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Robust Controller with a Supervisor Implemented on a Flexible Joint Robot

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Abstract— **In this paper a controller for flexible joint robots (FJR) in presence of actuator saturation is proposed, its robust stability is analyzed, and it is implemented on a laboratory FJR. This controller consists of a composite structure, with a PD controller on the fast dynamics and a PID controller on slow dynamics. The need of powerful actuator is released through decrease of fast controller bandwidth at critical occasions. This is done by means of a Fuzzy logic supervisory loop. The stability analysis of the overall system is then analyzed by Lyapunov Theory. It is proven that UUB stability of the overall system in presence of uncertainties is guaranteed, provided that the PD and the PID gains are tuned to satisfy certain conditions. Experimental studies are forwarded to verify the effectiveness and the performance of the proposed controller.**

I. INTRODUCTION

THE necessity of using *flexible joint robots* (FJR) is now an accepted fact in robotics community. This necessity has emerged new control strategies, for the traditional controllers implemented on FJRs have failed in performance [1]. Since 1980's many attempts have been made to remedy this shortcoming and now, several methods have been proposed such as various linear, nonlinear, robust, adaptive and intelligent controllers [2], [3]. However, the practical limitations such as actuator saturation are rarely considered in the controller synthesis [4] (a recent example among these few is [5]).

In order to come up with an online implementable controller for FJRs, a new composite controller, combined with a fuzzy logic supervisory loop has been proposed by authors in [6]. In this topology, the anti-saturation logic is set to be "out of the main loop", at a supervisory level, at the aim of preventing actuator saturation despite preserving the essential properties of the main controller. This idea in a general form has been first published by the authors in [7] and then has been modified to use for FJRs in [6]. In this structure a PD controller is used to stabilize the fast dynamics and a PID controller is proposed to robustly stabilize the slow dynamics of the FJR. Moreover, a supervisory control loop is added to the structure, in order to decrease the bandwidth of the fast controller during critical occasions. It is observed in various simulations that by including this supervisory loop into the controller structure, the steady state performance of the system is preserved, and moreover, the supervisory loop can remove instability caused by saturation. The robust stability of the overall system in presence of the modeling uncertainties is then analyzed in detail, and it is shown that UUB stability of the overall system is guaranteed, only if the PD gains of the fast controller and the PID gains are tuned to satisfy certain conditions [8]. This stability analysis is essential for susceptible applications of the FJRs such as space robotics, where the stability is a main concern. In this paper the effectiveness of the proposed method in practice is shown by providing experimental results.

This paper is organized as follows: In the next section the model and the control topology is reviewed. In section III the supervisory loop is introduced. Section IV is devoted to the stability analysis and simulation results are presented in section V.

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II. FJR MODEL AND COMPOSITE CONTROL

In order to model an FJR, the state vector is chosen to be:

$$\bar{Q} = [\theta_1, \theta_2, ..., \theta_n, \theta_{n+1} \mid \theta_{n+2}, ..., \theta_{2n}]^T = [\bar{q}_1{}^T, \bar{q}_2{}^T]^T \quad (1)$$

where θ_i : $i = 1,2,...,n$ represents the position of the i'th link and the position of the i'th actuator is represented by θ_{i+n} : $i=1,2,...,n$. The governing equations of motion would be [9]:

$$\begin{cases} \mathbf{M}(\bar{q}_1)\ddot{\bar{q}}_1 + \bar{N}(\bar{q}_1, \dot{\bar{q}}_1) = -\mathbf{K}(\bar{q}_1 - \bar{q}_2) \\ \mathbf{J}\ddot{\bar{q}}_2 - \mathbf{K}(\bar{q}_1 - \bar{q}_2) = \bar{u} \end{cases} \quad (2)$$

where **M** is the matrix of the link inertias and **J** is that of the motors, $\bar{u}$ is the vector of input torques, **K** is the diagonal matrix of spring constants and $\bar{N}$ is:

$$\bar{N}(\bar{q}_1, \dot{\bar{q}}_1) = \mathbf{V_m}(\bar{q}_1, \dot{\bar{q}}_1)\dot{\bar{q}}_1 + \bar{G}(\bar{q}_1) + \mathbf{F_d}\dot{\bar{q}}_1 + \bar{F}_S(\dot{\bar{q}}_1) + \bar{T}_d \quad (3)$$

in which matrix $\mathbf{V_m}(\bar{q}_1, \dot{\bar{q}}_1)$ consists of the Coriolis and centrifugal terms, $G(q)$ is the vector of gravity terms, $\mathbf{F_d}$ is the diagonal viscous friction matrix, F_s includes the static friction terms, and T_d is the vector of disturbance and unmodeled but bounded dynamics. Including this last term in the model, enables us to encapsulate the modeling uncertainties into the picture. The following uncertainty bounds hold [10] and will be used in robust stability analysis:

$$\underline{m}\,\mathbf{I} \le \mathbf{M}(\bar{q}_1) \le \bar{m}\,\mathbf{I} \tag{4}$$

$$\left\|\bar{N}(\bar{q}_1,\dot{\bar{q}}_1)\right\| \le \beta_0 + \beta_1\left\|\bar{\mathrm{E}}\right\| + \beta_2\left\|\bar{\mathrm{E}}\right\|^2 \tag{5}$$

$$\left\|\mathbf{V_m}(\bar{q}_1,\dot{\bar{q}}_1)\right\| \le \beta_3 + \beta_4\left\|\bar{\mathrm{E}}\right\| \tag{6}$$

where $\underline{m},\bar{m}$ and β_is are real positive constants and $\bar{\mathrm{E}} = [\bar{e}^T \quad \dot{\bar{e}}^T]$ is the vector of errors. It is assumed that all flexible elements are modeled by linear springs and without loss of generality, all springs are assumed to have the same spring constant *k*. the matrix $\mathbf{K}$ is defined as $\mathbf{K} = k\mathbf{I}$ The model can be changed to the following singular perturbation standard form:

$$\begin{cases} \ddot{\bar{q}} = -\mathbf{M}^{-1}(\bar{q})\bar{z} - \mathbf{M}^{-1}(\bar{q})\bar{N}(\bar{q},\dot{\bar{q}}) \\ \varepsilon\ddot{\bar{z}} = -(\mathbf{M}^{-1}(\bar{q}) + \mathbf{J}^{-1})\bar{z} - \mathbf{M}^{-1}(\bar{q})\bar{N}(\bar{q},\dot{\bar{q}}) - \mathbf{J}^{-1}\bar{u} \end{cases} \tag{7}$$

in which $\bar{q} = \bar{q}_1$, $\bar{z} = \mathbf{K}(\bar{q}_1 - \bar{q}_2)$ and $\varepsilon = 1/k$. Now if we choose $\varepsilon = 0$ then the slow behavior of $\bar{z}$ would be:

$$\begin{aligned} \bar{z}_S &= -(\mathbf{M}^{-1}(\bar{q}) + \mathbf{J}^{-1})^{-1}[\mathbf{M}^{-1}(\bar{q})\bar{N}(\bar{q},\dot{\bar{q}}) - \mathbf{J}^{-1}\bar{u}_S] \\ &= -(\mathbf{I} + \mathbf{M}(\bar{q})\mathbf{J}^{-1})^{-1}\bar{N}(\bar{q},\dot{\bar{q}}) - (\mathbf{J}\quad \mathbf{M}(\bar{q})^{-1} + \mathbf{I})^{-1}\bar{u}_S \end{aligned} \tag{8}$$

by substitution of this into (7) we reach to:

$$\ddot{\bar{q}}_S = -\mathbf{M}^{-1}(\bar{q})\bar{z}_f - [\mathbf{J} + \mathbf{M}(\bar{q})]^{-1}\bar{N}(\bar{q},\dot{\bar{q}}) + [\mathbf{J} + \mathbf{M}(\bar{q})]^{-1}\bar{u}_S \tag{9}$$

in which $\bar{z}_f = \bar{z} - \bar{z}_S$ represents the fast behavior of $\bar{z}$ whose dynamics could be found to be

$$\varepsilon\ddot{\bar{z}}_f = -(\mathbf{M}^{-1}(\bar{q}) + \mathbf{J}^{-1})\bar{z}_f - \mathbf{J}^{-1}\bar{u}_f \tag{10}$$

having $\bar{u}_S$ and $\bar{u}_f$ we can solve the last three equations to find $\bar{q}_S, \bar{z}_S$ and $\bar{z}_f$. Tikhonov theorem [11], provides some stability conditions, under which, the overall behavior of the system can be determined from these variables as follows:

$$\bar{q}(t) = \bar{q}_S(t) + O(\varepsilon) \qquad t \in [0,T]$$
$$\bar{z}(t) = \bar{z}_S(t) + \bar{z}_f(t) \qquad t \in [0,T]$$
$$\exists\ t_1 \ni \bar{z}(t) = \bar{z}_S(t) + O(\varepsilon) \qquad t \in [t_1,T]$$

in which, $O(\varepsilon)$ determines the terms whose order is as the order of ε. The stability conditions can be satisfied by proper selection of $\bar{u}_f$. Taking into account the dynamics of $\bar{z}_f$ (equation 10), a proper second order dynamics can be imposed to it by the use of a simple PD controller:

$$\bar{u}_f = [\mathbf{K_{Pf}}\bar{z}_f + \mathbf{K_{Df}}\dot{\bar{z}}_f] \tag{11}$$

Different control strategies can be used for the slow subsystem. However, a PID controller for the $\bar{u}_S$ has the benefits, that rate measurements and offline computations (especially derivations of the reference input) is not needed, and its robust stability in absence of saturation is guaranteed under certain conditions detailed in [12]. These characteristics make this structure attractive for practical implementation, and hence, it is proposed in our proposed structure.

$$\bar{u}_S = \mathbf{K_P}\bar{e} + \mathbf{K_D}\dot{\bar{e}} + \mathbf{K_I}\int_0^t \bar{e}(\tau)d\tau \tag{12}$$

in which, the error vector is defined as:

$$\bar{e} = \bar{q}_d - \bar{q} \tag{13}$$

The overall control system is shown in Fig. (1), through which the desirable performance can be achieved at the expense of high control effort, and may result in actuator saturation. This drawback is remedied by a supervisory loop added to the control structure which will be detailed in the proceeding section.

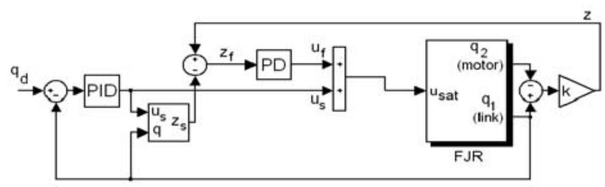

Fig. 1. The FJR control system

III. The Supervisory Loop

Without loss of generality one can assume that each element of the control vector has a saturation limit of 1. The proposed method is a two steps design procedure: first the compensator is designed without considering any saturation limit, then a time varying scalar gain $0<\lambda(t)\le 1$ is multiplied to the input of the fast controller to modify it. This gain is adjusted via a supervisory loop in order to cope with saturation as depicted in Fig. 2.

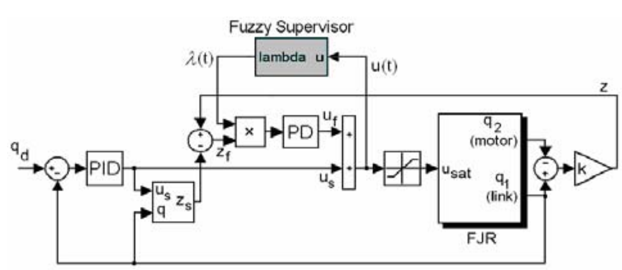

Fig. 2. The closed loop system with supervisor

Intuitively one can state the logic of adjustment as follows:

"If the system is near to experience saturation make λ smaller, Otherwise increase λ up to one."

This logic decreases the bandwidth of the fast subsystem when the system is near to experience saturation and in normal conditions it's effect is diminished by making λ=1. This configuration reduces the amplitude of the control effort as is done by saturation itself but there are some important differences: First, this is a dynamic compensator and not a hard nonlinearity as is the case with saturation. Second, this approach limits the control effort by affecting the controller states while saturation will limit the control effort independent of the controller states (note that the gain λ is multiplied before the controller but the saturation will affect the signal after the controller). In other words, it acts in a closed loop fashion rather than an open loop structure of a saturation block. Hence, it's dynamic behavior can be used to preserve stability despite limiting the control effort.

It is difficult to implement this logic with a rigorous mathematical model. However, fuzzy logic can be easily employed here. Details are as follows: In order to sense the value of closeness to saturation, the absolute value of the amplitude of the control effort $|u(t)|$ can be used as a good measure. To give a kind of prediction to the logic $\dot{u}(t)$ is also taken into account. Then a 2 input 1 output fuzzy inference system is designed to estimate proper lambda from these quantities. Fuzzy sets are definitions are given in [7].

Since the logic is based on a model free routine, the proposed method can be implemented not only on FJRs but also on a variety of systems experiencing actuator limitations, and the effectiveness of this structure has been verified in different applications [6], [7]. In addition it is observed that this method can preserve the stability of non-saturated system which may be lost due to saturation. Also the steady state behavior of the closed loop system remains unchanged. As mentioned in the previous section the composite controller composed of a fast PD controller and a slow PID controller as shown in [12] is robustly stable.

In the next section the robust stability of the system is studied in the presence of the fuzzy supervisor.

IV. Robust Stability Analysis

Recall the system equations

$$\ddot{\bar{q}}_S = -\mathbf{M}^{-1}(\bar{q})\bar{z}_f - \mathbf{M}_t(\bar{q})^{-1}\bar{N}(\bar{q},\dot{\bar{q}}) + \mathbf{M}_t(\bar{q})^{-1}\bar{u}_S \tag{14}$$

$$\varepsilon\ddot{\bar{z}}_f = -(\mathbf{M}^{-1}(\bar{q})+\mathbf{J}^{-1})\bar{z}_f - \mathbf{J}^{-1}\bar{u}_f \tag{15}$$

where $\mathbf{M}_t(\bar{q}) = \mathbf{J}+\mathbf{M}(\bar{q})$ and the control terms in presence of supervisor have the form of:

$$\bar{u}_S = \mathbf{K_P}\bar{e} + \mathbf{K_D}\dot{\bar{e}} + \mathbf{K_I}\int_0^t \bar{e}(\tau)d\tau \tag{16}$$

$$\bar{u}_f = \lambda(t).[\mathbf{K_{Pf}}\bar{z}_f + \mathbf{K_{Df}}\dot{\bar{z}}_f] \tag{17}$$

the error dynamics can be evaluated as:

$$\begin{aligned}\ddot{\bar{e}} &= \ddot{\bar{q}}_d + \mathbf{M}^{-1}(\bar{q})\bar{z}_f + \mathbf{M_t^{-1}}(\bar{q})\mathbf{N}(\bar{q},\dot{\bar{q}}) \\ &\quad -\mathbf{M_t^{-1}}(\bar{q})(\mathbf{K_P}\bar{e} + \mathbf{K_D}\dot{\bar{e}} + \mathbf{K_I}\int_0^t \bar{e}(\tau)d\tau)\end{aligned} \tag{18}$$

and the fast dynamics is

$$\varepsilon\ddot{\bar{z}}_f = -(\mathbf{M}^{-1}(\bar{q})+\mathbf{J}^{-1})\bar{z}_f - \mathbf{J}^{-1}\lambda(t).[\mathbf{K_{Pf}}\bar{z}_f + \mathbf{K_{Df}}\dot{\bar{z}}_f] \tag{19}$$

We can rewrite this dynamics as

$$\ddot{\bar{z}}_f = -\mathbf{K_1}\dot{\bar{z}}_f - \mathbf{K_2}\bar{z}_f \tag{20}$$

in which

$$\mathbf{K_1} = \frac{\lambda(t)}{\varepsilon}\mathbf{J}^{-1}\mathbf{K_{Df}} \tag{21}$$

$$\mathbf{K_2} = (1/\varepsilon)[\mathbf{M}^{-1}(\bar{q}) + \mathbf{J}^{-1}(\mathbf{I}+\lambda(t)\mathbf{K_{Pf}})] \tag{22}$$

These equations can be rearranged into the state space format as

$$\dot{\bar{x}} = \mathbf{A}\quad \bar{x} + \mathbf{B}\Delta\mathbf{A} + \mathbf{C}[\mathbf{I}\quad \mathbf{0}]\bar{y} \tag{23}$$

$$\dot{\bar{y}} = \mathbf{A_f}\bar{y} \tag{24}$$

in which

$$\bar{x} = \begin{bmatrix} \int_0^t \bar{e}(\tau)d\tau \\ \bar{e} \\ \dot{\bar{e}} \end{bmatrix}, \quad \bar{y} = \begin{bmatrix} \bar{z}_f \\ \dot{\bar{z}}_f \end{bmatrix} \tag{25}$$

and

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \\ -\mathbf{M_t^{-1}}(\bar{q})\mathbf{K_I} & -\mathbf{M_t^{-1}}(\bar{q})\mathbf{K_P} & -\mathbf{M_t^{-1}}(\bar{q})\mathbf{K_D} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{M_t^{-1}}(\bar{q}) \end{bmatrix}, \tag{26}$$

$$\Delta\mathbf{A} = (\mathbf{N}(\bar{q},\dot{\bar{q}}) + \mathbf{M_t}(\bar{q})\ddot{\bar{q}}_d), \quad \mathbf{C} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{M}^{-1}(\bar{q}) \end{bmatrix}$$

$$\mathbf{A_f} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{K_2} & -\mathbf{K_1} \end{bmatrix} \tag{27}$$

In the next subsection we study the effect of λ(t) on the stability of the fast subsystem.

A. Stability of the fast subsystem

To study the stability of the fast subsystem we consider the following Lyapunov function candidate:

$$V_f(\bar{y}) = \bar{y}^T\mathbf{S}\,\bar{y} \tag{28}$$

in which **S** is defined as

$$\mathbf{S} = 1/2\begin{bmatrix} 2\mathbf{I} & \mathbf{K_1}^{-1} \\ \mathbf{K_1}^{-1} & \mathbf{K_2}^{-1} \end{bmatrix} \tag{29}$$

Lemma 1: *The matrix* **S** *is positive definite.*

The proof is based on Shur complement and is the same as what can be found in [13] for the case $\lambda(t) = 1$. So the function V_f is positive. ■

Theorem 1: *The fast subsystem of* (24) *with the matrix* $\mathbf{A_f}$ *introduced in* (27) *is stable provided*

$$\mathbf{K_1}\mathbf{K_2}^{-1} - \mathbf{K_1}^{-1} > \mathbf{0} \tag{30}$$

Proof can be found in [8] and the condition can be met by assuming a lower bound on λ(t), increasing $\mathbf{K_{DF}}$, and decreasing $\mathbf{K_{PF}}$. ■

The stability of the fast subsystem is essential to be able to use the results of Tikhonov theorem. In addition, the Lyapunov function used above will be used again for stability analysis of the overall system.

B. Preliminary lemmas for stability analysis

To analyze the robust stability of the closed loop system in presence of modeling uncertainty, the Lyapunov direct method is used. Let V be the Lyapunov function candidate as follows

$$V(\bar{x},\bar{y}) = \bar{x}^T \mathbf{P}\,\bar{x} + \bar{y}^T \mathbf{S}\,\bar{y} \tag{31}$$

in which **S** is defined as before (equation (29)) and **P** is chosen to be

$$\mathbf{P} = \tfrac{1}{2}\begin{bmatrix} \alpha_2\mathbf{K_P} + \alpha_1\mathbf{K_I} + \alpha_2^2\mathbf{M} & \alpha_2\mathbf{K_D} + \mathbf{K_I} + \alpha_1\alpha_2\mathbf{M} & \alpha_2\mathbf{M} \\ \alpha_2\mathbf{K_D} + \mathbf{K_I} + \alpha_1\alpha_2\mathbf{M} & \alpha_1\mathbf{K_D} + \mathbf{K_P} + \alpha_1^2\mathbf{M} & \alpha_1\mathbf{M} \\ \alpha_2\mathbf{M} & \alpha_1\mathbf{M} & \mathbf{M} \end{bmatrix} \tag{32}$$

in which α_i s are real positive constants. The above function in Equation (33) has a quadratic form and it is positive definite due to positive definiteness of **P** and **S**. Positive definiteness of **S** has been shown in lemma 1 and the following lemma guarantees that **P** is also positive definite in presence of modeling uncertainties.

Lemma 2: *The matrix* **P** *is positive definite if*

$$\alpha_1 > 0\,,\ \alpha_2 > 0\,,\ \alpha_1 + \alpha_2 < 1 \tag{33}$$

$$\alpha_2(k_P - k_D) - (1-\alpha_1)k_I - \alpha_2(1+\alpha_1-\alpha_2)\overline{m} > 0 \tag{34}$$

$$k_P + (\alpha_1-\alpha_2)k_D - k_I - \alpha_1(1+\alpha_2-\alpha_1)\overline{m} > 0 \tag{35}$$

in which

$$\mathbf{K_P} = k_P\mathbf{I},\quad \mathbf{K_I} = k_I\mathbf{I},\quad \mathbf{K_D} = k_D\mathbf{I} \tag{36}$$

Proof is given in [14]. ■

Now for the stability analysis Differentiate V along trajectories (24) and (25), which yields to

$$\begin{aligned} \dot{V}(\bar{x},\bar{y}) &= \bar{x}^T\mathbf{P}\,\dot{\bar{x}} + \dot{\bar{x}}^T\mathbf{P}\,\bar{x} + \bar{x}^T\dot{\mathbf{P}}\,\bar{x} + \dot{\bar{y}}^T\mathbf{S}\,\bar{y} + \bar{y}^T\mathbf{S}\,\dot{\bar{y}} + \bar{y}^T\dot{\mathbf{S}}\,\bar{y} \\ &= \bar{x}^T[\mathbf{PA} + \mathbf{A}^T\mathbf{P}]\bar{x} + \bar{x}^T\mathbf{B}\Delta\mathbf{A} + \bar{x}^T\dot{\mathbf{P}}\,\bar{x} \\ &\quad + 2\bar{x}^T\mathbf{PC}[\mathbf{I}\quad \mathbf{0}]\bar{y} + \bar{y}^T[\mathbf{A_f}^T\mathbf{S} + \mathbf{S}\,\mathbf{A_f}]\bar{y} + \bar{y}^T\dot{\mathbf{S}}\,\bar{y} \end{aligned} \tag{37}$$

the following lemmas will be used to prove that V is a Lyapunov function.

Lemma 3: *For the matrices* **P**, **A**, **B** *and* $\Delta\mathbf{A}$ *defined previously, the following inequality holds*

$$\bar{x}^T[\mathbf{PA} + \mathbf{A}^T\mathbf{P}]\bar{x} + \bar{x}^T\mathbf{B}\Delta\mathbf{A} + \bar{x}^T\dot{\mathbf{P}}\,\bar{x} \ \le\ \|\bar{x}\|(\varepsilon_0 - \varepsilon_1\|\bar{x}\| + \varepsilon_2\|\bar{x}\|^2) \tag{38}$$

In this inequality ε_0, ε_1 and ε_2 are real positive constants that depend only on α_1, α_2 and the uncertainty bounds introduced in equations (4) to (6) as follows:

$$\varepsilon_0 = \lambda_1(\beta_0 + \overline{m}\lambda_3) \tag{39}$$

$$\varepsilon_1 = \gamma - \lambda_1\beta_3 - \overline{m}\lambda_2 - \lambda_1\beta_1 \tag{40}$$

$$\varepsilon_2 = \lambda_1\beta_4 + \lambda_1\beta_2 \tag{41}$$

in which

$$\begin{aligned} &\lambda_1 = \overline{\lambda}(\mathbf{R}_1),\ \lambda_2 = \overline{\lambda}(\mathbf{R}_2) \\ &\lambda_3 = \|\ddot{q}_d(t)\|_\infty\,,\quad \gamma = \underline{\lambda}(\mathbf{Q}) \end{aligned} \tag{42}$$

where

$$\mathbf{Q} = \begin{bmatrix} \alpha_2 k_I\mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & (\alpha_1 k_P - \alpha_2 k_D - k_I)\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & k_D\mathbf{I} \end{bmatrix} \tag{43}$$

$$\mathbf{R}_1 = \begin{bmatrix} \alpha_2^2\mathbf{I} & \alpha_1\alpha_2\mathbf{I} & \alpha_2\mathbf{I} \\ \alpha_1\alpha_2\mathbf{I} & \alpha_1^2\mathbf{I} & \alpha_1\mathbf{I} \\ \alpha_2\mathbf{I} & \alpha_1\mathbf{I} & \mathbf{I} \end{bmatrix} \tag{44}$$

$$\mathbf{R}_2 = \begin{bmatrix} \mathbf{0} & \alpha_2^2\mathbf{I} & \alpha_1\alpha_2\mathbf{I} \\ \alpha_2\mathbf{I} & 2\alpha_1\alpha_2\mathbf{I} & (\alpha_1^2 + \alpha_2)\mathbf{I} \\ \alpha_1\alpha_2\mathbf{I} & (\alpha_1^2 + \alpha_2)\mathbf{I} & \alpha_1\mathbf{I} \end{bmatrix} \tag{45}$$

Proof can be found in [8]. ■

Lemma 4: *For the matrix* **C** *defined previously the following inequality holds*

$$2\bar{x}^T\mathbf{PC}[\mathbf{I}\quad \mathbf{0}]\bar{y} \ \le\ 2\|\bar{x}\|\overline{\lambda}(\mathbf{P})\overline{\lambda}(\mathbf{M}^{-1})\|\bar{y}\| \tag{46}$$

Proof: Considering that

$$\mathbf{C}[\mathbf{I}\quad \mathbf{0}] = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{M}^{-1}(\bar{q}) & \mathbf{0} \end{bmatrix} \tag{47}$$

proof is straightforward. ■

C. Stability of the complete system

In this subsection we present the main result. To be compact we simply refer to the equations by their numbers in the body of the theorem.

Theorem 2: *Consider the flexible joint manipulator of equations* (14) *and* (15) *with the composite controller structure of equations* (16) *and* (17)*, under supervisory loop. Given that*

$$d = \frac{2\varepsilon_0}{\overline{\lambda}(\mathbf{R}) + \sqrt{\overline{\lambda}(\mathbf{R})^2 - 4\varepsilon_0\varepsilon_2}} \times \sqrt{\frac{\overline{\lambda}}{\underline{\lambda}}} \tag{48}$$

the overall closed loop system with governing equations of motion (23) *and* (24) *is UUB stable with respect to* **B**(0,*d*) *and the state variables converge to the origin under conditions* (30), (34), (35), (36) *and the following limits imposed on the fast (PD) and slow (PID) controller gains*

$$\overline{\lambda}(\mathbf{R}) > 2\sqrt{\varepsilon_0\varepsilon_2} \tag{49}$$

$$\overline{\lambda}(\mathbf{R})\left[\overline{\lambda}(\mathbf{R}) + \sqrt{\overline{\lambda}(\mathbf{R})^2 - 4\varepsilon_0\varepsilon_2}\right] > 2\varepsilon_0\varepsilon_2\left(1 + \sqrt{\frac{\overline{\lambda}}{\underline{\lambda}}}\right) \tag{50}$$

$$\overline{\lambda}(\mathbf{R}) + \sqrt{\overline{\lambda}(\mathbf{R})^2 - 4\varepsilon_0\varepsilon_2} > 2\varepsilon_2\|z_0\|\sqrt{\frac{\overline{\lambda}}{\underline{\lambda}}} \tag{51}$$

where $\|z_0\|$ denotes the initial condition and

$$\begin{aligned} &\overline{\lambda} = \max\{\overline{\lambda}(\mathbf{P}), \overline{\lambda}(\mathbf{S})\} \\ &\underline{\lambda} = \min\{\underline{\lambda}(\mathbf{P}), \underline{\lambda}(\mathbf{S})\} \end{aligned} \tag{52}$$

Proof is given in [8]. ■

This proof reveals an important aspect of the supervisory loop dynamics included in the proposed controller law.

Because of the dynamical gain adaptation of the controller and due to the negative definite influence it is adding to the derivative of Lyapunov function stated in Theorem 2, this adaptation preserves the robust stability of the system, without perturbing the stability conditions. This general idea which was observed through various simulations of implemented supervisory loop for different systems is technically proven here. Another aspect that can be concluded from this analysis is the robustness property of the stability, in presence of modeling uncertainty. Since the unmodeled but bounded dynamics of the system is systematically encapsulated in the system model (as stated in (4)-(6)), the only influence this will impose on the stability is the respective controller gains bounds depicted in the above mentioned conditions.

In the next section we will show the effectiveness of the proposed method in practice.

V. Experimental Results

The laboratory set up which has been considered for experimental study is shown in Fig. 3. It is a 2 DOF flexible joint manipulator. In the first joint a harmonic drive is used for power transmission. Its spring constant is empirically derived to be 6340 N.m/rad [15].

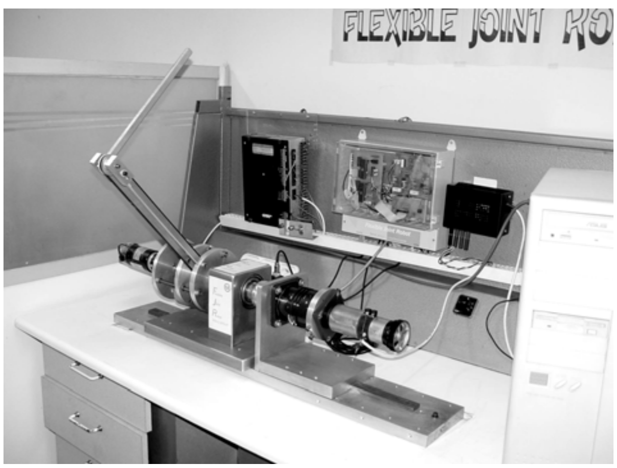

Fig. 3. Experimental Setup

The flexible element used in power transmission system of the second joint is shown in Fig. 4. It has been made from Polyurethane and is designed so that it has very high flexibility. Its equivalent spring constant is 8.5 N.m/rad which is a value that makes a challenging control problem. To show the effectiveness of the proposed algorithm in presence of actuator saturation despite low stiffness, the experimental results on the second joint are considered. Specifications of the second motor have been shown in Table I.

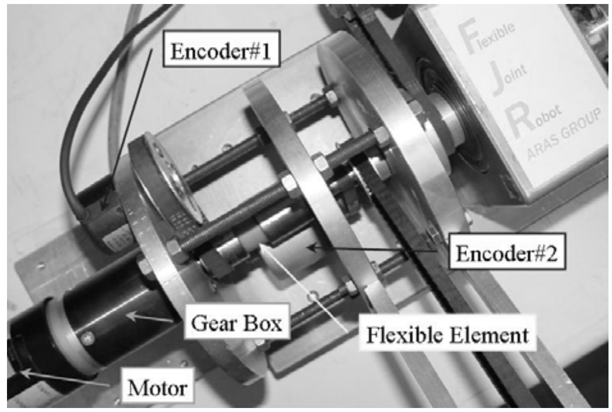

Fig. 4. The flexible element

TABLE I:
MOTOR SPECIFICATIONS

Continuous Torque (N.m)	13
Max. Rated Input (V DC)	12
Max. Continuous Power (W)	62
Rated Speed (rpm)	26

In order to control the system by means of a PC, a PCL-818 I/O card and a PCL-833 encoder handling card of the Advantech Company are used for hardware interfacing. The "Real Time Workshop" facilities of the MATLAB SIMULINK are used for user interface. A block diagram of the system is shown in Fig. 5.

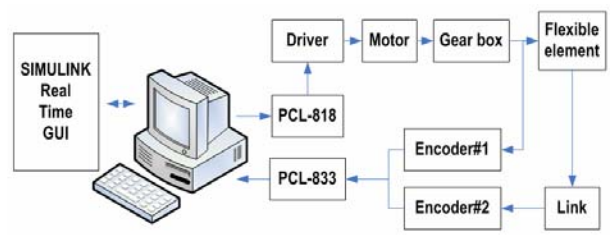

Fig. 5. Block diagram of the system

Experimental results are shown in figures 6 and 7. Fig. 6 shows the result for tracking a sine wave. During the interval (20.3, 30.4) seconds the supervisor is turned off i.e. the value of $\lambda(t)$ is set to be one. Results show that the control effort is increased and saturated resulting in high tracking error. The same result can be found for a pulse reference input in Fig. 7 in which the supervisor is turned off at time t=13 s. High resonance at this time can be observed.

These experimental observations confirm the theoretical proof given in this paper, which insures the ability of supervisory loop to diminish instabilities caused by actuator saturation.

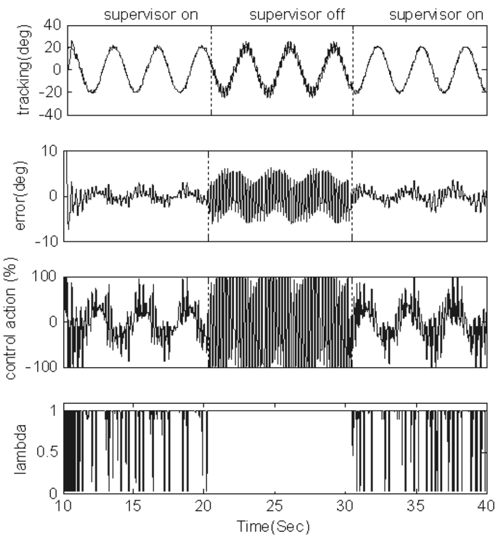

Fig. 6. experimental results for sine input

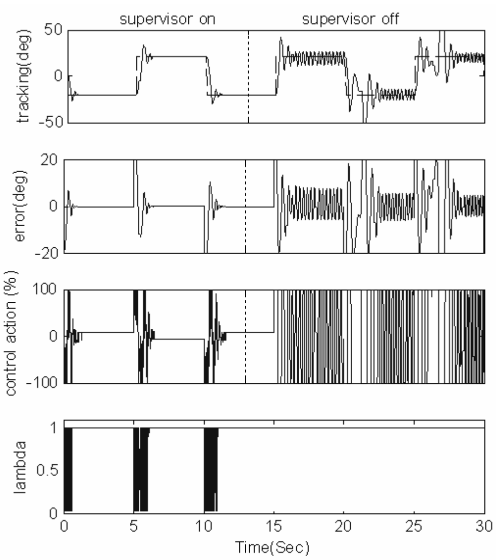

Fig. 7. Experimental results for pulse input

VI. CONCLUSIONS

In this paper the problem of controller synthesis for flexible joint robots in presence of actuator saturation is analyzed in detail. The singularly perturbed model of the system is first introduced briefly, and a composite controller structure is proposed for the system. The composite controller consists of a robust PID term for rigid (slow) and a PD controller for flexible (fast) dynamics. In order to remedy the limitations caused by actuator bounds, a supervisory loop is proposed, and it is shown that a model free fuzzy supervisory loop makes it possible to preserve stability, without great loss in performance. The supervisor will affect the signals *in prior to* the controller, and therefore, affecting the controller states. This is on contrary to the static saturation block which will be placed *after* the controller. It is shown through a Lyapunov based stability analysis, that due to the structure of the supervisory loop, since the controller adaptation gain is bounded and the overall variation of the system energy is dissipative, the stability condition of the composite controller remains unchanged. These all considerations have enabled us to offer an implementable controller with guaranteed robust stability which is an essential requirement for susceptible applications such as space robotics. These theoretical results are verified in practice and experimental results are provided at the last part of the paper.

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