

A Quasi-Closed Solution Method for Computing the Forward Kinematics of a Redundant Parallel Manipulator

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Abstract

In this paper, a quasi-closed solution method is presented to solve the forward kinematics of a three DOF actuator redundant hydraulic parallel manipulator. It is shown, that on the contrary to series manipulators, the forward kinematic map of the parallel manipulators involves highly coupled nonlinear equations, which are almost impossible to solve analytically. The proposed method uses a combination of analytical and numerical schemes to solve the problem. A simulation study is performed using a sample trajectory to identify the advantages and disadvantages of the proposed method in computing the forward kinematic map of the given mechanism. The results show that the proposed method provides us with a relatively fast solution and good tracking performance although being dependent on the initial conditions used in the solution process.

Key Words:

Parallel Manipulator, Forward Kinematics, Optimization, Numerical Solution, Closed-form Solution.

1. Introduction

Over the last two decades, parallel manipulators have been among the most considerable research topics in the field of robotics. These robots are now applied in real-life applications such as force sensing robots, fine positioning devices, and medical applications [1, 2].

In general robot manipulators with the end effector connected to the ground via several parallel-actuated legs, will have great load carrying capacity, high rigidity and good positioning capability. These interesting features as well as high speed and accuracy have led its main drawbacks, namely: smaller workspace and kinematic complexity, to be challenged. Kinematics is definitely of fundamental importance in design and control of robotic manipulators since the performance of a typical manipulator is achieved through the movement of the legs so the study of the geometry of leg motions would be crucial. As in the case of conventional serial robots, kinematics analysis of parallel manipulators is also performed in two phases. In forward or direct kinematics the position and orientation of the mobile platform is determined given the leg lengths. This is done

with respect to a base reference frame. In inverse kinematics we use position and orientation of the mobile platform to determine actuator lengths. This kind of kinematics analysis is sometimes referred to as position kinematics or kinematics in the position level. It is known that unlike serial manipulators, inverse position kinematics for parallel robots is usually simple and straightforward. In most cases joint variables (actuator displacements) may be computed independently using the given pose of the movable platform. The solution to this problem is in most cases uniquely determined.

But forward kinematics of parallel manipulators is generally very complicated. Its solution usually involves systems of nonlinear equations which are highly coupled and in general have no closed form and unique solution.

This topic has been of real interest to the robotics researchers and even mathematicians. Most of the research regards solving the forward kinematics in Gough-Stewart platform which has become a benchmark for both analytic and numerical computation schemes [3]. Different approaches are provided to solve this problem either generally or in special cases. There are also numerous cases in which the solution to this problem is provided for a special or novel architecture. In general, different solutions to this problem can be placed in one of the following [4]:

- Numerical approaches
- Analytical approaches
- Closed-form solution for special architectures

Most of the research regarding the closed form solution to the forward kinematics problem of parallel manipulators has assumed simplified or special conditions under which a closed form solution to the problem could be found [5]. Some researchers have focused on a special or a novel architecture [6, 7, 8]. Also, in [9] a closed form solution has been provided for parallel manipulators with planar base and mobile platform which is based on the use of three linear extra sensors to provide additional information.

In this paper, a quasi-closed form solution method is tested to solve the kinematics problem in a 3DOF actuator redundant hydraulic parallel manipulator. The paper is organized as following. Section 2 contains the mechanism description. Kinematic modeling of the manipulator is discussed in section 3, where inverse and

forward kinematics is studied in detail and the need for appropriate method to solve the forward kinematics is justified. In section 4, the proposed method to solve the forward kinematics problem is discussed.

Finally, in section 5, in order to identify the benefits and drawbacks of the proposed scheme, the method is simulated regarding the problem in hand.

2. Mechanism Description

A three DOF actuator redundant hydraulic parallel manipulator is used as the basis of our study. The mechanism is designed by Dr. V. Hayward [10, 11, and 12], borrowing design ideas from biological manipulators and specially the biological shoulder. The interesting features of the mechanism and its similarity to human shoulder have made it a unique design, which can serve as a basis for a good experimental setup for research. A schematic of the mechanism is shown in figure (1).

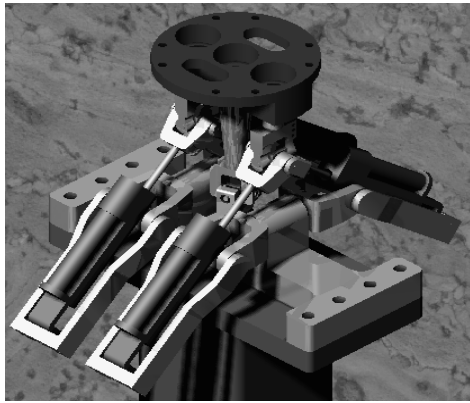


Fig (1): A schematic of the hydraulic shoulder manipulator

The mobile platform is constrained to spherical motions. Four high performance hydraulic piston actuators are used to give three degrees of freedom in the mobile platform. Each actuator includes a position sensor of LVDT type and an embedded force sensor (Hall Effect). Simple elements are used in the structure like spherical and universal joints. A complete analysis of such a careful design will provide us with good results regarding the structure itself and its performance. However, such an analysis requires a comprehensive study conducted in a systematic way as done for all robotic structures, namely kinematics (structural properties), kinetics and then identification and control problem must be regarded separately.

From the structural point of view, the shoulder mechanism which, from now on, we call it "the Hydraulic Shoulder" falls into an important class of robotic mechanisms called parallel robots. In these robots, the end effector is connected to the base through several closed kinematic chains. The motivation behind using these types of robot manipulators was to compensate for the shortcomings of the conventional serial manipulators such as low precision, low stiffness, error accumulation and load carrying capability. Parallel

structures are usually lighter and simpler than their serial counterparts. However, they have their own disadvantages, which are mainly smaller workspace and many singular configurations. Recently, hybrid structures are designed which combine the advantages of both serial and parallel robots.

The hydraulic shoulder, being a parallel structure, has the general features of these structures. It can be thought of as a shoulder for a light weighed seven DOF robotic arm, which can carry loads several times its own weight. Simple elements, used in this design, add to its lightness and simplicity. The workspace of such a mechanism can be considered as part of a sphere surface. The orientation angles are assumed to vary between $-\pi/6$ and $\pi/6$. Such an assumption is reasonable regarding the mechanism features, such as maximum actuator displacements and other mechanical constraints.

3. Kinematics

The hydraulic shoulder is kinematically over constrained. The inverse kinematics problem is easily solved, given the orientation of the mobile plate. This is also the case for general parallel robots. The inverse kinematics problem has a unique solution, in our case meaning that, the hydraulic shoulder cannot be optimized by choosing between inverse kinematics solutions. But, in contrast to serial structures, the forward kinematics is very complicated and there is no closed form solution in general. Figure (2) depicts a geometric model for the mechanism which will be used for its kinematics derivation.

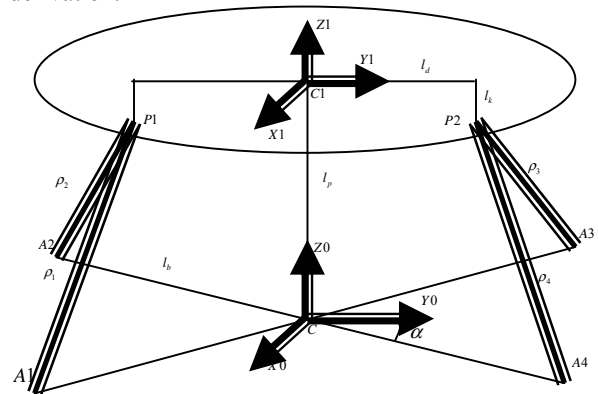


Fig (2): A geometric model for the hydraulic shoulder manipulator

The parameters used in kinematics can be defined as:

$$l_b : \left\| \overrightarrow{CA_i} \right\| \quad l_p : \left\| \overrightarrow{CC_1} \right\|$$

$$l_d : \left\| \overrightarrow{C_1P_i} \right\|_{z_1} \quad l_k : \left\| \overrightarrow{C_1P_i} \right\|_{z_1}$$

α : The angle between CA_4 and y_0

C : Center of the reference frame

C_1 : Center of the moving plate

ρ_i : Actuator lengths $i=1, 2, 3, 4$

P_i : Moving endpoints of the actuators

A_i : Fixed endpoints of the actuators

Two coordinate frames are defined. The base frame $X_0Y_0Z_0$ is centered at C (rotation center) with its Z_0 -axis perpendicular to the plane defined by $A_1A_2A_3A_4$ and an X_0 axis parallel to the bisector of angle $\angle A_1CA_4$. The second frame, namely $X_1Y_1Z_1$ is centered at C_1 (center of the moving plate) with its Z_1 axis perpendicular to the line defined by the actuators moving end points (P_1P_2) and horizontal Y axis along C_1P_2 .

3.1. Inverse kinematics

In modeling the inverse kinematics of the hydraulic shoulder we must determine actuator lengths (ρ_i) as the joint space variables given the task space variables, namely θ_x , θ_y and θ_z as the orientation angles of the moving platform. First we note that the fixed end points of the actuators (A_i) can be written in the base frame as:

$$\begin{aligned} A_1^0 &= (l_b \sin \alpha \quad -l_b \cos \alpha \quad 0) \\ A_2^0 &= (-l_b \sin \alpha \quad -l_b \cos \alpha \quad 0) \\ A_3^0 &= (-l_b \sin \alpha \quad l_b \cos \alpha \quad 0) \\ A_4^0 &= (l_b \sin \alpha \quad l_b \cos \alpha \quad 0) \end{aligned} \quad (1)$$

Also:

$$\begin{aligned} P_1^1 &= (0 \quad -l_d \quad -l_k) \\ P_2^1 &= (0 \quad l_d \quad -l_k) \end{aligned} \quad (2)$$

These must be transferred to the base frame using the rotation matrix R_i^0 ;

$$P_i^0 = R_i^0 P_i^1 \quad (3)$$

Where:

$$S_{3 \times 3} = R_1^0 = R_z(\theta_z)R_y(\theta_y)R_x(\theta_x) \quad (4)$$

As a result the rotation matrix components are computed as following:

$$\begin{aligned} S_{11} &= \cos(\theta_z)\cos(\theta_y) \\ S_{21} &= \sin(\theta_z)\cos(\theta_y) \\ S_{31} &= -\sin(\theta_y) \\ S_{12} &= \cos(\theta_z)\sin(\theta_y)\sin(\theta_x) - \sin(\theta_z)\cos(\theta_x) \\ S_{22} &= \sin(\theta_z)\sin(\theta_y)\sin(\theta_x) + \cos(\theta_z)\cos(\theta_x) \\ S_{32} &= \cos(\theta_y)\sin(\theta_x) \\ S_{13} &= \cos(\theta_z)\sin(\theta_y)\cos(\theta_x) + \sin(\theta_z)\sin(\theta_x) \\ S_{23} &= \sin(\theta_z)\sin(\theta_y)\cos(\theta_x) - \cos(\theta_z)\sin(\theta_x) \\ S_{33} &= \cos(\theta_y)\cos(\theta_x) \end{aligned} \quad (5)$$

The final step is to translate the resulting vectors P_i^0 by l_p along the Z axis. Having P_i^0 and A_j^0 in hand, the actuator lengths $\|\overrightarrow{P_iA_j}\|$ can be easily computed as:

$$\rho_j = \sqrt{(p_x - a_x)^2 + (p_y - a_y)^2 + (p_z - a_z)^2} \quad (6)$$

Where:

$${}^0P_i = [p_x \quad p_y \quad p_z] ; \quad {}^0A_j = [a_x \quad a_y \quad a_z] \quad (7)$$

Equations (6)-(7) completely model the IK of the hydraulic shoulder. As it is obvious from the equations the actuator lengths will be uniquely computed given the orientation angles θ_x , θ_y and θ_z . So the manipulator doesn't have any kinematics redundancy, meaning that reaching a specific point in the task space can't be satisfied through different combinations of the actuator lengths.

3.2. Forward kinematics

Equations (6)-(7) can also be used for the forward kinematics of the hydraulic shoulder but with the actuator lengths as the input and orientation angles θ_x , θ_y , θ_z as the unknowns. In fact, we have four nonlinear equations to solve for three unknowns. Obviously, solving such a system of nonlinear equations for a unique closed-form analytic solution to the FK problem is very complicated, although three equations of the four could be used. Several inconclusive attempts have been made in this direction; therefore, we propose using a combination of the numerical and analytic schemes to solve the FK problem as a basic element in modeling and control of the manipulator. This is studied in detail in the next section.

4. Forward Kinematics Solution

We introduce a quasi-closed form solution method to estimate the forward kinematic map of the hydraulic shoulder manipulator.

The kinematic equations of the mechanism can be rewritten as:

$$\begin{aligned} \rho_1 &= \sqrt{A + Bs_{12} + Cs_{13} - Ds_{22} - Es_{23} - Fs_{32} - Gs_{33}} \\ \rho_2 &= \sqrt{A + Bs_{12} + Cs_{13} - Ds_{22} - Es_{23} - Fs_{32} - Gs_{33}} \\ \rho_3 &= \sqrt{A + Bs_{12} + Cs_{13} - Ds_{22} - Es_{23} - Fs_{32} - Gs_{33}} \\ \rho_4 &= \sqrt{A + Bs_{12} + Cs_{13} - Ds_{22} - Es_{23} - Fs_{32} - Gs_{33}} \end{aligned} \quad (8)$$

Where parameters A, B, C, D, E, F and G depend on the geometric features of the mechanism and are measured as follows:

$$\begin{aligned} A &= .0268m, B = .0045m, C = .0083m, D = .0026m, \\ E &= .0048m, F = .0092m, G = .0169m \end{aligned}$$

The s_{ij} 's are the nine entries of the rotation matrix which represent the orientation of the moving platform, and $\rho_1, \rho_2, \rho_3, \rho_4$ are the actuator lengths. It is fairly easy to obtain the forward kinematic equations having the rotation matrix S in hand.

So, the problem reduces to solving for the rotation matrix instead, with nine entries as the unknowns. Noting that these entries are not independent, there would be no need to compute all nine unknowns.

Furthermore, The elements in the first column of S, namely: s_{11} , s_{21} and s_{31} are not present in the kinematic equations, which simplifies the problem as we can find the 2nd and 3rd columns of S and the 1st column will be simply computed as their cross product.

Hence, the problem is solving the kinematic equations (8) for the rotation matrix with the following constraints:

$$s_{12}^2 + s_{22}^2 + s_{32}^2 = 1 \quad (9)$$

$$s_{13}^2 + s_{23}^2 + s_{33}^2 = 1 \quad (10)$$

$$\text{col}(2) \cdot \text{col}(3) = 0 \quad (11)$$

Col (2) and Col (3) are defined as the second and third columns of the rotation matrix S, respectively.

We must note that S is an orthonormal matrix so the 2nd and 3rd columns must be orthogonal with unit lengths. As the cross product of two orthonormal vectors would also be orthonormal, the other constraints on the rotation matrix entries would be trivial.

From equations (8), we can solve for s_{12} , s_{13} as:

$$s_{12} = \frac{\rho_1^2 + \rho_3^2 - \rho_2^2 - \rho_4^2}{4B} \quad (12)$$

$$s_{13} = \frac{\rho_1^2 + \rho_4^2 - \rho_2^2 - \rho_3^2}{4C}$$

These equations are in the form of an analytic closed form solution. Unfortunately the high coupling of the forward kinematic equations makes the closed form computation of other entries of S complicated. Several inconclusive attempts were made to find an analytic solution for these entries; therefore we tried a new approach combining the analytic and numerical methods to solve for the remaining entries of the rotation matrix in a quasi-closed form.

We can relate the four remaining unknowns with the following equations:

$$s_{22} = \frac{4A - \rho_1^2 - \rho_2^2 - \rho_3^2 - \rho_4^2 - 4G s_{33}}{4D} \quad (13)$$

$$s_{23} = \frac{4F s_{32} + \rho_1^2 + \rho_2^2 - \rho_3^2 - \rho_4^2}{-4E}$$

Or much simpler:

$$s_{22} = \alpha + \beta s_{33} \quad (14)$$

$$s_{23} = \gamma s_{32} + \eta$$

Where:

$$\alpha = \frac{4A - \rho_1^2 - \rho_2^2 - \rho_3^2 - \rho_4^2}{4D}, \beta = -\frac{G}{D} \quad (15)$$

$$\eta = \frac{\rho_1^2 + \rho_2^2 - \rho_3^2 - \rho_4^2}{-4E}, \gamma = -\frac{F}{E}$$

Using constraint equations (9)-(10) we have:

$$s_{12}^2 + (\alpha + \beta s_{33})^2 + s_{32}^2 = 1 \quad (16)$$

$$s_{13}^2 + (\gamma s_{32} + \eta)^2 + s_{33}^2 = 1$$

The remaining problem will be to determine s_{32} , s_{33} using the above system of nonlinear equations, given s_{12} , s_{13} . Having s_{32} , s_{33} in hand, we can easily find the remaining entries s_{22} , s_{23} to obtain the 2nd and 3rd columns of the

rotation matrix S, from which the first column would be determined by a cross product.

The two entries s_{32} , s_{33} were obtained numerically from equation (16) solving a constrained optimization problem. In the next step the rotation matrix S was determined from which the orientation angles were easily obtained. Hence the forward kinematics problem was solved using the rotation matrix of the end effector as a means of simplifying the equations.

It should be noted that the degree of accuracy of the numerical optimization scheme to find an orthonormal rotation matrix is considered very important. The accuracy of the proposed method will definitely depend on the initial conditions used in the optimization process. We should also note that the proposed method uses a combination of analytic and numerical computation schemes, hence called a quasi-closed form solution method.

5. Simulation Results

To show how the results of the previous section apply in practice to estimate the forward kinematics map of the hydraulic shoulder manipulator, a simulation study was performed in which a sample trajectory was considered in the reachable workspace of the manipulator. The provided method was tested along such a trajectory and the results were compared. By this means, a good judgment could be made considering the advantages and the drawbacks of the proposed method.

5.1. Sample Trajectory Generation

We consider a smooth motion specified in terms of a desired pose of the moving platform of the hydraulic shoulder manipulator. We know that for the motion to be smooth, at least four constraints must be met [13]. The first two constraints involve the selection of the initial and final points and also the time to reach the final point, namely:

$$\theta(0) = \theta_0, \theta(t_f) = \theta_f \quad (17)$$

The other two constraints ensure that the function is continuous in velocity so that:

$$\dot{\theta}(0) = 0, \dot{\theta}(t_f) = 0 \quad (18)$$

A cubic polynomial of the form

$$\alpha_0 + \alpha_1 t + \alpha_2 t^2 + \alpha_3 t^3 \quad (19)$$

is considered to meet the above constraints by choosing the coefficients α_i as follows: [13]

$$\alpha_0 = \theta_0, \alpha_1 = 0$$

$$\alpha_2 = \frac{3}{t_f^2}(\theta_f - \theta_0), \alpha_3 = -\frac{2}{t_f^3}(\theta_f - \theta_0) \quad (20)$$

Thus, the sample trajectory is easily defined given the initial and final points and the time to reach the final point. This has been done to generate a sample trajectory in the reachable workspace of the hydraulic shoulder manipulator. Figure (4) shows this trajectory for each orientation angle in the task space.

Note that the actuator lengths profile corresponding to the desired task space trajectory could be easily obtained through the inverse kinematics map of the manipulator.

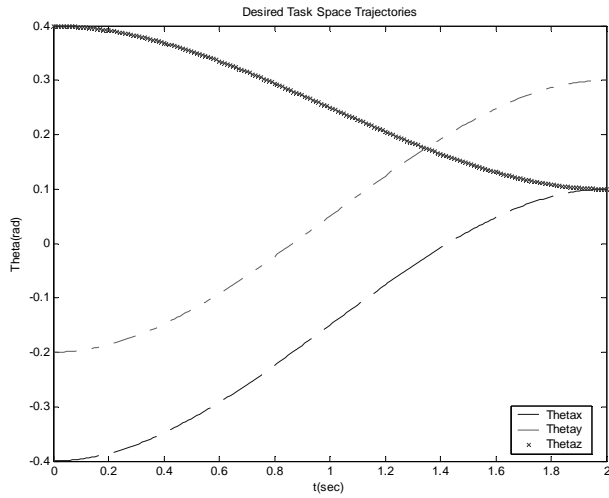


Fig (4): Sample Trajectory for Orientation Angles

5.2. Quasi-Closed Method Simulation

Figures (5-a)-(5-c), show the simulation results for the Quasi-Closed method applied to follow the sample trajectory along each orientation angle. The tracking performance is seen to be really good which shows the efficiency of the proposed method.

Table (1) summarizes different measures of estimation errors along the test trajectory in which PE stands for Prediction Error, SSE stands for sum of square of error, MSE stands for mean square error, and MAE stands for mean absolute error.

	Max PE	SSE	MSE	MAE
θ_x	.086	.225	.0011	.0193
θ_y	.014	.0072	3.6e-5	.0041
θ_z	.025	.0171	8.5e-5	.0053

Table (1): Measures of Prediction Errors

6. Conclusions

In this paper, a new quasi-closed approach was presented to solve the forward kinematics problem in a three DOF actuator redundant hydraulic parallel manipulator. The method used the rotation matrix of the end effector to determine the corresponding orientation angles needed to solve the forward kinematics map. The proposed method uses a combination of analytic and numerical computation schemes; hence called a quasi-closed form solution method. The degree of accuracy of the proposed method mainly depends on the numerical optimization scheme to find an orthonormal rotation matrix which is considered very important. Different measures were used for the prediction errors along the sample trajectory to test the tracking performance of the proposed method. In

general, the method also provides us with some external erroneous solutions due to the nonlinear nature of the forward kinematics map. In a separate research the results obtained using numerical approaches to solve the same problem is presented [14].

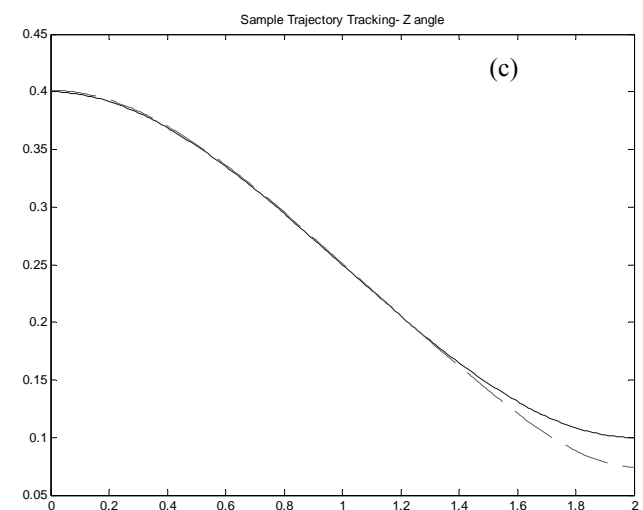
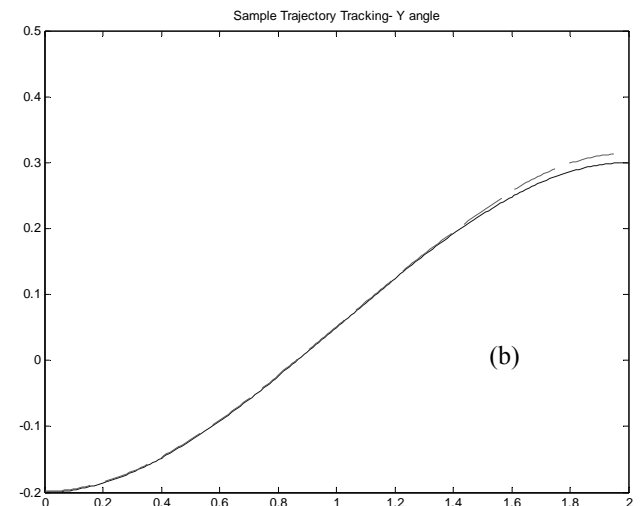
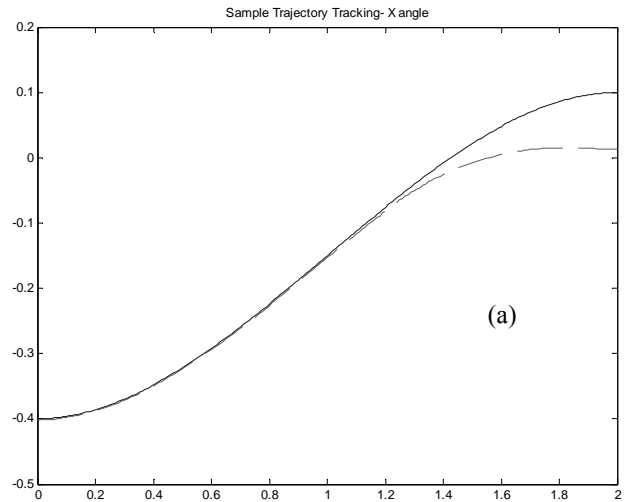


Fig (5): Sample Trajectory Tracking

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