

Designing and Implementation of Mixed H_2/H_∞ Controller for Flexible Joint Robot to Encounter Actuator Saturation

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Abstract: *Mixed H_2/H_∞ method is proposed to design and implement a controller for flexible joint robots considering actuator saturation by considering the control action in the mixed sensitivity. A more advanced method based on frequency weighting of the control action contribution in the mixed sensitivity function is considered here which may result higher bandwidth. But this method is also limited and to get better specifications the H_2/H_∞ method is proposed. This methods are shown to be very good remedies for problems caused by actuator saturation in practice. Experimental studies are forwarded to verify the effectiveness and the performance of the proposed controllers in practice.*

Keywords: Flexible Joint Robot, Actuator Saturation, Mixed H_2/H_∞ , Mixed Sensitivity.

1. Introduction

Multiple-axis robot manipulators are widely used in industrial and space applications. The success in reaching high accuracy in these robots is due to their rigidity, which make them highly controllable. After the inception of harmonic drive in 1955, and its wide acceptance, and use in the design of many electrically driven robots, the rigidity of the robot manipulators are greatly affected. In early eighties researchers showed that the use of control algorithms developed based on rigid robot dynamics on real non-rigid robots is very limited and may

even cause instability [1].

Several new applications such as space manipulators [2] and articulated hands [3] necessitate using flexible joint robots (FJR). This necessity has emerged new control strategies required, because the traditional controllers implemented on FJRs have failed in performance [4]. Since 1980's many attempts have been made to encounter this problem and several methods have been proposed including various linear, nonlinear, robust, adaptive and intelligent controllers [5, 6]. Among these, only a few researchers have considered practical limitations such as actuator saturation in the controller synthesis, as a real drawback to achieve good performance [7].

On the other hand actuator saturation has been considered by the control community from early achievements of control engineering and over the last decade researchers have shown a new interest in the study of the effects of saturation on the performance of the systems. In fact in the past, researchers were encountered a *drawback* identified as actuator saturation and they developed methods to avoid it, while now, researchers develop methods to achieve a desirable performance in the presence of actuator saturation encountered as a *limitation*. Saturation may cause two types of performance degradations. First, it may cause inevitable limitations such as slow responses, undesirable transitions, etc. Second, it may turn into removable problems such as instability, undesired steady state performance, etc. The goal in considering saturation in controller synthesis is to decrease or to remove the latter, especially, to remove the instability.

A few papers have considered this matter in the controller synthesis of FJR's. In an earlier paper [8] the authors proposed a fuzzy supervisory control to encounter this problem and in their next work [9], they have used the fuzzy supervisory loop with a composite PID+PD controller for FJR. The robust methods used in this present paper are simpler than that method in structure, and moreover, need only the link position to be fed back. Use of this approach for reduction of the control action for FJRs has been first proposed in [10].

In this paper using a frequency weighted penalty function of the control action is recommended in the mixed sensitivity minimization. Furthermore, in order to decrease the amplitude of the control action, while keeping the desired bandwidth, a mixed H_2/H_∞ minimization approach is proposed, whose solution is obtained using LMI. Simulation studies have been verified the superior performance of this method compared to that in the composite PID+PD controller, and H_∞ mixed sensitivity controller [10]. In the present paper this approach is verified in practice. This paper is organized as follows: Section 2 presents the mixed sensitivity optimization and Section 3 is devoted to description of the mixed H_2/H_∞ approach. Sections 4 and 5 are allocated for the experimental studies, and finally the conclusions are presented in Section 6.

2. Mixed Sensitivity Approach

In an optimal H_∞ design procedure the controller is designed to meet an upper bound on the infinity norm of the (weighted) system output. This can be employed to limit the control action in a mixed sensitivity formulation in which the amplitude of control is considered in the vector to be limited. The problem formulation is as follows. Suppose that the plant model belongs to a family of models Σ as follows:

$$\Sigma = \{P(j\omega) | P(j\omega) = P_0(j\omega)[1 + \Delta(j\omega)W_{unc}(j\omega)]\} \quad (1)$$

Where $P_0(j\omega)$ is the nominal transfer function of the plant and $\Delta(j\omega)W_{unc}(j\omega)$ encapsulates all perturbations of the real plant from its nominal model in which $\|\Delta(s)\|_\infty < 1$. These perturbations may come from nonlinearities, unmodeled dynamics, changes of parameters and operating points, etc. Nominal plant P_0 can be evaluated experimentally through a series of frequency response estimates of the system in the operating regime and then to find the best fit to the average of these models. Linear

identification for the system can be applied with different parameter values in different operating points to estimate a set of linear models which can be considered as Σ . Now from equation (1) we have

$$\frac{P(j\omega)}{P_0(j\omega)} - 1 = \Delta(j\omega)W_{unc}(j\omega) \quad (2)$$

and as $\|\Delta(s)\|_\infty < 1$ this yields

$$\left| \frac{P(j\omega)}{P_0(j\omega)} - 1 \right| \leq |W_{unc}(j\omega)|, \quad \forall \omega \quad (3)$$

The uncertainty bound W_{unc} can be obtained from the results of identification by finding a curve to satisfy the above equation in each frequency. Now the problem can be formulated as follows:

Problem 1. Design a controller $C(s)$ to be used in a feedback loop to control a plant $P(s)$ which belongs to the set Σ which satisfies the performance index $\|W_{perf}(s)S(s)\|_\infty \leq 1$ whilst the control action $u(t)$ remains limited such that $\|W_U(s)U(s)\|_\infty \leq 1$. ■

Where $S(s)$ denotes the sensitivity function defined as $S(s) = [1 + P(s)C(s)]^{-1}$.

Note that the robustness condition (or the fact that the plant model belongs to the set Σ) can be met by satisfaction of the following condition [11]

$$\|W_{unc}(s)T(s)\|_\infty \leq 1 \quad (4)$$

Where $T(s)$ denotes the complementary sensitivity function which is defined as $T(s) = 1 - S(s)$. So the above problem can be changed to the following problem [11].

Problem 1a. Design a controller $C(s)$ to be used in a feedback loop to control a plant $P_0(s)$ such that the following condition be met

$$\left\| \begin{array}{l} W_{unc}(s)T(s) \\ W_{perf}(s)S(s) \\ W_U(s)U(s) \end{array} \right\|_\infty \leq 1 \quad (5)$$

■

This problem can be solved numerically using the hinfmi function from the LMI toolbox of the MATLAB [12].

In this manner we can use the weight function $W_U(s)$ to manipulate the infinity norm of $U(s)$ that affects uniformly the amplitude of $u(t)$; So this method can be used to decrease the needed saturation level. The effectiveness of this method for FJRs has been shown for the first time in [10].

Note that the function $W_U(s)$ can be chosen simply

to be a constant level as $W_U(s) = \Omega$ which may cause $\|U(s)\|_\infty < 1/\Omega$. This selection may limit the magnitude of $U(s)$ in all frequencies which may limit the resultant bandwidth. As an alternative we can shape $W_U(s)$ in the frequency domain. Obviously reducing the function $W_U(s)$ in high frequencies will result in decreasing the level of the control action fast transients (or jumps) and reducing its integral in frequency domain will result in decreasing the total amplitude of $u(t)$ during all times. This fact can also be seen empirically in simulation studies.

Instead of frequency shaping of the $W_U(s)$ one can limit the 2 norm of $U(s)$. According to the parseval's relation [13] this limits the 2 norm or the energy of $u(t)$. This idea will be elaborated in the next section.

3. Mixed H_2/H_∞ controller design

As said, another approach to limit the control action is to limit its energy. This can be done via an H_2 optimal controller design. In addition in order to impose robustness to uncertainties and in order to get a desired level of performance a simultaneous H_∞ controller design must be done. This will lead to the following formulation.

Problem 2. Design a controller $C(s)$ to be used in a feedback loop to control a plant $P(s)$ which belongs to the set Σ which satisfies the performance index $\|W_{perf}(s)S(s)\|_\infty \leq 1$ whilst minimizing the energy of the control action $u(t)$.

■

This problem can be changed in the same manner as we did for problem 1 to the following problem:

Problem 2a. Design a controller $C(s)$ to be used in a feedback loop to control a plant $P_0(s)$ such that the following condition be met

$$\frac{\|W_{unc}(s)T(s)\|}{\|W_{perf}(s)S(s)\|_\infty} \leq 1 \quad (6)$$

and simultaneously minimizes $\|U(s)\|_2$.

■

The above problem can be changed to a Linear Matrix Inequality [12] and can be solved numerically using the `hinfmix` function from the LMI toolbox of the MATLAB. The performance of this method will be tested via experiments in the next section.

4. The laboratory setup and experimental problem formulation

The laboratory set up which has been considered for experimental study is shown in Figure 1. It is a 2 DOF flexible joint manipulator. In the first joint a harmonic drive is used for power transmission. Its spring constant is empirically derived to be 6340 N.m/rad [14].

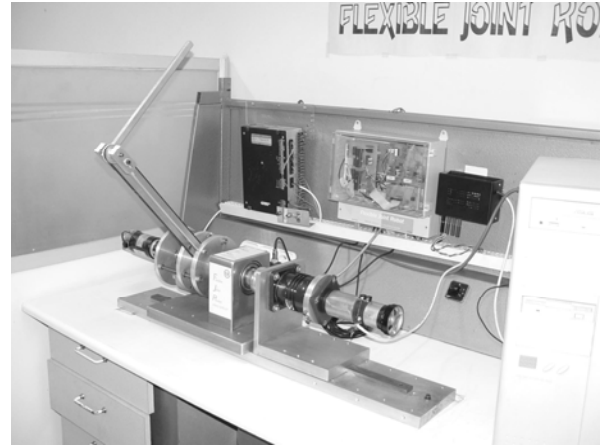


Figure 1: Experimental Setup

The flexible element used in power transmission system of the second joint is shown in Figure 2. It has been made from Polyurethane and is designed so that it has very high flexibility. Its equivalent spring constant is 8.5 N.m/rad which is a value that makes a challenging control problem. To show the effectiveness of the proposed algorithm in presence of actuator saturation despite low stiffness, the experimental results on the second joint are considered. Specifications of the second motor have been shown in Table I.

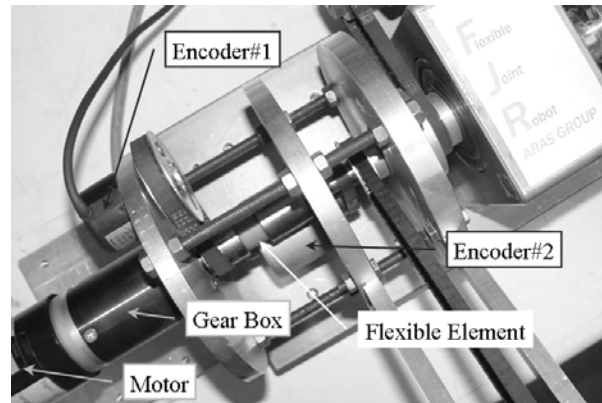


Figure 2: The flexible element

Table I: Second Motor specifications

Continuous Torque (N.m)	13
Max. Rated Input (V DC)	12
Max. Continuous Power (W)	62
Rated Speed (rpm)	26

In order to control the system by means of a PC, a PCL-818 I/O card and a PCL-833 encoder handling card from the Advantech Company are used for hardware interfacing. The ‘‘Real Time Workshop’’ facility of the MATLAB SIMULINK is used for user interface. A block diagram of the system is shown in Figure 3.

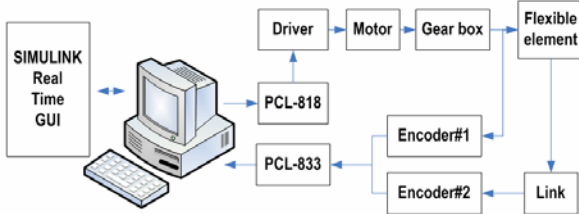


Figure 3: Block diagram of the system

In order to determine a linear model for the system and simultaneously determine the uncertainty bound which encapsulates the nonlinear terms and variations of the parameters, the real system has been operated for several times with different inputs and data has been logged. Then least-square estimation method (the system identification toolbox of the MATLAB) has been used to find the frequency response estimates. Figure 4 and Figure 5 illustrate some frequency response estimates of the system and their relating uncertainty profile. The nominal model of the system P_o is determined by averaging the estimated models, which is as follows:

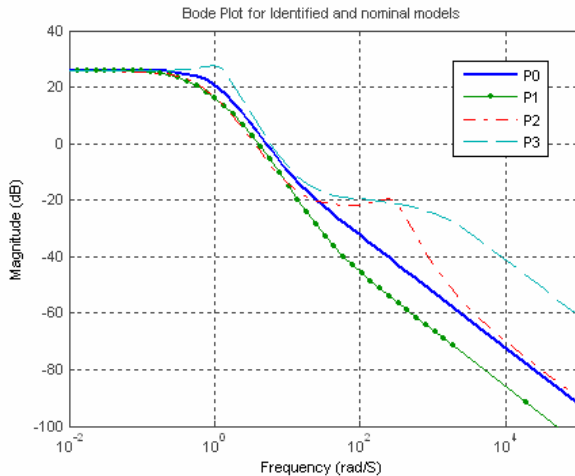


Figure 4: Identified models (P_1 to P_3) and selected proper nominal model (P_0).

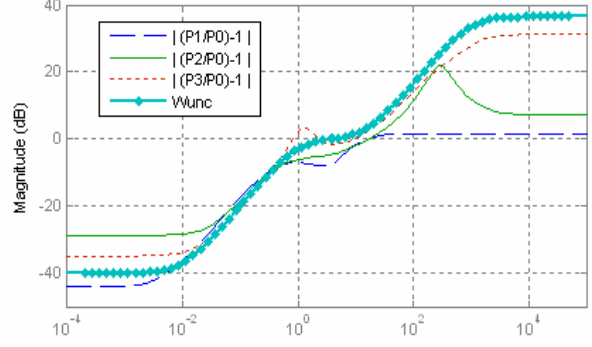


Figure 5: Determination of uncertainty weighting function, $W_{unc}(s)$

$$P_0(s) = 2.4 \frac{(s + 10)}{(s + 1.1)^2} \quad (8)$$

The uncertainty profile is estimated as:

$$W_{unc}(s) = 67 \frac{(s + 0.01)(s + 15)}{(s + 1)(s + 1000)} \quad (9)$$

The sensitivity weighting function $W_{perf}(s)$ is determined such that a desirable tracking error performance in frequency domain is obtained, while a solution to the mixed sensitivity problem be existed. In order to accomplish this, first a desired sensitivity function $S_{des}(s)$, is nominated for the system and the performance weighting function $W_{perf}(s)$ is evaluated as its inverse in the frequency domain. This systematic approach can be tuned further, taking into account the obtained performance characteristics in time domain. Through several iterations the following performance weighting function is nominated for the optimization problem.

$$W_{perf}(s) = \frac{0.7(s^2 + 0.2s + 9)}{(s + 0.1)^2} \quad (10)$$

This selection indicates that the steady state error to a unit step is smaller than 1%, while the designed bandwidth is about 0.1 rad/sec.

Finally, the weighting function on $u(t)$ has been selected as following:

$$W_U(s) = \frac{0.025(s + 100)}{(s + 1)} \quad (11)$$

In the next subsection the H_∞ mixed sensitivity controller, referred to as ‘‘MixSen’’, and the H_2/H_∞ controller, referred to as ‘‘ H_2/H_∞ ’’ are designed and their closed loop performance is compared to that of a PD controller. The PD controller is designed to have good tracking while control effort not to be very large.

5. Designing controllers and experimental results

Numerical solution of the H_∞ mixed sensitivity problem (Problem 1) by means of LMI toolbox of the MATLAB would result the H_∞ controller with following poles and zeros:

$$P_{\text{MixSen}} = \{-36000, -4300, -14, -2.2, -0.1, -0.1, -0.01\}$$

$$Z_{\text{MixSen}} = \{-1000, -1.9, -1.1, -1.1, -1.0, -0.01\}$$

which its DC gain is $C_{\text{MixSen}}(0) = 32$.

The mixed H_2/H_∞ controller has been also designed as the numerical solution of Problem 2, which has the following poles and zeros:

$$P_{H_2/H_\infty} = \{-15, -10, -2.7, -0.1, -0.08, -0.01, -8200 \pm 4800i\}$$

$$Z_{H_2/H_\infty} = \{-1000, -15, -1.9, -1.2, -0.02, -1.0 \pm 0.12i\}$$

and its DC gain equals $C_{H_2/H_\infty}(0) = 77$.

The Bode plots of these controllers are shown in Figure 6. Note that they are both stable and minimum phase.

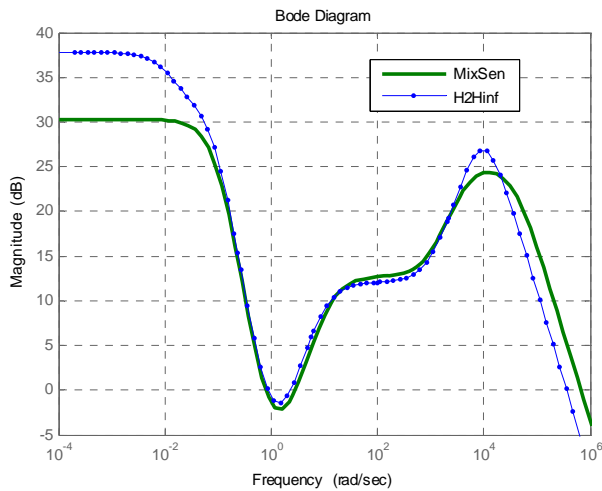


Figure 6: Bode plots of the two controllers

The mid range frequency response of the two controllers resemble, which guarantees the robustness requirement of the solution in this region where the uncertainty peaks occur. At low frequencies, H_2/H_∞ solution provides much higher DC gain, and hence, a better disturbance attenuation, and steady state tracking performance is expected. The high frequency response of H_2/H_∞ controller is also better than that of the mixed sensitivity controller but in practice this will not affect because the high frequency poles could not be implemented.

Two samples of experimental results are shown in the following figures. Upper part of Figure 7 shows the result for tracking a sine wave with frequency 3.75 rad/s. This result shows that the tracking

performance of all three controllers (PD, “MixSen” and H_2/H_∞) is the same. The lower part of this figure shows the result for the same reference input when the saturation level is lowered to the 30% of its normal value. It can be seen that the PD output is much more affected. This shows the effectiveness of using the proposed methods in practice when there would be serious limitation on control amplitude.

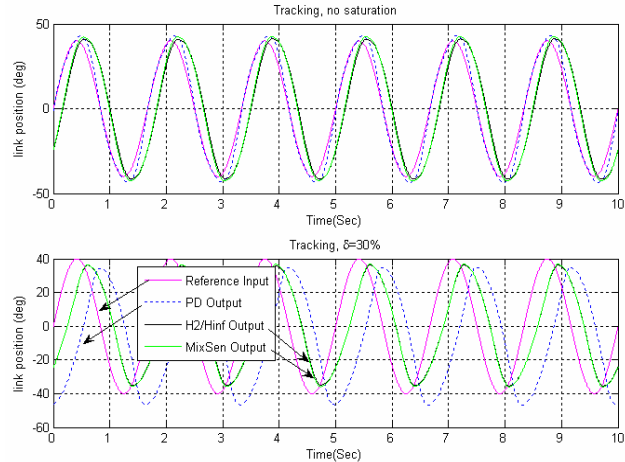


Figure 7: Tracking for sine reference ($40\sin(3.75t)$)

At Figure 8 a faster reference input is considered. It can be seen that when the saturation level is adjusted to 70% of its maximum (upper part of the figure) tracking performance of the mixed sensitivity and H_2/H_∞ controllers is the same and is much better than that of the PD controller. Moreover if the saturation limit is decreased to 50% of its maximum (see the lower part of the figure, this is done at $t = 2$ Sec) the H_∞ optimal controllers will continue tracking (although tracking is bad) but the PD controller will lost the reference input.

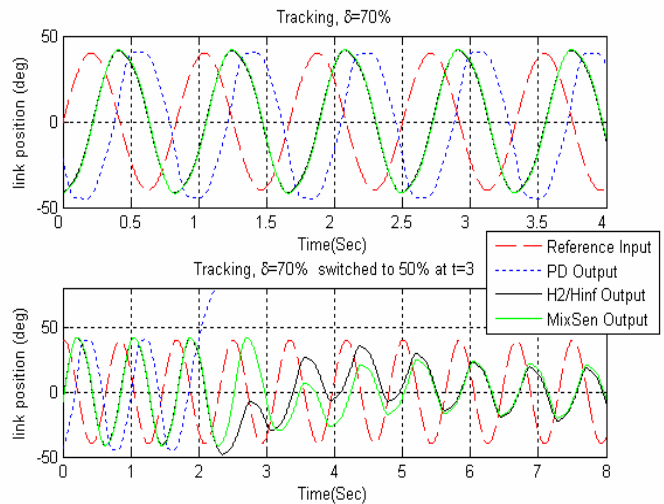


Figure 8: Tracking for sine reference ($40\sin(7.5t)$)

These experimental observations confirm the theoretical expectations and the simulation results given in the previous work [10] which insures the effectiveness of using multipurpose optimization methods to encounter the problems caused by actuator saturation. Therefore, in susceptible applications such as space robotics, where robust stability of the closed loop systems in presence of actuator limitations is an essential requirement, these methods seem to be more appropriate.

6. Conclusions

In this paper the problem of design and implementation of a controller for flexible joint robots in presence of actuator saturation is considered. Two approaches based on H_∞ optimization are presented to design controller: the mixed sensitivity approach and the H_2/H_∞ approach. These approaches in comparison with most approaches presented in the literature are simpler in the sense that they need only link position to be fed back. In addition they can reduce the amplitude of control effort hence they are proper to encounter the problem of actuator saturation. The proposed controllers are numerically designed by solving an LMI problem and the performance of them is tested by experiment. Results show that these methods can be used well for controlling FJR's when there is a control action amplitude limitation.

References

- [1] Cesareo G., Marino R., "On the controllability properties of elastic robots", in *Proc. Int. Conf. Analysis and Optimization of Systems*, 1984
- [2] Machida K., Nishida H., Akita K., "Precise telerobotic system for space experiment on ETS-VII", 1998
- [3] Schaffer A. A., Hirzinger G., "State Feedback Controller for FJR : A Globally Stable Approach Implemented on DLR's Light Weight Robots", in *Proc. IEEE/RSJ Conf. on Intelligent Robots and Systems*, 2000
- [4] Sweet L.M., Good M.C., "Re-definition of the robot motion control problems: Effects of plant dynamics, drive system constraints, and user requirements", in *Proc. IEEE Int. Conf. on Decision and Control*, 1984
- [5] Spong M.W., "The control of FJR's: A survey", in *New Trends and Applications of Distributed Parameter control systems*, G.Chen, E.B.Lee, W.Littman, L.Markus, 1990
- [6] Ozgoli S., Taghirad H.D., "A survey on the control of flexible joint robots", *Asian Journal of Control*, Vol. 8, No. 4, 2006 (in Press).
- [7] Ozgoli S., "Position control for flexible joint robots in presence of actuator saturation", PhD Dissertation, K.N.Toosi University, 2005.
- [8] Ozgoli S., Taghirad H.D., "Fuzzy supervisory loop as a remedy for actuator saturation drawbacks", in *Proc. 16th Int. Conf. System Eng.*, UK, 2003.
- [9] Ozgoli S., Taghirad H.D., "Design of Composite Control for Flexible Joint Robots with Saturating actuators", in *Proc. IEEE Conf on Mechatronics and Robotics*, Germany, 2004.
- [10] Ozgoli S., Taghirad H.D., "Designing H_2/H_∞ Controller for FJR to Remove Instabilities due to Saturation", in *Proc. 13th Int. Mechanical Engineering Conference*, Isfahan, Iran, May 2005.
- [11] Zhou K., Doyle J.C., *Essentials of robust control*, Prentice Hall, 1998.
- [12] Gahinet P., Nemirovski A., Laub A.J., Chilali M., *LMI Control Toolbox User's Guide*, The mathworks, Inc., 1995.
- [13] Oppenheim A.V., Willsky A.S., *Signals and Systems*, Prentice-Hall, 1983.
- [14] H.D. Taghirad and P.R. Belanger, "Modeling and parameter identification of harmonic drive systems", *Journal of Dynamic Systems, Measurements, and Control*, Vol. 120, No. 4, pp 439-444, ASME Pub., Dec. 1998.