Nonlinear H∞ Controller Design for Flexible Joint Robots

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Abstract— **In this paper the design of a nonlinear H∞ controller for flexible joint robot (FJR) is presented. An approximate solution based on Taylor Series expansion is considered for the Hamilton-Jacobi-Isaac (HJI) inequality. A two-degree-of-freedom controller combined of nonlinear H∞ controller and inverse dynamics controller is proposed to tackle the regulation as well as tracking problem in FJR. The proposed nonlinear H∞ controller attenuates the disturbance with a minimum achievable control effort, despite system parameter uncertainty. Simulation comparisons for single and multiple joint manipulators, show that the proposed controller yields to superior performance such as larger domain of attraction and smaller control effort as well as better tracking characteristics, compared to that of the other methods.**

I. INTRODUCTION

THE desire for higher performance from the structure and mechanical specifications of robot manipulators has encouraged designers to use flexible joints in robots. Several new applications such as space manipulators [1] and articulated hands [2], necessitate using FJRs. This necessity has emerged new control strategies, while the traditional controllers used for FJRs have failed in performance [3,4]. Since 1980's many attempts have been made to encounter this problem and several methods have been proposed including various linear, nonlinear, robust, adaptive and intelligent controllers for this means [5,6]. Among these controllers, the robust controller possesses some advantages such as acceptable performance [7], and robust stability against the uncertainties [8]. One of the drawbacks of the aforementioned algorithms is their relatively high control efforts needed to accomplish a good performance.

In this paper the nonlinear H∞ controller design for the FJR is studied in detail. The nonlinear H∞ control theory is a newly emerging nonlinear design approach in practical applications. Since in most physical systems nonlinear nature is observed, one of main advantage of the nonlinear H∞ theory over the linear control theories is the systematic consideration of system nonlinearities and uncertainties together, and thus robust stability and performance can be analyzed over a large operating region [9]. Therefore, the nonlinear H∞ controller provides better potential for handling uncertain nonlinear control problems [10, 11]. Since the nonlinear H∞ controller does not support tracking requirements, a combinational method is proposed to

accommodate this requirement. Moreover, the minimization of the control effort that is needed for regulation as well as tracking, is systematically encountered into controller synthesis.

In this paper first the modeling procedure for an uncertain FJR is elaborated. Next through a short overview of nonlinear H∞ control methods, suitable controllers are designed and simulated for FJR. The obtained results show the effectiveness of the proposed controller in tracking performance, despite model uncertainties and external disturbances.

II. FJR MODELING

To model an FJR both the link and actuator positions are chosen as state vector. Consider that the position of the i'th link is shown with θ_i: $i = 1,2,...,n$ and the position of the i'th actuator with θ_{i+n}: $i=1,2,...,n$. It is usual in the FJR literature to arrange these angles in a vector as follows:

$$\bar{Q} = [\theta_1, \theta_2,...,\theta_n, \theta_{n+1} \mid \theta_{n+2},...,\theta_{2n}]^T = [\bar{q}_1^T, \bar{q}_2^T]^T \quad (1)$$

Using this notation and taking into account some simplifying assumptions, Spong has proposed a model for FJRs [12] as following:

$$I(\bar{q}_1)\ddot{\bar{q}}_1 + \bar{C}(\bar{q}_1,\dot{\bar{q}}_1) + K(\bar{q}_1 - \bar{q}_2) = 0$$
$$J\ddot{\bar{q}}_2 - K(\bar{q}_1 - \bar{q}_2) - \bar{u} = 0 \quad (2)$$

Where, I is the matrix of the link inertias and J is that of the motors, C is the vector of all gravitational, centrifugal and Coriolis torques and u is the input vector. Furthermore, without loss of generality [12] it is assumed that all flexible elements are modeled by linear springs with the same spring constant k and the matrix $K = k I_{3x3}$.

III. NONLINEAR H∞ CONTROL

A. Problem formulation

We consider a system of the form

$$\dot{x} = X(x,w,u), \quad z = Z(x,w,u) \quad (3)$$

Where x is defined in a neighborhood of the origin in R^n and $w \in R^{m_1}, u \in R^{m_2}, z \in R^{p_1}$. It is assumed that X and Z are smooth mapping of class C^k, while k being sufficiently large. In addition to smoothness, the following is assumed:

A_1: The linear H∞ controller must exist.

A_2: For any bounded trajectory of system (3) with the following input we have:

$$w = 0 \; \forall t \;\; Z(x,0,u) = 0 \;\; \forall t \;\; \Rightarrow \;\; \lim_{t\to\infty} x(t) = 0 .$$

The controller has to achieve two goals: it must stabilize the closed-loop system and attenuate the influence of the

exogenous input signal w on the controlled signal z, i.e., it has to limit its L_2 gain by a given value γ. The disturbance attenuation property can be characterized by dissipativity [13,14].

B. State Feedback Controller

In this section, a feedback law $u = a_2(x)$ is sought, which renders the closed-loop system

$$\dot{x} = X(x, w, a_2(x)), \quad z = Z(x, w, a_2(x))$$

(locally) dissipative with respect to the supply rate $s(w, z) = \gamma^2 \|w\|^2 - \|z\|^2$. For this problem let's define :

$$v = \begin{bmatrix} w \\ u \end{bmatrix}$$

By which, the system equations read

$$\dot{x} = \widetilde{X}(x, v), \quad z = \widetilde{Z}(x, v) \tag{4}$$

Then,

$$\gamma^2 \|w\|^2 = v^T \begin{bmatrix} \gamma^2 I_{m_1} & 0 \\ 0 & 0 \end{bmatrix} v$$

and, the Hamiltonian function for this differential game can be found to be [11,13]

$$H(x, \lambda, v) = \lambda^T \widetilde{X}(x, v) + \left\| \widetilde{Z}(x, v) \right\|^2 - v^T \begin{bmatrix} \gamma^2 I_{m_1} & 0 \\ 0 & 0 \end{bmatrix} v .$$

Define a smooth, nonnegative function $V : R^n \to R$ in a neighborhood of $x = 0$ such that $V(0) = 0$ and the Hamilton-Jacobi-Isaacs inequality

$$H(x, V_x^T(x), v_*(x, V_x^T(x)) \le 0 \tag{5}$$

holds for each x in a neighborhood of zero. Then,

$$u = u_*(x, V_x^T(x)) \tag{6}$$

yields a closed-loop system satisfying

$$V_x(x) X(x, 0, u_*) + \left\| Z(x, 0, u_*) \right\|^2 - \gamma^2 \|w\|^2 - \gamma^2 \|w\|^2 \le 0$$

Which means that, a system which has the required dissipativity property in a neighborhood of $(x, w) = (0,0)$.

Due to the Assumption $\mathbf{A_2}$, the feedback law (6) stabilizes the system locally asymptotically, provided $V(x)$ is positive definite. This can be seen by a Lyapunov type of argument. For $w = 0$, expression (5) reads

$$V_x(x) X(x, 0, u_*) + \left\| Z(x, 0, u_*) \right\|^2 \le 0$$

where, $Z(x, 0, u_*)$ can only be zero for asymptotically stable trajectories of x (Assumption $\mathbf{A_2}$). Thus, $V(x)$ being positive definite and

$$\frac{d}{dt} V(x) = \frac{d}{dt} V(x) + V_x(x) \frac{\partial x}{\partial t} = V_x(x) X(x, 0, u_*)$$

being negative proves asymptotic stability of the closed-loop system.

C. Design of approximate controller

The HJI inequalities cannot be solved explicitly, in general. However, it is possible to compute an approximated solution based on Taylor Series expansions, which can be used for finding an approximate control law [14]. First order

approximation corresponds to the linear H_∞ problem. It yields an approximation of first order for the control law and of second order for the storage function $V(x)$. Higher order approximations lead to nonlinear H_∞ controller [10,16].

D. Incorporation of uncertainties in the FJR model

To incorporate the effects of the uncertainties due to indistinctness of parameters, one may rewrite (3) in the form:

$$\dot{x} = f(x, u, w) \tag{7}$$

where, w is a vector of uncertainty that represent the deviation of parameters from their nominal values. To consider the uncertainty one may assume that:

$$P_i = P_{io}(1 + w_i) \tag{8}$$

where P_i stands for parameter in the system having uncertain values and P_{io} is the nominal value of the parameter. w_i is an L_2 bounded disturbance. The suitable design approach considered for FJR is the nonlinear H_∞ technique, where the objective is to attenuate the disturbances on the controlled output so that the exogenous inputs are selected to have bounded energy. Therefore, any bounded signal support can enter the system as a disturbance. Consequently, in the model (7) with respect to the exogenous input w, all deviation must be L_2 bounded. It should be noted, however, that not all the parameter perturbations are necessarily L_2 bounded. Constant deviation from the nominal values of the parameters is very common in practice. Obviously, constant deviation is not L_2 bounded. As a matter of fact model (7) cannot handle constant deviation from the nominal values of the parameters. To circumvent this difficulty the system equation can be modified to the following form:

$$\dot{x} = \overline{f}(x) + \Delta f(x) + (\overline{G} + \Delta G)u \tag{9}$$

where Δf, ΔG denote the deviations of f and G from their nominal values $\overline{f}, \overline{G}$, respectively. Equations (9) can be rewritten in the form

$$\dot{x} = \overline{f}(x) + \overline{G}u + v$$

where, the plant uncertainties due to parameter deviation are handled as a part of the disturbed input signal $v = \Delta f + \Delta G \times u$. Note that this signal can be expressed as:

$$v = \begin{pmatrix} \Delta G & \Delta(x) \end{pmatrix} \begin{pmatrix} u \\ x \end{pmatrix} \quad \Delta(x) := \int_0^1 \frac{\partial \Delta f(\lambda x)}{\partial x} d\lambda$$

These model perturbations due to parametric uncertainties are schematically illustrated in figure (2) as pointed out in [10]. It follows from the small gain theorem that the closed loop system in figure (2) will be stable (in the sense of L_2) for all perturbations ΔP with $\gamma_\Delta < 1/\gamma$. The H_∞ control problem requires that disturbance v be L_{2e} signal.

E. Tracking controller design

The nonlinear H_∞ controller described above can only regulate the system to the equilibrium point (origin). However, in robotic applications, tracking of a desired trajectory is required, and therefore, it is important to reconfigure the controller to provide such control action.
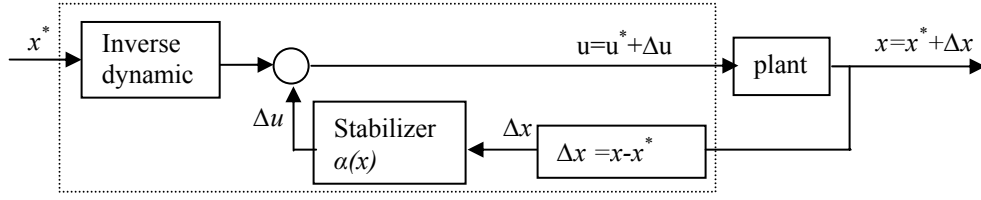
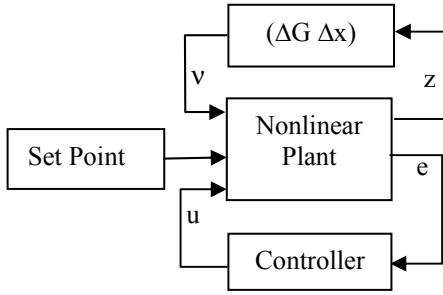Figure 1. controller with two structural degrees of freedom

Figure 2. Control structure under model perturbations

To this end, a two step procedure is used:

1. Linearize the system about a trajectory that satisfies the state equations.
2. Design a control strategy to generate incremental control in response to small deviations from the trajectory.

To do this, suppose that state space equations of the system are as follow:

$$\dot{x} = f(x,u) \tag{10}$$

For $u = u^*$, the desired trajectory x^* should be tracked, provided that:

$$\dot{x}^* = f(x^*,u^*) \tag{11}$$

To design the tracking controller for any arbitrary trajectory, we rewrite equation (10) for incremental variables, i.e., for deviations from desired trajectory. Let

$$x = x^* + \Delta x \tag{12}$$

$$u = u^* + \Delta u$$

Substitution in equation (10) yields:

$$\Delta \dot{x} + \dot{x}^* = f(x^* + \Delta x, u^* + \Delta u) \tag{13}$$

Considering a stabilizer with u=α(x), we can use block diagram shown in Figure (1) to solve the tracking problem for the system that involves such stabilizer. As it is shown in this figure the controller consists of two parts; the first part u=α(x) stabilizes the system forcing Δx to zero by corresponding Δu. The second part uses the inverse dynamic equations of the plant and produces proper control signal u for following the desired trajectory. By designing a robust stabilizer in the first part, the closed loop performance of the system is improved, since the dependence of the inverse dynamics to the plant uncertainties are reduced.

Note that the control input, which will be minimized by the nonlinear H∞ controller design process, is only the Δu, and not the total amount of control input. However, the control signal needed for tracking u* is uniquely calculated and is

not related to the control strategy used in the stabilizer. Hence, the total control effort, u=u*+Δu, becomes minimal for the tracking objective as well. This is promising for the applications where actuator saturation is a practical limitation. This combined topology of the inverse dynamic controller and robust nonlinear H∞ controller is not previously proposed in the literature for robotic applications. The details of implementation and the effectiveness of the proposed method are evaluated through computer simulations on FJR in the proceeding sections.

IV. SIMULATIONS ANALYSIS

The effectiveness of the proposed method is verified through some simulations on a typical FJR. First, a single degree-of-freedom FJR is considered as shown in figure 3. The parameter values are selected as in [12], to be: m=1, I=1, J=1, L=1, g=9.8, k=100.
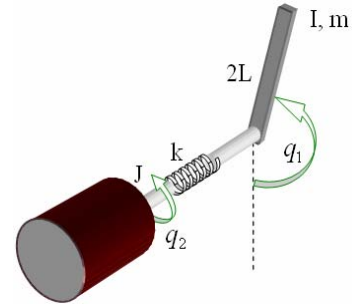

Figure 3. The single degree of freedom FJR

The reference input is chosen as q_d=Sin (8t), which is the largest bandwidth required in such applications [8]. The state variables are considered as follows

$$x_1 = q_1, x_2 = q_2, x_3 = \dot{q}_1, x_4 = \dot{q}_2$$

The design of the controller consists of three parts. First the nominal model of the system and its uncertainty are determined as depicted in Figure 2. Then the inverse dynamics equations are calculated from the nominal model and the desired trajectory by analytical solution of Equation 11. Finally, the nonlinear H∞ controller is derived through approximate solution of corresponding HJI inequality depicted in Equation 5, [14]. Considering the penalty variable as Z=[x u]^T, the results for both linear and 3rd order nonlinear controllers starting from initial condition x_o=[0 5 0 4.24] which is calculated from inverse dynamics equation 11, are shown in figures 4 and 5. As it can be shown in figure (4), the error is converging to zero with a robust performance in both cases, while nonlinear H∞ shows a

relatively better performance, despite even smaller control effort (figure (5)).

To make a qualitative assessment of the control effort and the tracking errors, L_2 and L_∞ norms of these variables are given in Table I, for both linear and nonlinear controllers. As it is clear from these results, nonlinear controller provides relatively better tracking performance with lower control effort. However, the difference is relatively small. In order to further investigate the performance of the controllers, consider larger deviations in initial conditions illustrated in figure (6). This simulation result is achieved considering initial condition as $x_o = [0\ 0\ 0\ 0]$.
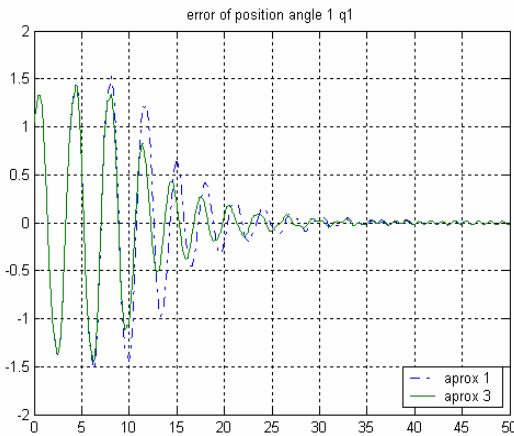

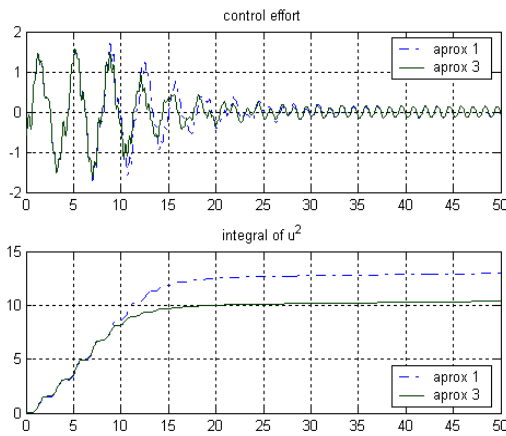Figure 4.   state convergence to the origin for small initial condition

Figure 5.   Control effort and cost function for small initial condition

The difference of two controllers are apparently illustrated in figure 6, in which for this initial values, the linear controllers becomes unstable, while the nonlinear controller preserves its tracking performance. On the other hand, from a robotic tracking requirement, the settling time of this system is too slow. In order to remedy this draw back, the controlled output is weighted with respect to the disturbance for obtaining a faster response. Since the cost function is of quadratic type, increasing the weighting on the output should result in a more damped response.

TABLE I.   TRACKING ERROR AND CONTROL EFFORT MEASURES FOR
                      LINEAR VS NONLINEAR CONTROLLER

CONTROLLERS	$\|\Delta u\|_2$	$\|\Delta u\|_\infty$	$\|e\|_2$	$\|e\|_\infty$
L H_∞	25.86	1.79	27.74	1.52
NL H_∞	23.02	1.63	24.03	1.44

Weighted penalty variable is chosen so that fast modes of system, i.e. x_3 and x_4, will be damped rapidly and a great amount of emphasis be taken on x_1. By this means, the fast mode damping advantage of the composite control strategies proposed for FJR in the literature [7,8], is used into our design, without need of adding any composite structure to the controller. More emphasis on rise time, however, may result in a smaller region of attraction. By increasing the weighting on the output, the algebraic Riccati equation for the linearized model may have not a positive semi-definite solution, and in this case, the attenuation factor γ must be increased. Moreover, the higher the attenuation factor γ, the smaller the reign of attraction is obtained. Hence a practical compromise shall be found through simulation for performance and stability.

Figures 7 and 8 illustrate the results obtained using the proposed correction for linear and nonlinear 3rd order controllers with the following weighted penalty variable:
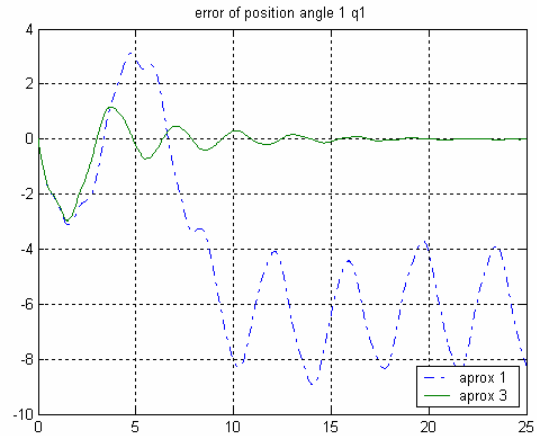$$z = [10000x_1, 100x_2, 100x_3, 100x_4, u]$$

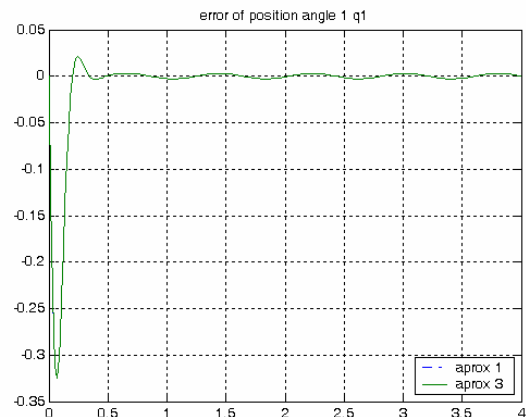Figure 6.   Tracking error for initial condition xo= [0 0 0 0]

Figure 7.   The effect of the linear weight and γ on the performance
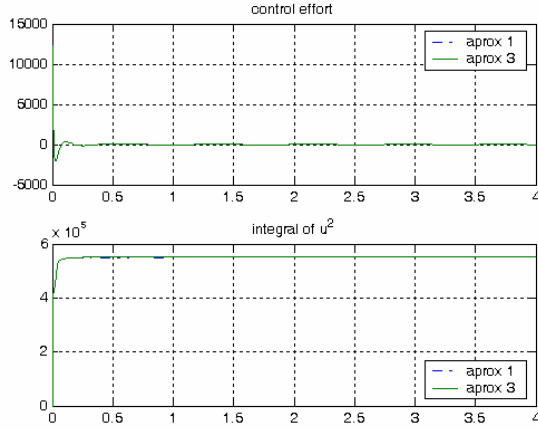
Figure 8.   The effect of the linear weight and γ on the cost function

As it is shown in the obtained results, faster response can be obtained with the expense of higher control effort. Comparing this response with the similar controllers for the FJR proves the effectiveness of this method for reaching to the desired performance. On the other hand, the difference of the linear vs. nonlinear controller is minimal in this case, and the results show that the performance of these two cases is practically identical. This is due to the linear weighting of the variables in the penalty variable. The nonlinear algorithm provides the ability to select weightings in a nonlinear fashion, and therefore shape the output more tractable. As an example the penalty variable is chosen as:

$$z = \left\lfloor 5000x_1 + 12000x_1^2 + 15000x_1^3, 100x_2, 100x_3, 100x_4, u \right\rceil$$

and, the result are illustrated in figure (9). It is quite clear from these simulations that relatively better performance is achievable, using nonlinear controller with nonlinear weightings. The quantitative norms of control effort and tracking errors are given in table (2), which shows the significant reduction of control effort upper bound. To investigate the robustness of the controller in presence of uncertainties, we considered up to 10% uncertainty in the following parameters of the model: *I, J, m, L,* $\varepsilon = 1/k$. The controllers are quite robust to the parameter perturbations and the tracking performance is preserved and quite similar to figure (9) despite the parametric uncertainties. There is only a slight increase in tracking error, in which the 2-norm of the error is increased to the amount of 0.17, and the infinity norm of it reaches 0.44. Different desired trajectories are simulated for the perturbed system, and similar results are obtained, which provides confidence on the robust performance of the system.

Finally, in order to compare the proposed controller performance to that of other controllers for FJR, composite PID controller [7], and composite H∞ controller [8] are considered. Simulations are conducted to reproduce the tracking performance and control effort of them for a sin(8t) reference command as proposed in [8], and the simulation results are given in figure (10). The quantitative measures of tracking errors and control efforts are given in table (3). The

results reveal that the proposed algorithm in this paper provides superior tracking performance with much smaller control effort compared to that of composite H∞. As it is clearly illustrated in figure (10) the general tracking performance of the nonlinear H∞ controller is better than that of the others, except for the initial transient, with much smaller control effort. Although the tracking error infinity norm is relatively higher than that in composite PID, the need of much smaller control input, the superior energy norm of the tracking error, and the surprising steady state tracking performance, makes the application of the proposed controller in the presence of actuator saturation much favorable.
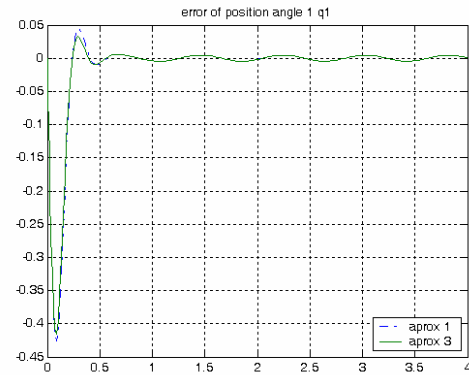

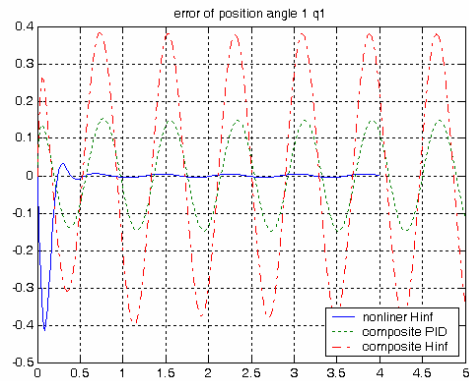Figure 9.   The effect of the nonlinear weight and γ on the performance

Figure 10. Ttracking error comparision of composite PID, composite H∞ and nonlonear H∞

TABLE II.   TRACKING ERROR AND CONTROL EFFORT MEASURES FOR CONTROLLERS DESIGNED FOR PERFORMANCE

CONTROLLERS	$\|\Delta u\|_2$	$\|\Delta u\|_\infty$	$\|e\|_2$	$\|e\|_\infty$
L H∞	362	5375	0.137	0.42
NL H∞	353	4030	0.132	0.41

TABLE III.   TRACKING ERROR AND CONTROL EFFORT MEASURES FOR DIFFERENT CONTROL ALGORITHMS

CONTROL METHOD	$\|e\|_2$	$\|e\|_\infty$	$\|u\|_\infty$
Composite PID	0.21	0.152	10.7×10^5
Composite H∞	0.52	0.39	0.54×10^5
Nonlinear H∞	0.132	0.41	0.04×10^5

This promising result emerged from the fact that not only the nonlinear model of the system encapsulating uncertainties are considered in this method, optimal solution for controller is also achieved through the controller synthesis. The only drawback of the proposed system is the requirement of more complicated offline algorithms to determine the controller. However, the nonlinear approach to the controller design makes it suitable for the case of multiple degrees of freedom. This is accomplished for a two-degrees-of freedom FJR, and in order to have a fair comparison, the resulting closed loop performance is compared to that of reference [17], with the same model and conditions. The model of the system has the form of Equation (2) with nonlinear coupled terms in mass matrices I, and J as well as the term C. The system parameters are as following [17]:

$$m_1 = m_2 = 1\,kg, \ \ l_1 = l_2 = 1\,m, g = 9.8\,m/s^2\,,$$

$$J_1 = J_2 = 0.1\,kg - m^2, k_1 = k_2 = 100\,Nm/rad$$

With these parameters, the flexibility of the typical FJRs is exaggerated and the natural frequency of the model is much lower than that in a typical system. Moreover, since no structural damping is considered into the model, the control of this model is more challenging. The same procedure is considered to design the nonlinear H_∞ for this system as explained in the beginning of section IV, with the difference of assuming the reference command for the joints as the following [17]:

$$q_{l_1} = 1 - 0.5\sin(t), q_{l_2} = 1.2 - 0.4\sin(t)$$

The closed loop performance of both joints is given in figure 11. In this figure, first the performance of the proposed nonlinear adaptive scheme [17] is illustrated, and the resulting closed loop performance of our system is given with the same scale for the ease of comparison. The nonlinear H_∞ tracking performance proved to be much faster than that in [17]. Moreover, because of the robustness properties of our proposed method no instability drawbacks are observed in our system such as what has been reported in case of adaptive control in [17].
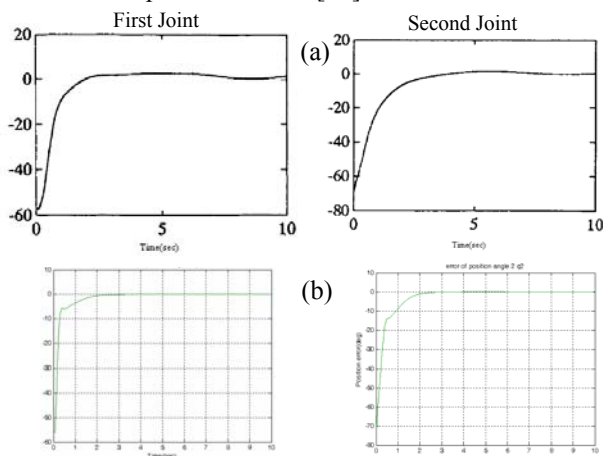

Figure 11. The tracking erors of two link manipulators a) Nonlinear adaptive control [17] , b) The proposed method

V. CONCLUSIONS

In this paper, the performance of a nonlinear H_∞ controller for tracking objective of an FJR under parametric uncertainties is thoroughly investigated. A two-degree-of-freedom controller combined of nonlinear H_∞ controller and inverse dynamics controller is proposed to tackle the tracking problem in FJR. It is shown how the level of the required performance can be adjusted through relative linear and nonlinear weighting of the controlled output with respect to the disturbance, and how the above factors can influence the region of attraction. Therefore, it is concluded that achieving a desired performance can be accomplished through a compromise between performance requirement and stability. This compromise is performed for a single and multiple joint manipulators, and simulation results illustrated the superiority of the nonlinear H_∞ controller over linear H_∞, composite H_∞, and composite PID controllers, respectively.

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