Composite QFT Controller Design for
Flexible Joint Robots

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Abstract — **In this paper, a practical method to design a robust controller for a flexible joint robot (FJR) using quantitative feedback theory (QFT) is proposed. In order to control fast and slow dynamics of the FJR separately, composite control scheme is considered as the basis for the design. A simple PD controller is used to stabilize the fast dynamics, and a QFT controller is used in addition to an integral manifold corrective term to perform on the slow dynamics. Because of the nonlinear dynamics of FJR and the proposed controller scheme, linear time invariant equivalent (LTIE) technique is used to assign a nominal model for the system with uncertainties templates. Design of the QFT controller, as slow part of the composite control law, is performed to compromise between the required bandwidth and the controller order. Comparisons with previous works on FJR, such as robust PID and composite H_∞, illustrate the effectiveness of the proposed controller to reduce the tracking errors despite actuator limitations.**

I. INTRODUCTION

Multiple-axis robot manipulators are widely used in industrial and space applications. The success in reaching high position accuracy in robots is due to their rigidity, which make them highly controllable. After the inception of harmonic drive in 1955, and its wide acceptance, the rigidity of the robot manipulators is greatly affected. In early eighties researchers showed that the use of control algorithms developed based on rigid robot dynamics, on real non-rigid robots is very limited, and may even cause instability [1]. Since then a large number of researchers have been working on the development of control algorithms for flexible joint robots (FJR). The singular perturbation theory is used as the basic method to encapsulate the flexible joint robot dynamics. By this means and through the use of two-time scale behavior, FJR dynamics is divided into fast and slow subsystems [2, 3]. As shown in [4] for a three-axis flexible robot the system is not feedback linearizable, and the use of methods such as computed-torque methods for flexible manipulators is not directly implementable. By neglecting the effects of link motion on the kinetic energy of the rotor, Spong has derived a mathematical model for such systems in which the system is feedback linearizable [5]. However, in order to linearize the system acceleration and jerk feedback is required whose measurements are costly. To avoid the need of acceleration and jerk in this method the idea of integral

manifolds is employed. In this method instead of using the zero order approximation of the model extracted from the singular perturbation theory, higher order models can be used, and hence, a series of corrective terms is added to the control algorithm [1, 6]. In adaptive methods many algorithms are developed for FJR, in most of which a term due to the fast subsystem is added to the adaptive algorithm based on rigid models [6, 7]. In robust methods by considering model uncertainties the stability of the fast subsystem is analyzed, and through robust control synthesis, a robust controller is designed for the slow subsystem [5, 8], [9, 10].

In this paper nonlinear QFT approach is applied for controller design of FJR. It is first observed that the use of a stand alone QFT controller for the system causes large uncertainty templates, and hence, loop shaping within the bounds is infeasible. Hence, the use of a composite controller similar to that in [7, 10] is proposed, but with a nonlinear QFT controller for slow subsystem. The structure of controller consists of three parts. A simple PD controller is used to stabilize the fast dynamics, and a QFT controller is used in addition to an integral manifold corrective term to perform on the slow dynamics. Because of the nonlinear dynamics of FJR and the proposed controller scheme, linear time invariant equivalent (LTIE) technique is used to assign a nominal model for the system with uncertainties templates. Design of the QFT controller, as slow part of the composite control law, is performed to compromise between the required bandwidth and the controller order. It is observed that this proposed structure is insensitive to structured variation of the plant, and only one design is sufficient for the full envelope. Moreover, any design limitations and the structure of the controller are apparent up front, and there is less development time for a full envelope design. Furthermore, one can determine what specifications are achievable early in the design process, and the changes in the specifications can be accomplished quickly in the redesign. The effectiveness of the proposed control topology is verified through various simulations.

II. FJR MODELLING AND CONTROL BACKGROUND

A. Modelling

To model a FJR, the link positions are assumed as the state vector similar to the case with solid robots. Actuator positions must be also considered in the state vector, for these are dynamically related to the link positions through the flexile element. Assume that the position of the i'th link is shown with $\theta_i : i = 1,2,...,n$ and the position of the i'th

actuator with θ_{i+n} : i=1,2,…,n , It is usual to arrange these angles in a vector as follows:

$$\bar{Q} = [\theta_1, \theta_2, ..., \theta_n, \theta_{n+1} \mid \theta_{n+2}, ..., \theta_{2n}]^T = [\bar{q}_1^T, \bar{q}_2^T]^T \quad (1)$$

Using this notation and taking into account some simplifying assumptions, Spong has proposed a model for FJRs as follows [12]:

$$I(\bar{q}_1)\ddot{\bar{q}}_1 + \bar{C}(\bar{q}_1, \dot{\bar{q}}_1) + K(\bar{q}_1 - \bar{q}_2) = 0 \quad (2)$$
$$J\ddot{\bar{q}}_2 - K(\bar{q}_1 - \bar{q}_2) - \bar{u} = 0$$

Where, I is the matrix of the link inertias and J is that of the motors, C is the vector of all gravitational, centrifugal and Coriolis forces and torques and u is the input vector. Furthermore, without loss of generality it is assumed that all flexible elements are modeled by linear springs with the same spring constant k [12], and the matrix $K = k\, I_{3x3}$. The inertia matrices are non-singular so the model can be changed to the following singular perturbation standard form:

$$\ddot{\bar{q}} = -A(\bar{q})\bar{z} - G(\bar{q}, \dot{\bar{q}}) \quad (3)$$
$$\varepsilon \ddot{\bar{z}} = -(A(\bar{q}) + B(\bar{q}))\bar{z} - G(\bar{q}, \dot{\bar{q}}) - B(\bar{q})\bar{u}$$

in which: $\bar{q} = \bar{q}_1$, $\bar{z} = K(\bar{q}_1 - \bar{q}_2)$ and $\varepsilon = 1/k$.

As seen from the model, dynamics of FJR possesses a two-time-scale behavior due to the presence of the small parameter ε as a multiplier in the second differential equation. This causes the system to have fast and slow variables. In the sequel we will use the concept of integral manifold and composite control to design a suitable controller for this requirement [12].

B.   Composite Control

It is shown in [12], for FJR with any given input u_s that there exists an integral manifold in the (q, z) space, described as follows:

$$z_s = h(q_1, \dot{q}_1, u_s, \varepsilon) \quad (4)$$

by which the fast dynamics becomes asymptotically stable. The above condition, if initially violated, will be nearly satisfied after the decay of the fast transients, i.e. z will approach to the z_s. The unknown function h can be found by solving the following partial differential equation which is obtained by substitution of h and its derivatives in equation (3):

$$\varepsilon \ddot{h} = -(A(q) + B(q))h - G(q, \dot{q}) - B(q)u_s \quad (5)$$

This equation referred to as the *manifold condition* is hard to solve analytically. Spong *et al.* have proposed a method to solve this equation approximately to any order of ε by expansion of terms as done in equations (8) and (9), [12]. Using the concept of composite control a fast term could be added to the control input to make the fast dynamics to be asymptotically stable [13]:

$$u = u_s + u_f(z_f, \dot{z}_f) \quad (6)$$

Where, $z_f = z - z_s$, represents the deviation of the fast variables from the manifold. The fast control is designed such that $u_f(0,0)=0$ so on the manifold $u=u_s$ and no more modification is needed to be applied on the manifold condition (5). By subtracting (5) from (3) the fast dynamics derived as:

$$\varepsilon \ddot{z}_f = -(A(q) + B(q))z_f - B(q)u_f \quad (7)$$

Hence, a PD controller can be used to stabilize the fast dynamics. To solve the manifold condition and simultaneously design a corrective term in the controller, expansion of h and u_s can be used as follows:

$$u_s = u_0 + \varepsilon u_1 + \varepsilon^2 u_2 + ... \quad (8)$$
$$h = h_0 + \varepsilon h_1 + \varepsilon^2 h_2 + ... \quad (9)$$

Substituting these in the manifold condition and equating the terms with the same order will result in:

$$h_0 = -\frac{G(q) + B(q)u_0}{A(q) + B(q)} , \quad h_i = -\frac{\ddot{h}_{i-1} + B(q)u_i}{A(q) + B(q)} \quad (10)$$

By substitution of these results in the differential equation of q we will have:

$$\ddot{q} = -\frac{B(q)G(q)}{A(q) + B(q)} + \frac{A(q)B(q)}{A(q) + B(q)}u_0 - A(q)\sum_{i=1}^{\infty}\varepsilon^i h_i \quad (11)$$

Now if we choose:

$$u_1 = -\frac{\ddot{h}_0}{B(q)} \quad (12)$$
$$u_i = 0 , \quad i = 2, 3, ...$$

Then, the h_i's will vanish except for h_0 and equation (11) will reduce to the solid model. Therefore, using the corrective term u_1 will enable us to design the u_0 as usual as that for solid robots. What is remaining in the control algorithm is u_o, which will be derived thoroughly using QFT method in the sequel.

III.   QUANTITATIVE FEEDBACK THEORY

As a robust controller design method QFT has been an ordinary method during recent two decades. According to its name in this method all demands, including robust stability and robust performance, are quantized and translated into some limitative bounds in Nichols chart; while uncertainties (structured and unstructured) are translated into areas in Nichols chart called templates. Finally the controller results from loop shaping of nominal loop transfer function $L_o(s)=G(s)\cdot P_o(s)$, such that satisfies the sketched bounds. One of exceptional specifications of QFT is that we are able to specify certain upper and lower tolerances for desired output in time domain and ensure that the final output of the system will be between these two tolerances. To satisfy this characteristic, there are specific bounds on Nichols chart which guarantee that the variation in the closed-loop transfer function is less than or equal to that allowed. Therefore, a pre-filter, F(s), is required to bring the response within the

area between upper and lower tolerances. Hence, QFT is a two degree of freedom (2 DOF) design algorithm [14].

QFT is inherently a linear method, for the design is eventually performed in Nichols Chart. Thus, in nonlinear plants like FJR, nonlinearities have to be translated into defined concepts in QFT like templates or bounds. For this purpose, literature on QFT offers two different techniques [15], namely Linear Time Invariant Equivalent (LTIE) of nonlinear plant, and Non-Linear Equivalent Disturbance Attenuation (NLEDA) techniques. In both techniques, limited accepted output is the main tool to translate nonlinearities of the plant into templates for the first technique, or disturbance bounds for the second technique. In this paper the first technique is used as follows. The main idea in this technique is to substitute the nonlinear plant with an equally family of LTI plants [14]. Consider the nonlinear and/or time varying plant with a general nonlinear function $w \in \Psi$ such that all acceptable plant outputs to be achieved is $y_a \in Y$. The equivalent plant family is defined by:

$$P_0 = \{p_0 \mid p_0 = \frac{L[y_a]}{L[x_a]}, y_a \in Y, x_a = w^{-1}(y_a), w \in \Psi\} \quad (13)$$

Where, L[·] denotes the Laplace transform. It is useful to add that, Y contains all desired output of the system to be delivered, and Ψ contains all system uncertainties and plant variations. Once the acceptable plant outputs are selected by the designer, the family of equivalent transfer functions could be generated analytically or numerically from the plant mathematical model. In this case since a specific nonlinear model for FJR (Spong model) is considered, the LTI family is easily achieved. For this means first, the acceptable plant input-output sets for a finite time interval [0,T] are determined. Then the Golubev method [16] is applied for each input-output set, in order to reach directly to a linear time-invariant transfer function, relating acceptable plant input-output data set. Once the equivalent family is derived, QFT method can be employed to develop a single robust controller. If a controller provides an acceptable response for the equivalent family, then it is claimed that the same controller provides an acceptable response for the original nonlinear plant (for proof see [17] and [18]).

IV. QFT CONTROLLER SYNTHESIS

In order to apply the proposed method, it is essential to perform the linear identification routines onto the system. For this purpose consider the single link flexible joint manipulator introduced in [12] and illustrated in figure 1, with ordinary nominal values of *m=1, I=1, J=1, L=1, g=9.8, k=100*. This system proves to be a typical model for flexible joint manipulators, where the flexibility characteristics of it is exaggerated, since there is no structural damping modeled onto the system. Therefore, the control of this typical model is more challenging. This model is considered here in order to compare the closed loop performance of the proposed controller to that reported in the literature [5, 10].
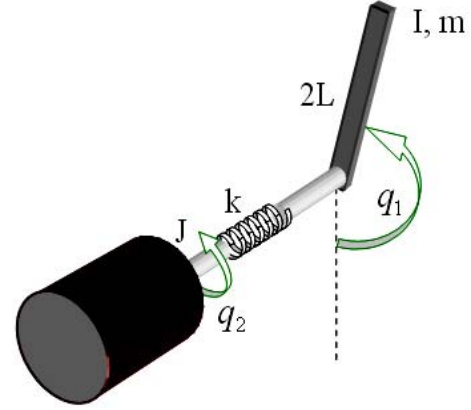

Fig. 1 The single degree of freedom FJR

Consider the desired control objectives are assigned experimentally as follows:

(i) - Closed-loop robust constraint is given by

$$\left| \frac{L(jw)}{1 + L(jw)} \right| \le M = 1.2 \qquad \forall \omega \in [0, \infty) \quad (14)$$

(ii) - For tracking performance requirement, the controller should satisfy the following inequality:

$$|T_l(i\omega)| \le |T(i\omega)| \le |T_{ll}(i\omega)| \qquad \forall \omega \in [0, \infty) \quad (15)$$

where,

$$T(s) = \frac{FGP_e(s)}{1 + GP_e(s)}$$

$$T_u(s) = \frac{23100(s+10)}{(s+20)(s+21)(s+22)(s+25)}$$

and

$$T_l(s) = \frac{1155(s+200)}{(s+20)(s+21)(s+22)(s+25)}$$

These transfer functions are systematically derived from the desired step response bounds for the system with suitable rise time and overshoot. Some factors are considered to choose these transfer functions are:

(1)- Format: FJR model equations show that it is equivalent to a 4th order transfer function.

(2)- Frequencies: Previous works on FJR show that its desired maximum bandwidth is about 8 rad/sec. Fixing 4 poles around 20 rad/sec will result in a bandwidth of about 8 rad/sec in practice, if a variable and suitable zero between 10 and 200 rad/sec is added to the system.

(3)- Templates: In addition to the above analytical factors there is another condition to find suitable tolerances for output which tunes the domain iteratively. The acceptable outputs should be chosen such that templates in Nichols Chart are small enough to permit desired loop-shaping.

As described before the composite control strategy is proposed for the system with the following control signal:
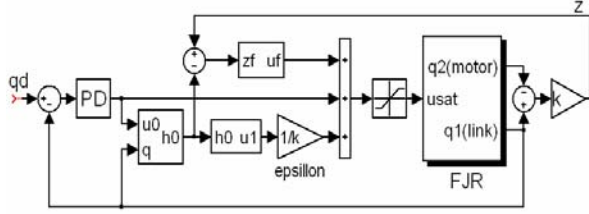
Fig. 2 The FJR Composite Control Topology

$$u = u_f + u_0 + \varepsilon u_1$$

where,

$$u_1 = \ddot{h}_0$$

and h_0 is integral manifold [10]:

$$h_0 = -4.9 \sin(q) + 1/2\ u_0$$

and the fast control law is a simple PD controller satisfying the robust stability conditions such as:

$$u_f = \eta + \dot{\eta}$$

in which η indicates the variation of z from the integral manifold h. The overall control system for an FJR using composite control with a corrective term is shown in figure 2. After adding these terms to the model we can achieve the equation relating u_0 as our modified input and q as our output. By this means the family of linear time invariant equivalent for the uncertain system is identified considering the composite control structure affecting the system.

Therefore, if we choose the acceptable plant outputs for unit step reference input as:

$$Y_a(s) = \frac{231000(s+a)/a}{s(s+20)(s+21)(s+22)(s+25)} \qquad a \in [10\ \ 200],$$

where, these outputs are drawn in time domain in figure 3, the LTI family can be obtained by calculating control signals for these outputs and getting Fourier Transform of these pairs. Note that $T_u(s)$ and $T_l(s)$ are upper and lower bounds of:

$$T_a(s) = \frac{Y_a(s)}{R(s)} \quad , \quad R(s) = \frac{1}{s}$$

Sketching the templates for test frequencies of

$$w = \{.1, 1, 5, 8, 20, 50, 100, 200, 500, 1000\},$$

the other steps of QFT design are just like linear cases. By this means the nonlinear behavior of the FJR is encapsulated in the uncertainty templates. Eventually, with the experience of the designer as usual, the loop-shaping is performed, where the designed nominal loop has shown in figure 4, and is given bellow.

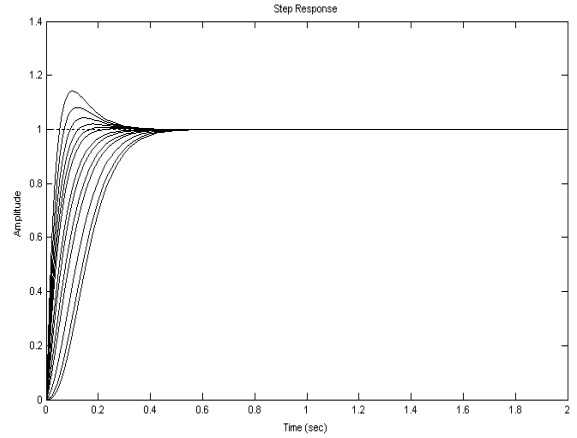$$G(s) = \frac{2889567.08\ (s+69.3)\ (s+36.63)}{(s+1861)\ (s+577.2)}$$

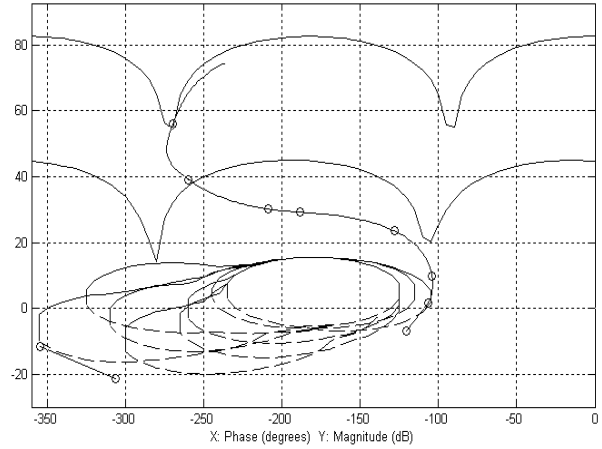Fig. 3 Area of acceptable outputs in time domain

Fig. 4 Loop-shaping in Nichols chart

In design of the pre-filter, it is designer's choice to decide, in which frequencies transfer function should locate between two bounds.

One choice is to locate transfer function between bounds in all frequencies. The resulted pre-filter of this choice is:

$$F_1(s) = \frac{1012079.}{(s+53.91)\ (s+47.84)\ (s+28.29)\ (s+13.87)}$$

Figure 5 shows step response and control effort of the system with these controllers. As shown the output is between two tolerances, with much smaller control effort compared to that to previous designs [5], [10].

To evaluate the robustness of controller in presence of uncertainties, up to 10% uncertainty in the following parameters of the model is considered: I, J, m, L, $\varepsilon = 1/\tilde{k}$.

The controllers are quite robust to the parameter perturbations and the tracking performance is preserved despite the parametric uncertainties.

V. SIMULATION COMPARISONS

In order to fairly compare the proposed controller performance to that of the other proposed controllers for FJR, composite PID controller [12], and composite H_∞ controller [11] are nominated.

Simulations are conducted to reproduce the tracking performance and control effort of them for a sin(8t) reference command as proposed in [11]. It is observed that for this bandwidth requirement, if we apply $F_1(s)$ as the pre-filter response delay is too much as expected, and therefore tracking errors are not acceptable. However, the pre-filter can be designed such that to tune only low frequencies up to the required 8 rad/sec bandwidth. Since in this case, this requirement is automatically satisfied for frequencies less than 10 rad/sec, the pre-filter can be chosen as a pure gain $F_2(s) = 0.9803$.

Figure 6 shows the tracking error for the proposed closed loop system compared to that of the robust PID [10], and composite H_∞ [5]. These results are more elucidated in Table(1), where 2-norm and infinity-norm of tracking error and infinity-norm of control effort, are numerically given. As clarified, in QFT design tracking performance is excellent, with only high peaks of control at first moments. This is caused clearly by high frequencies components of the controller, and therefore, this drawback can be remedied by suitable tuning of the pre-filter.

In the final design of the pre-filter, these high frequencies must be attenuated without affecting low frequencies, particularly, frequencies less than 8 rad/sec. After some analytical trials the final design is:

$$F_3(s) = \frac{.9965 * 10^{\wedge}8}{(s+100)^4}$$

This design is performed using a low pass filter structure with 4 poles at 100. Experimentally the tuning of the pre-filter poles in this case causes rejection of high frequency components of the control effort without effecting low frequencies up to 8 rad/sec. Although, poles with larger values can be used to reduce the effect on low frequencies, but more poles is required which makes the order of controller not pleasant. According to our engineering judgment, this pre-filter is a good compromise between the order of controller and the obtained performance, among the many trials performed in this case. The obtained closed loop results with this pre-filter are illustrated in Figures 7 and 8.

As it is illustrated the tracking performance is better than that of the composite H_∞, with better control effort. However, the tracking error of the proposed controller is almost twice as that in robust PID but with almost 100 times less control effort. These results are quantitatively given in the Table 1. The results reveals that infinity-norm of control effort with this design gets about 5 times lower than that of composite H_∞.
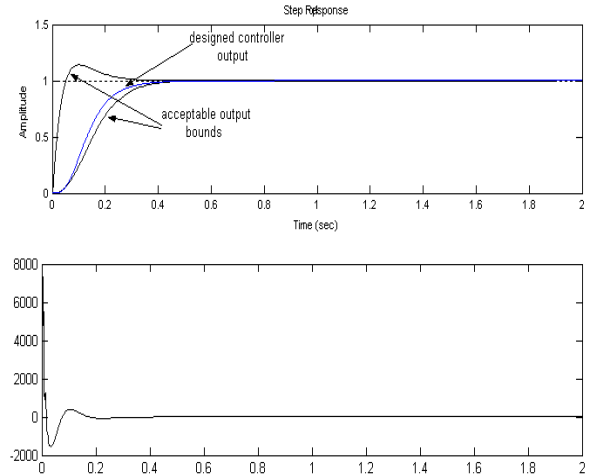

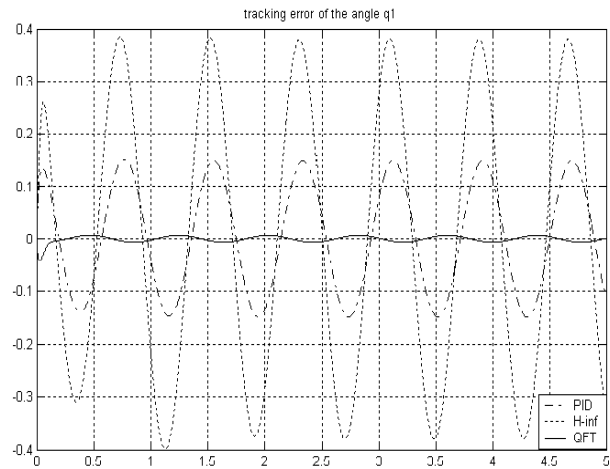Fig. 5 Step response and control effort of system with G(s) and F1(s)

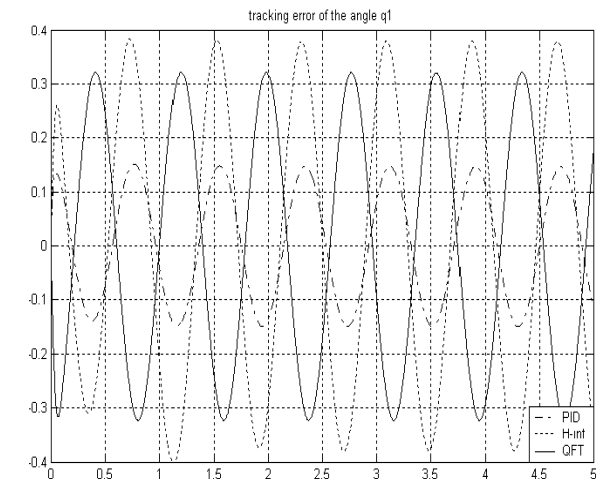Fig.6 Tracking error comparison of robust PID, composite H∞ and QFT (with F2(s))

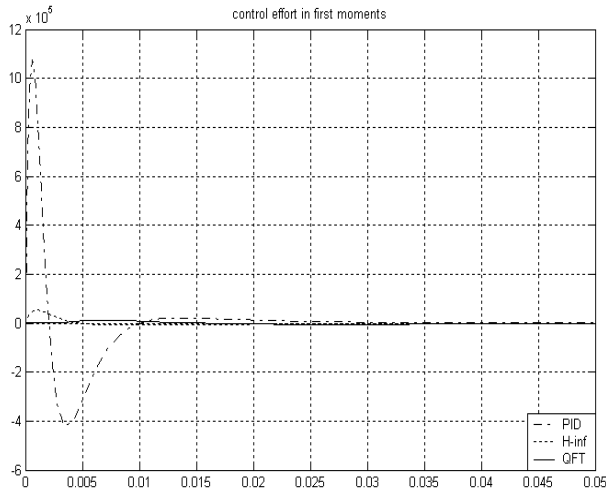Fig.7 Tracking error comparison of robust PID, composite H∞ and QFT (with F3(s))

Fig.8 Control effort comparision of composite PID, H∞ and QFT (with F3(s))

TABLE I.   CLOSED LOOP PERFORMANCE COMPARISON

CONTROL METHOD	$\|e\|_2$	$\|e\|_\infty$	$\|u\|_\infty$
Robust PID	0.232	0.153	0.106×10^7
Composite H_∞	0.586	0.399	0.005×10^7
QFT(with F2(s))	0.014	0.041	1.99×10^7
QFT (with F3(s))	0.507	0.323	0.001×10^7

VI.   CONCLUSIONS

In this paper the ability of quantitative feedback theory to design a well performed controller for an uncertain FJR in presence of actuator limitations is thoroughly investigated.

In order to control fast and slow dynamics of the FJR separately, composite control scheme is considered as the basis for the design. A simple PD controller is used to stabilize the fast dynamics, and a QFT controller is used in addition to an integral manifold corrective term to perform on the slow dynamics. It is shown how the nonlinear model of an uncertain FJR in this structure can be encapsulated into a family of LTI models using linear time invariant equivalent (LTIE) technique. By this means and by use of Glubev method in obtaining LTI models which relates plant input-output data set, usual linear loop shaping method in the QFT can be performed for the FJR. It is observed that the characteristics of QFT make it feasible to obtain suitable tracking performance through a design compromise between the control effort and the controller order. Although loop shaping technique used in this paper requires design experience like all QFT designs, the obtained QFT controller provides an appropriate and practical solution for the system. The comparison of the proposed closed loop system performance to that of other reported results in the literature

reveals the effectiveness of the controller to reduce the tracking error of the closed loop system, in the presence of actuator limitations.

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