13th Annual (International) Mechanical Engineering Conference – May 2005
Isfahan University of Technology, Isfahan, Iran

Designing H_2/H_∞ Controller for Flexible Joint Robot to Remove Instabilities due to Saturation

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Abstract

Mixed H_2/H_∞ method is proposed to design a controller for flexible joint robots (FJR) considering actuator saturation. By considering the control action in the mixed sensitivity function one can reduce the amplitude of the control action but this may affect all frequencies. A more advanced method based on frequency weighting of the control action contribution in the mixed sensitivity function is considered here which may result in higher bandwidth. But this method is also limited and to get wider bandwidth the H_2/H_∞ method is proposed. This method is shown to be a very good remedy to remove instabilities caused by actuator saturation.

Keywords: Flexible Joint Robot - Actuator Saturation – Mixed H_2/H_∞ – Mixed Sensitivity.

Introduction

The problem of position control for *rigid robots* is a well known issue, and rigid manipulators are extensively used in industries. The desire for higher performance from the structure and mechanical specifications of robot manipulators has been spurred designers to come up with *flexible joint robots* (FJR). Several new applications such as space manipulators [1] and articulated hands [2] necessitate using FJRs. This necessity has emerged new control strategies required, since the traditional controllers implemented on FJRs have failed in performance [3,4]. Since 1980's many attempts have been made to encounter this problem and now, several methods has been proposed including various linear, nonlinear, robust, adaptive and intelligent controllers [5, 6]. Among these, only a few researchers have considered practical limitations such as actuator saturation in the controller synthesis, as a real practical drawback to achieve a good performance [7].

On the other hand actuator saturation has been considered by the control community from early achievements of control engineering. During 50's and 60's at the beginning era of optimal control, researchers have been working on saturation, introducing bang-bang control methods. Over the last decade the control research community has shown a new interest in the study of the effects of saturation on the performance of systems. In fact it can be said that in the past, researchers were encountered a *drawback* identified as actuator saturation and developed methods to avoid it, while now, researchers develop methods to achieve a desirable performance in the presence of actuator saturation encountered as a *limitation*.

It can be said that saturation may cause two types of performance degradations:

1) Inevitable limitations such as slow responses, undesirable transitions, etc.

2) Removable problems such as instability, undesired steady state performance, etc.

The goal in considering saturation in controller synthesis is to decrease or remove the latter and the most important of them is instability. A few papers have been considered this. In an earlier paper [8] we propose to use a fuzzy supervisory control to encounter this problem and in the next work [9] we have been used that fuzzy supervisory loop with a composite PID+PD controller for FJR. The robust methods proposed in this present paper is simpler than that and moreover need only the link position to be feedback. Use of the mixed sensitivity

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approach for reduction of the control action for FJRs has been first proposed in [10]. In this paper we have proposed a more advanced version of that method based on frequency weighting of the control action contribution in the mixed sensitivity function. The more important proposition in this paper is to propose a mixed H_2/H_∞ method in order to decrease the amplitude of the control action. This method has not only a very better result than the composite PID+PD controller but also a better result than the mixed sensitivity approach.

This paper is organized as follows: Section 2 presents the mixed sensitivity approach and Section 3 is devoted to description of the H_2/H_∞ approach; Section 4 is allocated for simulation study and finally the conclusions are presented in Section 5.

Mixed Sensitivity Approach

In an optimal H_∞ design procedure the controller is designed to meet an upper bound on the infinity norm of the (weighted) system output. This can be employed to limit the control action in a mixed sensitivity formulation in which the amplitude of control is considered in the vector to be limited. The problem formulation is as follows. Suppose that the plant model belongs to a of family of models Σ

$$\begin{aligned}\Sigma = \{P(j\omega)\,|\\ P(j\omega) = P_0(j\omega)[1+\Delta(j\omega)W_{unc}(j\omega)\}\end{aligned} \quad (1)$$

Where $P_0(j\omega)$ is the nominal transfer function of the plant and $\Delta(j\omega)W_{unc}(j\omega)$ encapsulates all perturbations of the real plant from its nominal model in which $\|\Delta(s)\|_\infty < 1$. These perturbations may come from nonlinearities, unmodeled dynamics, changes of parameters and operating points, etc. Nominal plant P_0 can be evaluated experimentally through a series of frequency response estimates of the system in the operating regime and then to find the best fit to the average of these models. Linear identification for the system can be applied with different parameter values in different operating points to estimate a set of linear models which can be considered as Σ. Now from equation (1) we have

$$\frac{P(j\omega)}{P_0(j\omega)} - 1 = \Delta(j\omega)W_{unc}(j\omega) \quad (2)$$

and as $\|\Delta(s)\|_\infty < 1$ this yields

$$\left|\frac{P(j\omega)}{P_0(j\omega)} - 1\right| \le |W_{unc}(j\omega)|\ ,\quad \forall\omega \quad (3)$$

The uncertainty bound W_{unc} can be obtained from the results of identification by finding a curve to satisfy the above equation in each frequency. Now the problem can be formulated as follows:

Problem 1. Design a controller C(s) to be used in a feedback loop to control a plant P(s) which belongs to the set Σ which satisfies the performance index $\|W_{perf}(s)S(s)\|_\infty \le 1$ whilst the control action u(t) remains limited such that $\|W_U(s)U(s)\|_\infty \le 1$. ■

Where S(s) denotes the sensitivity function defined as $S(s) = [I+P(s)C(s)]^{-1}$.

Note that the robustness condition (or the fact that the plant model belongs to the set Σ) can be met by satisfaction of the following condition [11]

$$\|W_{unc}(s)T(s)\|_\infty \le 1 \quad (4)$$

Where T(s) denotes the complementary sensitivity function which is defined as T(s) = I-S(s). So the above problem can be changed to the following problem [11].

Problem 1a. Design a controller C(s) to be used in a feedback loop to control a plant $P_0(s)$ such that the following condition be met

$$\left\|\begin{matrix} W_{unc}(s)T(s) \\ W_{Perf}(s)S(s) \\ W_U(s)U(s)\end{matrix}\right\|_\infty \le 1 \quad (5)$$

■

This problem can be solved numerically using the hinflmi function from the LMI toolbox of the MATLAB software [12].

In this manner we can use the weight function $W_U(s)$ to manipulate the infinity norm of U(s) that affects uniformly the amplitude of u(t); So this method can be used to decrease the needed saturation level. The effectiveness of this method for FJRs has been shown for the first time in [10].

Note that the function $W_U(s)$ can be chosen simply to be a constant level as $W_U(s) = \Omega$ which may cause $\|U(s)\|_\infty < 1/\Omega$. This selection may limit the magnitude of U(s) in all frequencies which may limit the resultant bandwidth. As an alternative we can shape $W_U(S)$ in the frequency domain. Obviously reducing the function $W_U(S)$ in high frequencies will result in decreasing the level of the control action fast transients (or jumps) and reducing its integral in frequency domain will result in decreasing the total amplitude of u(t) during all times. This fact can also be seen empirically in simulation studies.

Instead of frequency shaping of the $W_U(s)$ one can limit the 2 norm of U(s). According to the parseval's relation [13] this limits the 2 norm or the energy of u(t). This idea will be elaborated in the next section.

Mixed H_2/H_∞ controller design

As said, another approach to limit the control action is to limit its energy. This can be done via an H_2 optimal controller design. In addition in order to impose robustness to uncertainties and in order to get a desired level of performance a simultaneous H_∞ controller design must be done. This will lead to the following formulation.

Problem 2. Design a controller C(s) to be used in a feedback loop to control a plant P(s) which belongs to the set Σ which satisfies the performance index $\left\|W_{perf}(s)S(s)\right\|_\infty \le 1$ whilst minimizing the energy of the control action u(t).

■

This problem can be changed in the same manner as we did for problem 1 to the following problem:

Problem 2a. Design a controller C(s) to be used in a feedback loop to control a plant $P_0(s)$ such that the following condition be met

$$\left\| \begin{matrix} W_{unc}(s)T(s) \\ W_{Perf}(s)S(s) \end{matrix} \right\|_\infty \le 1 \tag{6}$$

and simultaneously minimizes $||U(s)||_2$.

■

The above problem can be changed to a Linear Matrix Inequality [12] and can be solved numerically using the hinfmix function from the LMI toolbox of the MATLAB software. The performance of this method will also be tested via simulations in the next section.

Case study

In this section the effectiveness of the proposed methods is verified through simulations. To do this a single degree of freedom flexible joint manipulator has been considered and the weighting functions are determined in the following subsection.

A. *The plant and selection of the weights*

The 1DOF FJR considered is as shown in figure 1. Its dynamics can be found to be

$$\begin{cases} I\ddot{q}_1 = -mgL\sin q_1 - k(q_1 - q_2) - b_1\dot{q}_1 \\ J\ddot{q}_2 = k(q_1 - q_2) + u_{sat} - b_2\dot{q}_2 \end{cases} \tag{7}$$

Where g is the gravity acceleration and other parameters are defined in the figure. u_{sat} is the saturated input defined as

$$u_{sat}(u) = \begin{cases} -\delta & u \le -\delta \\ u & -\delta < u < \delta \\ \delta & \delta \le u \end{cases} \tag{8}$$

Numerical values are selected as *g=9.8, k=100* and all other values are selected to be 1 in order to be comparable with a benchmark problem in the literature [8, 14, 15, 16]. The values are considered to vary 10 percent. To determine a linear model for the system and simultaneously determine the uncertainty bound which encapsulates the nonlinear $\sin(q_1)$ term and variations of the parameters, the nonlinear system has been simulated 20 times with parameters (except than g) randomly varying. Then a linear model is identified for each set of input output data when $u(t)$ is considered as input and $q_1(t)$ as the output. Result is shown as bode plot in figure 2. The nominal model is selected such that it would be proper

$$P_0(s) = 4.2935\times 10^{-5}\,\frac{(s^2 + 2660s + 2329000)}{(s^2 + s + 4.78)\,(s^2 + s + 205)}$$

Now the uncertainty bound can be found from equation (3). (see figure 3) which is

$$W_{unc}(s) = 4.7261\times 10^{-7}\,\frac{(s+0.8)^2\,(s+900)^2}{(s+1.4)^2}$$

To select a proper performance weighting function W_{Perf}(s) note that it is an upper bound for the sensitivity function S(s). Thus we propose to select W_{Perf}(s) such that it is a little more than the desired sensitivity function $S_{des}(s)$. In this way we hope that the numerical design procedure would result a sensitivity function S(s) that is clipped from top to the $S_{des}(s)$ so it would be very similar to $S_{des}(s)$. This idea has been tested via several simulations and seems to be a very good starting point. The following performance weighting function has been selected in this way

$$W_{Perf}(s) = \frac{0.70423\,(s+26.15)\,(s+4.054)}{(s+30)\,(s+0.2)}$$

The weighting function on u(t) has been selected considering the facts that has been said about its effect on the amplitude of control action (see the second section of the paper) and it is

$$W_U(s) = \frac{0.0006\,(s+50)}{(s+300)}$$

In the next subsection the H_∞ mixed sensitivity controller (referred to as MixSen) and the H_2/H_∞ controller (referred to as H_2/H_∞) are designed and compared to each other and to a composite PD+PID controller (referred to as CPID) which has been widely used for this problem [8, 16].

B. Controllers and Simulation results

The H_∞ mixed sensitivity controller would have the following poles and zeros

$$P_{MixSen} = \{-7838.1, -908.5, -891.6, -12.6 \pm 42i, -54.6 \pm 19.9i, -30, -1.7, -1.3, -0.2\}$$

$$Z_{MixSen} = \{-903.48, -896.55, -300, -44.59, -0.5 \pm 14.31i, -0.5 \pm 2.13i, -1.4 \pm 0.00i\}$$

And its dc gain equals $C_{MixSen}(0) = 151.9237$.

The H_2/H_∞ controller would have the following poles and zeros

$$P_{H2/H\infty} = \{-50.49, -30.06, -0.02, -1.61, -1.27, -903.82 \pm 10.60i, -20.67 \pm 13.94i, -2.81 \pm 21.74i, -0.80 + 0.00i\}$$

$$Z_{H2/H\infty} = \{-909.91, -891.53, -302.14, -35.01, -0.50 \pm 14.31i, -0.50 \pm 2.13i, -1.40 \pm 0.00i, -0.80 \pm 0.01i\}$$

And its dc gain equals $C_{H2/H\infty}(0) = 1318.8$

The Bode plot of these controllers is shown in figure 4. Note that they are both stable and minimum phase. Tracking performance of the controllers is shown in figure 5. As seen, the CPID controller tracks very good (even it is not distinguishable from the reference input), tracking of the H_2/H_∞ controller is acceptable and that of the MixSen controller is poor. But this is not the end of everything. A look at figure 6 shows that the CPID has a huge control torque in the beginning (it increases up to 13500 which is clipped in the figure) and the MixSen produces a large and chattering control torque but that of the H_2/H_∞ is excellent. A much more confusing result is shown in figure 7 where the saturation limit, δ, is selected equal to 9. The H_2/H_∞ controller is the only one that will remain stable for this level of saturation. It will be stable even for $\delta = 1$. Note that the amplitude of the control action in steady state is also 1 and this means that the saturation is not a limitation for this case. To have a quantitative measure the following definition will be helpful:

Definition: a *minimum acceptable limit*, δ_{min} is defined as the saturation limit that preserves stability itself but a reduction of 5 percent in it will cause instability.

According to this definition the rows of table 1 have been filled for 4 various reference inputs. In this table the smoothed step reference input means

$$q_{des} = 1+5e^{-t}-6e^{(-t/1.2)}$$

for which the tracking performance is shown in figure 8. From table 1 one can deduce that the wider bandwidth of the reference input the larger saturation level may retain stability.

Conclusions

In this paper two approaches based on H_∞ optimization are presented to design controller: the mixed sensitivity approach and the H_2/H_∞ approach. These approaches are compared with a composite PID+PD approach which has been proposed in the literature. The proposed approaches are simpler in the sense that they need only link position to be feedback. In addition they can reduce (the MixSen approach) or entirely remove (the H_2/H_∞ approach) the instability caused by actuator saturation. The proposed controllers are numerically designed by solving an LMI problem and the performance of them is tested by simulation. Simulations show that the H_2/H_∞ method can entirely remove the instabilities due to control action amplitude limitations.

Tables and Figures

TABLE 1
MINIMUM ACCEPTABLE LIMIT FOR DIFFERENT CASES

Input	Steady State Amplitude	δ_{min} for Composite PID + PD	δ_{min} for H_∞ Mixed Sensitivity	δ_{min} for Mixed H_2/H_∞
Step	8.5	370	55	-
0.1 Sin(4t)	1	42	24	2.5
0.1 Sin(t)	1	11	10	-
Smoothed Step	8.5	-	25	-

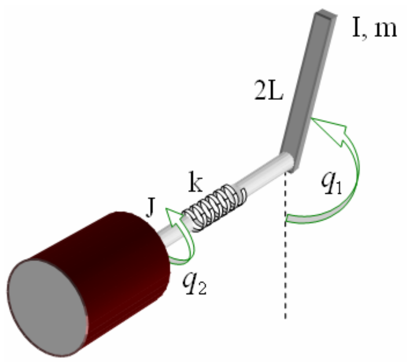

Figure 1: The single degree of freedom FJR

Note: All bode plots are db mag vs frequency in Rad/Sec and for time plots the time is in Seconds.

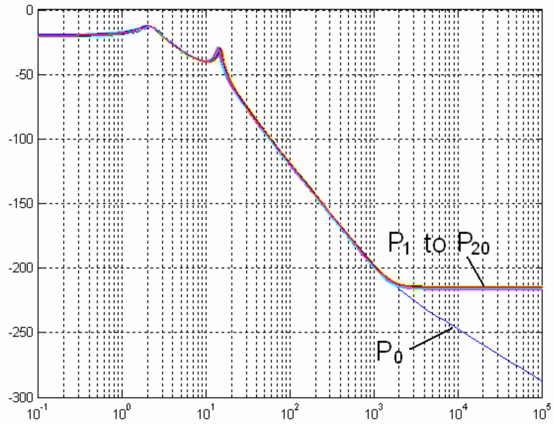

Figure 2: Identified models (P_1 to P_{20}) and selected proper nominal model (P_0).

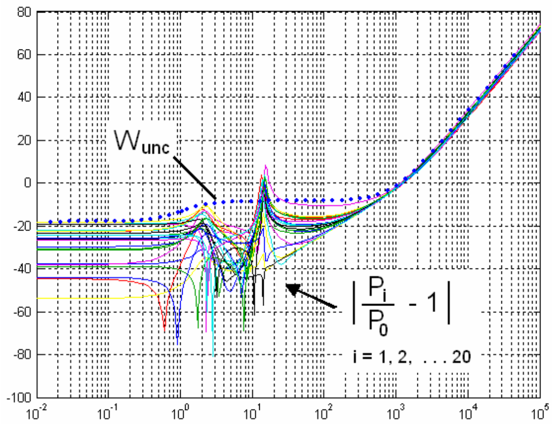

Figure 3: Determination of W_{unc}

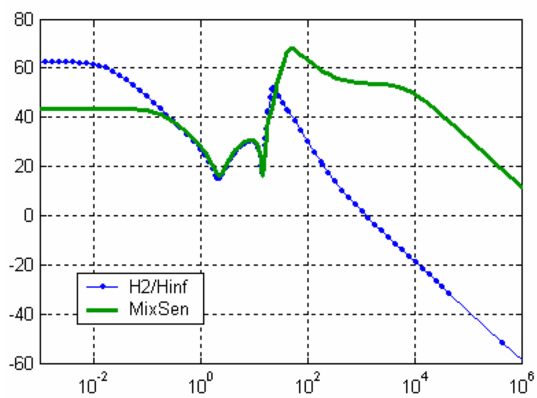

Figure 4: Bode plots of the two controllers

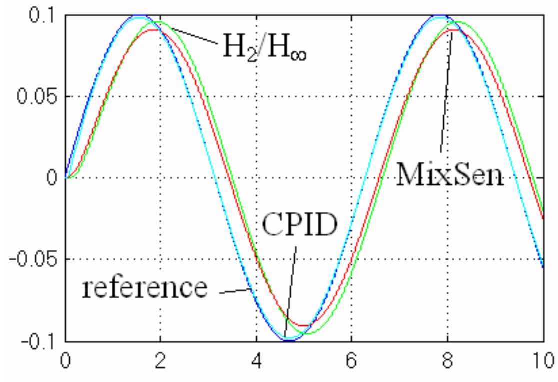

Figure 5: Tracking for Sine reference, δ= 12

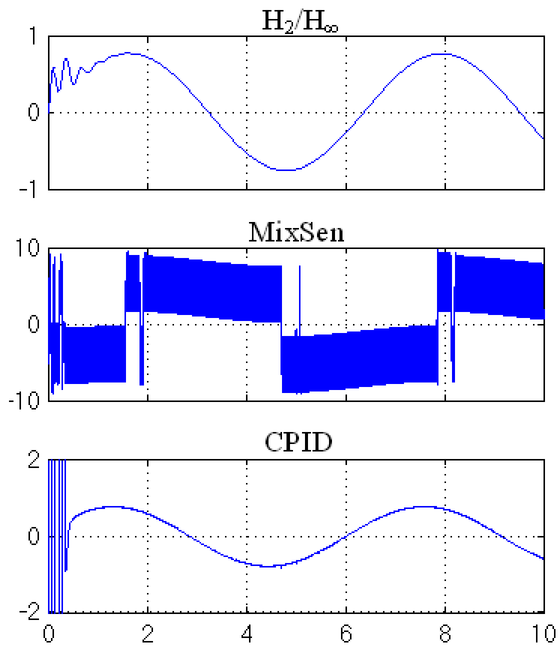

Figure 6: Control action for Sine reference, δ= 12

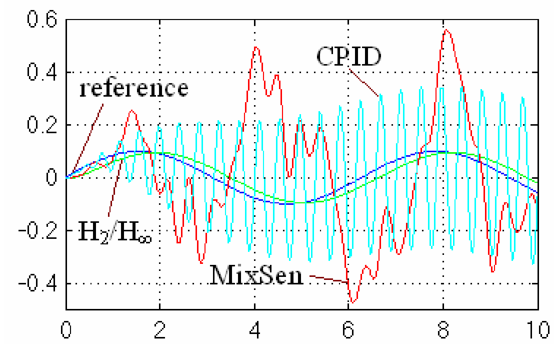

Figure 7: Instability for Sine reference for δ= 9

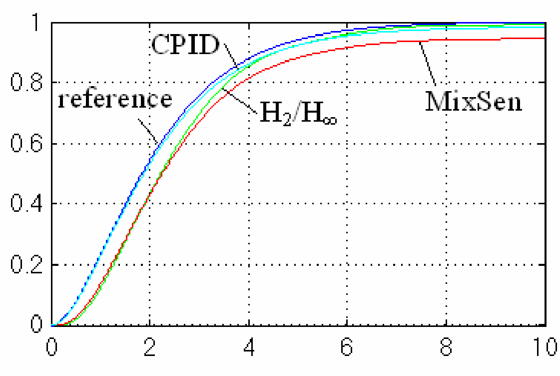

Figure 8: Tracking, Smoothed Step, $\delta = 12$

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