} - B_2u_1) \qquad (35)$$

To calculate u_1 refer to reduced flexible model 22 and approximate it to the first power of ϵ:

$$\ddot{q} = a_1(q,\dot{q}) + A_1(q)H_0 + \epsilon A_1(q)A_2^{-1}(\ddot{H}_0 - \Delta a_{20} - B_2u_1)$$

By factoring the equal powers of ϵ we reach to:

$$u_1 = B_2^{-1}(\ddot{H}_0 - \Delta a_{20}) \qquad (36)$$

The only condition on robust control design is that u_0 must be at least twice differentiable. Finally, the control law for slow subsystem has the form:

$$u_s = u_0 + \epsilon u_1 \qquad (37)$$

In which u_1 is called the corrective term which is derived through this subsection and u_0 is the robust control based on the rigid model elaborated in the next section.

C. Robust PID Control for Rigid model

In this section we first propose a robust PID controller based on the rigid model of the system and then prove its robust stability with respect to the model uncertainties. Recall the rigid model of the system from Equation 15, choose a PID controller for u_0:

$$u_0 = K_V\dot{e} + K_Pe + K_I\int_0^t e(s)ds = Kx \qquad (38)$$

in which

$$\begin{cases} e = q_d - q \\ K = [K_I \quad K_P \quad K_V] \\ x = [\int_0^t e^T(s)ds \quad e^T \quad \dot{e}^T]^T \end{cases}$$

Similar to [2, 11] and [12], assume:

$$\underline{m}_t I \le M_t(q) \le \overline{m}_t I \qquad (39)$$

and put some limits on:

$$\|N_t\| \le \beta_0 + \beta_1\|L\| + \beta_2\|L\|^2 \quad ; \quad \|V_m\| \le \beta_3 + \beta_4\|L\| \qquad (40)$$

in which $\|.\|$ is the Euclidean norm and $L = [e^T \quad \dot{e}^T]$. Implement the control law u_0 in 15 to get:

$$\dot{x} = Ax + B\Delta A \qquad (41)$$

where

$$A = \begin{bmatrix} \emptyset & I_n & \emptyset \\ \emptyset & \emptyset & I_n \\ -M_t^{-1}K_I & -M_t^{-1}K_P & -M_t^{-1}K_V \end{bmatrix} \quad B = \begin{bmatrix} \emptyset \\ \emptyset \\ M_t^{-1} \end{bmatrix}$$

$$\Delta A = N_t + M_t\ddot{q}_d \qquad (42)$$

To analyze the system robust stability consider the following Lyapunov function:

$$V(x) = x^TPx = \frac{1}{2}[\alpha_2\int_0^t e(s)ds + \alpha_1e + \dot{e}]^T.M_t.$$
$$[\alpha_2\int_0^t e(s)ds + \alpha_1e + \dot{e}] + w^TP_1w \qquad (43)$$

in which

$$w = \begin{bmatrix} \int_0^t e(s)ds \\ e \end{bmatrix} \quad P_1 = \frac{1}{2}\begin{bmatrix} \alpha_2K_P + \alpha_1K_I & \alpha_2K_V + K_I \\ \alpha_2K_V + K_I & \alpha_1K_V + K_P \end{bmatrix}$$

Hence,

$$P = \frac{1}{2}\begin{bmatrix} \alpha_2K_P + \alpha_1K_I + \alpha_2^2M_t & \alpha_2K_V + K_I + \alpha_1\alpha_2M_t & \alpha_2M_t \\ \alpha_2K_V + K_I + \alpha_1\alpha_2M_t & \alpha_1K_V + K_P + \alpha_1^2M_t & \alpha_1M_t \\ \alpha_2M_t & \alpha_1M_t & M_t \end{bmatrix}$$

Since M_t is a positive definite matrix, P is positive definite, if and only if, P_1 is positive definite.
Lemma 1: Assume the following inequalities hold:

$$\alpha_1 > 0 \quad \alpha_2 > 0 \quad \alpha_1 + \alpha_2 < 1$$
$$s_1 = \alpha_2(k_P - k_V) - (1-\alpha_1)k_I - \alpha_2(1+\alpha_1-\alpha_2)\overline{m}_t > 0$$
$$s_2 = k_P + (\alpha_1 - \alpha_2)k_V - k_I - \alpha_1(1+\alpha_2-\alpha_1)\overline{m}_t > 0$$

Then P is positive definite and satisfies the following inequality (Rayleigh-Ritz)[9]:

$$\underline{\lambda}(P)\|x\|^2 \leq V(x) \leq \overline{\lambda}(P)\|x\|^2 \qquad (44)$$

in which,

$$\underline{\lambda}(P) = min\{\frac{1-\alpha_1-\alpha_2}{2}\underline{m}_t, \frac{s_1}{2}, \frac{s_2}{2}\}$$
$$\overline{\lambda}(P) = max\{\frac{1+\alpha_1+\alpha_2}{2}\overline{m}_t, \frac{s_3}{2}, \frac{s_4}{2}\}$$

and

$$s_3 = \alpha_2(k_P + k_v) + (1+\alpha_1)k_I + (1+\alpha_1+\alpha_2)\alpha_2\overline{m}_t$$
$$s_4 = \alpha_1\overline{m}_t(1+\alpha_1+\alpha_2) + (\alpha_1+\alpha_2)k_V + k_P + k_I$$

Proof is based on Gershgorin theorem and is similar to that in [11]. with some manipulations we can show [5]:

$$\gamma = min\{\alpha_2 k_I, \alpha_1 k_P - \alpha_2 k_V - k_I, k_V\}$$

Now considering Equations 40, 42 and $\|L\| \leq \|x\|$ then,

$$\begin{aligned} \xi_0 &= \alpha_2^{-1}\lambda_1\beta_0 + \alpha_2^{-1}\lambda_1\lambda_3\overline{m}_t \\ \xi_1 &= \gamma - \lambda_1\beta_3 - \lambda_2\overline{m}_t - \alpha_2^{-1}\lambda_1\beta_1 \\ \xi_2 &= \lambda_1\beta_4 + \alpha_2^{-1}\lambda_1\beta_2 \end{aligned}$$

in which

$$\begin{cases} \lambda_1 = \lambda_{max}(R_1) \\ \lambda_2 = \lambda_{max}(R_2) \\ \lambda_3 = sup\|\ddot{q}_d\| \end{cases}$$

and $\lambda_{Min}, \lambda_{Max}$ are the least and largest eigenvalues, respectively, and

$$R_1 = \begin{bmatrix} \alpha_2^2 I & \alpha_1\alpha_2 I & \alpha_2 I \\ \alpha_1\alpha_2 I & \alpha_1^2 I & \alpha_1 I \\ \alpha_2 I & \alpha_1 I & I \end{bmatrix}$$
$$R_2 = \frac{1}{2}\begin{bmatrix} \emptyset & \alpha_2^2 I & \alpha_1\alpha_2 I \\ \alpha_2 I & 2\alpha_1\alpha_2 I & (\alpha_1^2+\alpha_2)I \\ \alpha_1\alpha_2 I & (\alpha_1^2+\alpha_2)I & \alpha_1 I \end{bmatrix}$$

According to the result obtained so far, we can proof the stability of the error system based on the following theorem.
Theorem 2: The error system 41 is stable of the form of UUB, if ξ_1 is chosen large enough.
The conditions are:

$$\begin{aligned} \xi_1 &> 2\sqrt{\xi_0\xi_2} \\ \xi_1^2 + \xi_1\sqrt{\xi_1^2 - 4\xi_0\xi_2} &> 2\xi_0\xi_2(1+\sqrt{\frac{\overline{\lambda}(P)}{\underline{\lambda}(P)}}) \\ \xi_1 + \sqrt{\xi_1^2 - 4\xi_0\xi_2} &> 2\xi_2\|x_0\|\sqrt{\frac{\overline{\lambda}(P)}{\underline{\lambda}(P)}} \end{aligned}$$

These conditions can be simply met by making ξ_1 large enough by choosing large enough control gains K_P, K_v, and K_I. (Proof in [17])

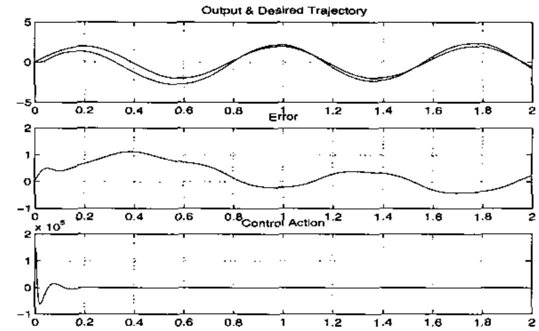

Fig. 2. Poor tracking performance of the closed loop system for perturbed model; Spong algorithm

V. Stability Analysis of the Complete Closed-loop System

The stability of the fast, and slow subsystems are separately analyzed in previous sections. However, the stability of the complete closed-loop system may not be guaranteed through these separate analysis [8]. In this section the stability of the complete system is analyzed. Recall the dynamic equations of the FJR Equation 11. The integral manifold and the control effort are chosen as:

$$\eta = z - H$$
$$H = H_0 + \epsilon H_1$$
$$u = u_s + u_f = u_0 + \epsilon u_1 + u_f$$

Combine these equations to Equation 11, 35, 31 and 38, and consider, $x = \left[\int_0^t e(s)^T ds \quad e^T \quad \dot{e}^T\right]^T$; $y = [\eta^T \quad \dot{\eta}^T]^T$ then,

$$\dot{x} = Ax + B\Delta A + C[I \quad \emptyset]y \qquad (45)$$
$$\epsilon\dot{y} = \tilde{A}y \qquad (46)$$

in which,

$$A = \begin{bmatrix} \emptyset & I & \emptyset \\ \emptyset & \emptyset & I \\ -M_t^{-1}K_I & -M_t^{-1}K_P & -M_t^{-1}K_V \end{bmatrix} ; B = \begin{bmatrix} \emptyset \\ \emptyset \\ M_t^{-1} \end{bmatrix}$$
$$\Delta A = N_t + M_t\ddot{q}_d$$
$$C = \begin{bmatrix} \emptyset \\ \emptyset \\ -A_1 \end{bmatrix} \quad ; \quad \tilde{A} = \begin{bmatrix} \emptyset & \epsilon I \\ A_2 + B_2K_{pf} & -\epsilon J^{-1}D + B_2K_{vf} \end{bmatrix}$$

Theorem 3: There exist diagonal and positive definite matrices K_{pf} and K_{vf} such that the closed loop system 46 becomes globally asymptotically stable. (Proof in [17])
Theorem 4: The closed-loop system of Equations 45 and 46 is UUB stable if K_{pf}, K_{vf}, and ξ_1 are chosen large enough. (Proof in [17])
The detail conditions on the PID controller parameter bounds to preseve the closed-loop stability, are given in [17]. However, the stability conditions met if the controller gains are selected high enough.

VI. Simulations

In order to verify the effectiveness of the algorithm a simulation study has been forwarded next. In the following simulation study, the results of the closed loop performance of a single, [14], and a two link flexible joint manipulator, [1], examined in the literature is compared to that of the proposed control algorithm.

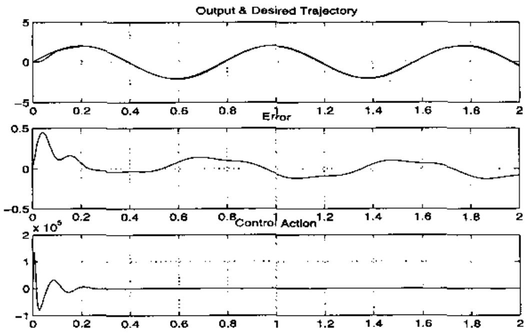

Fig. 3. Suitable tracking performance of the closed loop system for perturbed model; Proposed algorithm

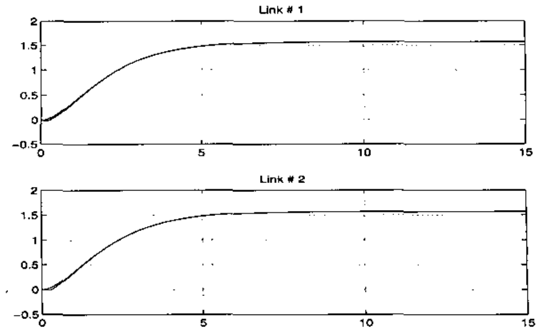

Fig. 4. Tracking performance of the closed loop system and perturbed model; Proposed algorithm

A. Single Link Flexible Joint Manipulator

Consider the single link flexible joint manipulator introduced in [14].The dynamic equation of motion of this system is as following:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{-MgL}{I}\sin(x_1) - \frac{K}{I}(x_1 - x_3) \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{K}{J}(x_1 - x_3) + \frac{1}{J}u \end{aligned} \tag{47}$$

in which $x_1 = q_1$ and $x_2 = q_2$. By choosing $q_1 = q$ and $z = K(q_1 - q_2)$ as the elastic force, the model of the system can be rewritten in a singular perturbation form:

$$\begin{aligned} \ddot{q} &= \frac{-MgL}{I}\sin(q) - \frac{1}{I}z \\ \epsilon\ddot{z} &= \frac{-MgL}{I}\sin(q) - (\frac{1}{I} + \frac{1}{J})z - \frac{1}{J}u \end{aligned} \tag{48}$$

in which $\epsilon = \frac{1}{K}$.

Spong has proposed a composite control law for this system, in which there exists two control components corresponding to the fast and slow dynamics. As it is illustrated in [14], the closed loop system became unstable, provided that only the corresponding rigid control effort u_0 is applied on the system. Moreover, the system becomes stable and the desired trajectory $q_d = \sin(8t)$ is well tracked, implementing the proposed composite control on the nominal model of the system. However, this algorithm is not robust to the model parameter variations. As illustrated in Figure 2 the tracking performance is getting quite poor for the maximum perturbation values for the parameters I, J, M, and L. For the sake of comparison, the proposed robust PID controller may be now applied on the same system. The proposed control law is composed of three terms, in which the rigid control law is a PID controller whose coefficients satisfies the robust stability conditions elaborated in Theorem (4) as following:

$$u_o = 200\dot{e} + 500e + 100\int_0^t e(s)ds.$$

The integral manifold would be:

$$H_o = -4.9\sin(q) - \frac{1}{2}u_o$$

and the corrective term corresponds to

$$u_1 = \ddot{H}_o.$$

The fast control law is a simple PD controller satisfying the robust stability conditions such as:

$$u_f = 5\eta + 5\dot{\eta}$$

in which η indicates the variation of z from the integral manifold H.

It is observed that by implementing the proposed control law, not only the system is well tracking the desired trajectory for the nominal parameters of the model [5], but also the robust stability and tracking performance of the system with maximum variation in its model parameters are preserved (Figure 3).

B. Multiple Link Flexible Joint Manipulator

Consider the two link Flexible Joint manipulator illustrated in Figure 1. In this manipulator Joint flexibility is modeled with a linear torsional spring with stiffness k. The equation of motion of this system and its parameters is given in [1]. Our proposed algorithm is applied to the system for comparison of the results. Hence, the reduced order first order model is evaluated as following:

$$\begin{bmatrix} 2.25 & 0.5\cos(\epsilon H_1^o) \\ 0.5\cos(\epsilon H_1^o) & 1.25 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} H_1^o + \epsilon H_1^1 \\ H_2^o + \epsilon H_2^1 \end{bmatrix}$$
$$\begin{bmatrix} 14.7\cos(\theta_1) + 0.5\dot{\theta}_2^2\,\sin(\epsilon H_1^o) \\ 4.9\cos(\theta_2) - 0.5\dot{\theta}_1^2\,\sin(\epsilon H_1^o) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

In order to evaluate the fast dynamics caused by the joint flexibility, the normalized time variable $\tau = \frac{t}{\sqrt{\epsilon}}$ is used. Hence,

$$H_1^o = \ddot{\theta}_1 - u_1^o \;\; ; \;\; H_2^o = \ddot{\theta}_2 - u_2^o \tag{49}$$

$$H_1^1 = -\ddot{H}_1^o - u_1^1 \;\; ; \;\; H_2^1 = -\ddot{H}_2^o - u_2^1 \tag{50}$$

With expanding Equation 49 to the first order of ϵ we have:

$$H_1^1 = -0.5\dot{\theta}_2^2\, H_1^o \;\; ; \;\; H_2^1 = -0.5\dot{\theta}_1^2\, H_1^o \tag{51}$$

And from Equation 50 we get:

$$u_1^1 = -0.5\dot{\theta}_2^2\, H_1^o - \ddot{H}_1^o \;\; ; \;\; u_2^1 = -0.5\dot{\theta}_1^2\, H_1^o - \ddot{H}_2^o \tag{52}$$

Finally, the slow part of the control law will be calculated from:

$$u_{1s} = u_1^o + \epsilon u_1^1 \;\; ; \;\; u_{2s} = u_2^o + \epsilon u_2^1 \tag{53}$$

The u_1^o, u_2^o are the rigid part of the control law and as elaborated before is robustly designed as a PID controller.

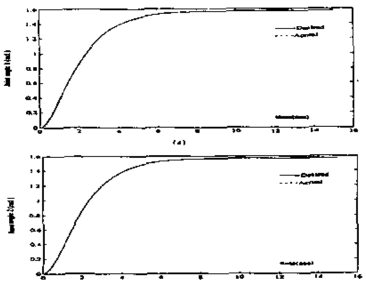

Fig. 5. Tracking performance of the closed loop system for nominal model; Al-Ashoor et al algorithm

In here we design the PID gains as following which satisfies the robust conditions:

$$u_1^o = 500e + 50\dot{e} + 50\int_0^t e(s)ds \qquad (54)$$
$$u_2^o = 200e + 50\dot{e} + 50\int_0^t e(s)ds$$

The fast control law is also designed as a PD controller as:

$$u_{1f} = \dot{\eta}_1 + \eta_1 \;\; ; \;\; u_{2f} = \dot{\eta}_2 + \eta_2 \qquad (55)$$

Finally the control law is composed from the east and slow parts:

$$u_1 = u_{1s} + u_{1f} \;\; ; \;\; u_2 = u_{2s} + u_{2f} \qquad (56)$$

To have simulation results compared to [1], the reference signal is considered as:

$$\theta_i = 1.57 + 7.8539e^{-t} - 9.428e^{-t/1.2} \qquad i = 1,2 \qquad (57)$$

in which the joint angles reach to a final value of $\theta_i = \frac{\pi}{2}$ from zero initial state. Figure 4 illustrates the response of the perturbed system to our proposed composite control law. The system becomes stable, and the tracking performance is quite desirable, despite the 50% variation in model parameters. The control is limited to a maximum allowable bounds by adding a saturation block in the simulation. Al-Ashoor et al have used a robust–adaptive control law in addition to the composite law we introduced in this paper. Figure 5 illustrates the results obtained for the reference signal introduced in Equation 57 in [1]. This figure illustrates the tracking performance despite the bounded control effort illustrated in Figure 6. Comparing it to our result (Figure 4), similar performances are obtained. The only limitation exists in our proposed law compared to that in [1], is the amplitude of the control law in the initial time of the simulation. The adaptive law have smaller control effort in the beginning of the simulation, due to the adaptive nature of the algorithm, and using the information of the identified model of the system in the control law. This issue is under current investigation, and promising results are obtained by a $\mathcal{H}_\infty$ –based composite controller, in which the control effort can be limited to desirable bounds, [16].

VII. Conclusions

In this paper the control of flexible joint manipulators is examined in detail. In order to achieve the required performance a composite control algorithm is proposed, consisting of corresponding control law for fast and slow subsystems. A simple PD control is proposed for the fast subsystem, and it is proven that the fast subsystem becomes asymptotically stable. The slow subsystem itself is controlled through a robust PID controller designed based on the rigid model, and a correction term designed based on the reduced flexible model. The stability of the complete closed–loop system is analyzed and it is shown that the proposed controller is capable of robustly stabilizing the uncertain flexible joint manipulator. Finally, the effectiveness of the proposed control law is verified through simulations. are compared to that given in the literature, and the effectiveness of preserving the robust stability, and performance of the system is verified and compared relative to them.

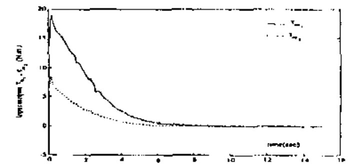

Fig. 6. Control effort for the closed loop system and nominal model; Al-Ashoor et al algorithm

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