

## H-Infinity Controller Design for A Flexible Joint Robot With Phase Uncertainty

Sh. Dadashi<sup>†</sup>, H. D. Taghirad<sup>††</sup>

Advanced Robotics and Automated Systems (ARAS)

Department of Electrical Engineering

K.N. Toosi University of Technology

<sup>†</sup>Sh.dadashi@eece.kntu.ac.ir

<sup>††</sup>Taghirad@kntu.ac.ir

**Abstract** –In this paper, the design and implementation of the  $H_\infty$  controller for flexible joint robot (FJR) is presented and the capability of the controller to deal with actuator saturation is investigated in practice. The new procedure of design is introduced to avoid an instability caused by unmodeled phase behaviour which can not be encapsulated in multiplicative uncertainty. In order to avoid instability caused by unmodeled phase behaviour, the robust controller design is divided into two stages:  $H_\infty$  controller design and checking closed loop sensitivity function. Simulation results reveal the capability of the controller to stabilize the closed loop system and to reduce the tracking error in the presence of the actuator limitation.

acceleration and jerk feedback is required whose measurement is very costly. To avoid the acceleration and jerk feedback the idea of composite control is employed [3]. In adaptive methods many algorithms are developed for FJR's, in most of which a term due to the fast subsystem is added to the adaptive algorithm based on rigid models [3]. In robust methods considering model uncertainties the stability of fast subsystem is first analyzed and, by the use of robust control synthesis, a robust controller is designed for the slow subsystem [4,5].

### I. INTRODUCTION

Joint flexibility is one of the main reasons of robotic systems complexities. In practical implementations, in order to achieve better tracking performance, joint flexibility must be taken into account in both modeling and control [1]. However, joint flexibility and nonlinearity, in addition to the increasing complexity of robot modeling, is a potential factor of system uncertainty that can affect favorable features of system and even in some case, leads to instability. Due to existing joint flexibility, actuators position (for angle of motors' shaft) does not depend directly on the driven arms position. This is not favorable for applications with high precision. Moreover unwanted oscillations due to joint flexibility, imposes bandwidth limitation on all algorithms designed based on rigid robots and may create stability problems for feedback controls that neglect joint flexibility. In addition, the actuator saturation has been considered by the designers as an important practical limitation. Over the last decade, the control research community has shown a new interest in the study of the effects of saturation on the performance of the closed loop system [2, 3]. The saturation can deteriorate the performance of the closed loop system by causing some limitations such as slow responses, undesirable transitions, or even it can cause instability. Most researches on FJR's have concentrated on nonlinear control schemes. In order to linearize the system,

The simulation results, has been illustrated the capability of the robust linear controller [6], the  $H_\infty$  controller and the composite  $H_\infty$  controller [7], and the composite QFT controller [8] to control the motion of FJR's. And the anti-saturation strategy is proposed for practical issues [2]. The special flexibility added to KNTU FJR's joint makes the control problem more challenging than the usual flexibility caused by applying harmonic drives. In addition, due to added joint flexibility, the precise structure of the system is unknown and we respect various peaks in model of the system. This makes it suitable to be identified by nonparametric methods and be controlled by  $H_\infty$  controller. Due to this high flexibility and complicated dynamical behavior of the joint, the characteristic of the phase behavior is impossible to model precisely while it can not be encapsulated in uncertainty bound to be applied in  $H_\infty$  controller design. Therefore, a two stage procedure is proposed to encounter this type of uncertainty.

### II. EXPERIMENTAL SETUP: KNTU FJR

The laboratory set up, which is a 2 DOF flexible joint manipulator and has been used for implementation, is shown in figure 1. The flexible element used in power transmission system of the 2<sup>nd</sup> joint, shown in figure 2, is made of polyurethane which has a very high flexible characteristic. The equivalent spring constant, 8.5 N.m/rad makes a challenging control problem. In order to control the system by means of a

PC, a PCL-818 I/O card and a PCL-833 encoder handling card of the ADVANTECH Company, are used for hardware interfacing. The “Real Time Windows Target” facilities of MATLAB SIMULINK are used as user interface. The “Real Time Windows Target” is a PC solution for prototyping and testing real time systems. It can be used when a single computer is as a host and target. The sampling time of the blocks in RTW is regulated as 0.01 sec. The block diagram of the system is shown in figure 3.

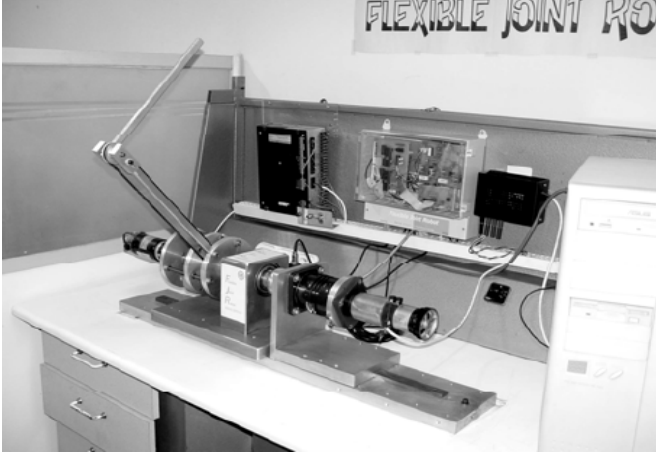


Figure 1: *Experimental Setup: KNTU FJR.*

Similar to any other practical systems, we have a limitation on the amplitude of control signal. The hardware implementation of the controller and using I/O cards, imposes the bounds on  $u(t)$ , that means  $u(t)$  should be in the range of  $[-128 \ 128]$ . To avoid the control signal being out of this range, we used a saturation block in RTW interface.

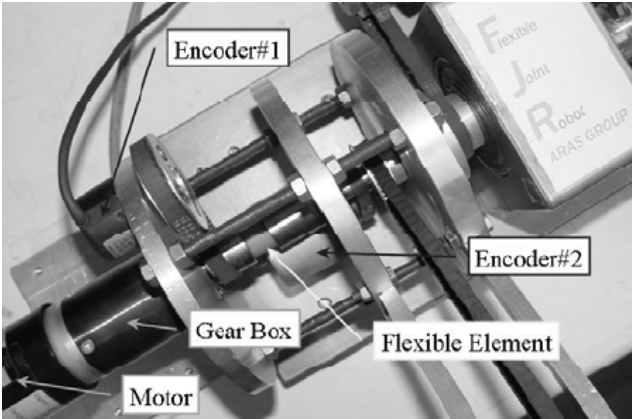


Figure 2: *The Flexible Element*

### III. CONTROLLER DESIGN

#### A. $H_\infty$ SYNTHESIS

Since the required objectives of robust stability, fast and suitable tracking response and disturbance attenuation despite the limited control effort, are well suited into an  $H_\infty$  design framework, in this section the FJR controller design is reformulated such that this methodology can be applied. In order to apply  $H_\infty$  synthesis to this problem, the nonlinear model of FJR is represented by a linear model and the multiplicative uncertainty, using spectral identification scheme. In this representation the nominal model replicated the dynamic behavior of the system, only at nominal conditions, and all nonlinear interactions, unmodeled dynamics and disturbances are encapsulated via an unstructured uncertainty representation [9].

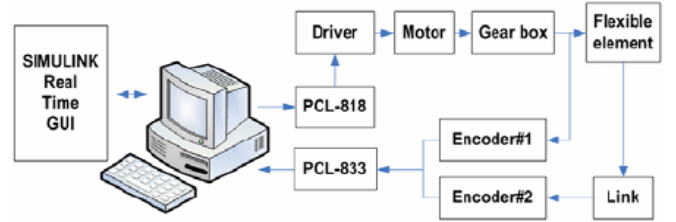


Figure 3: *Block Diagram of the System*

In order to represent a system into this form, suppose the true system belongs to a family of plants  $\Pi$ , which is defined by using the following perturbation to the nominal plant  $P_o$ :

$$\forall P(s) \in \Pi \quad P(s) = (1 + \Delta(s)W(s))P_o(s) \quad (1)$$

In this equation  $W(s)$  is a stable transfer function indicating the upper bound of uncertainty and  $\Delta(s)$  indicates the admissible uncertainty block, which is a stable but unknown transfer function with  $\|\Delta\|_\infty < 1$ . In this general representation  $\Delta(s)W(s)$  describes the normalized perturbation of the true plant from nominal plant, and is quantitatively determined through identification at each frequency:

$$\frac{P(j\omega)}{P_o(j\omega)} - 1 = \Delta(j\omega)W(j\omega) \quad (2)$$

In which  $\|\Delta\|_\infty < 1$ ; hence,

$$\left| \frac{P(j\omega)}{P_o(j\omega)} - 1 \right| \leq |W(j\omega)|, \quad \forall \omega \quad (3)$$

Where,  $|W(j\omega)|$  represents the amplitude of the uncertainty profile with respect to frequency. Nominal plant  $P_o$ , can be evaluated experimentally, through a series of frequency response estimates of the system in the operating regime [9].

The objectives of controller design are robust stability and good tracking performance in presence of torque disturbances, despite the *limited control effort*. All these objectives can be simultaneously optimized by the solution of a mixed sensitivity problem formulated on the generalized plant illustrated in figure 4. The robust stability is guaranteed by minimizing the infinity norm of weighted transfer function from  $y_d$  to  $z_1$ , which is equivalent to the weighted

complementary sensitivity function:  $\|WT\|_{\infty} < 1$  (small gain theorem) [10]. The tracking performance and disturbance attenuation is obtained by minimizing the infinity norm of  $Y_d$  to  $Z_2$ , or the weighted sensitivity function  $\|W_s S\|_{\infty} < 1$ .

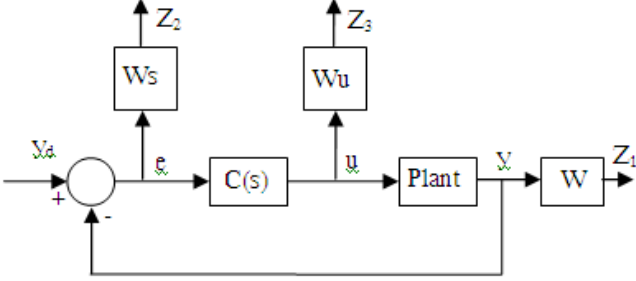


Figure 4: Block diagram representation of mixed sensitivity solution for the system.

## B. $H_{\infty}$ CONTROLLER DESIGN FOR FJR

In a linear system, different frequencies pass through the system independently of each other. Hence we can use the following equation to calculate the estimate of the transfer function:

$$\hat{G}_N(e^{j\omega}) = \frac{Y_N(\omega)}{U_N(\omega)} \quad (4)$$

The estimate of (4) is called the empirical transfer function estimate (ETF). Due to poor variance properties of the ETF, we assume that the values of the true transfer function are related. So the best way to estimate the transfer function by ETF is to form a weighted average of measurements. Each measurement is weighted according to its inverse variance,  $\alpha(\zeta)$ . If the transfer function is not constant over the interval  $\omega_0 - \Delta\omega \leq \omega \leq \omega_0 + \Delta\omega$ , it is reasonable to use an additional weighting that pays more attention to frequencies close to  $\omega_0$ .

$$\hat{G}_N(e^{j\omega_0}) = \frac{\int_{\xi=\omega_0-\Delta\omega}^{\omega_0+\Delta\omega} W_{\gamma}(\xi-\omega_0) |U_N(\xi)|^2 \hat{G}_N(e^{j\xi}) d\xi}{\int_{\xi=\omega_0-\Delta\omega}^{\omega_0+\Delta\omega} W_{\gamma}(\xi-\omega_0) |U_N(\xi)|^2 d\xi} \quad (5)$$

Here, the  $W_{\gamma}(\omega)$  is a weighting function concentrated around  $\zeta=0$  and  $\gamma$  is a shape parameter. There are some standard shapes of weighting functions in literature [11]. In spectral analysis,  $W_{\gamma}(\omega)$  is often called frequency window. The amount of  $\gamma$  describes the length of the frequency window, so a large value of  $\gamma$  corresponds to a narrow window. The "wide" window leads to a small variance of estimate. At the same time, it will involve frequency estimates farther away from  $\omega_0$ . This will cause large bias [11]. This procedure is used to identify the family of plants we need in  $H_{\infty}$  control. The amount  $\gamma$  is called frequency resolution in Matlab. This technique is implemented through the Matlab function 'spa' to achieve a trustful result in KNTU FJR's 2<sup>nd</sup> link identification by selecting spectral model estimation in system identification tool of Matlab and regulate the 'Frequency Resolution' by an experimental procedure.

There is a trade-off between the bias and the variance of the estimate which is controlled by the frequency resolution. In other words, the trade-off is between the frequency resolution and the uncertainty of the estimate: the better the resolution, the more uncertain the system. By using this technique, from the set of input-output information, a set of linear models are estimated for the system, which can be considered as the set  $\Pi$ . Figure 5 illustrates some frequency response estimates of the system, chirp functions with different amplitudes and frequencies are used as input.

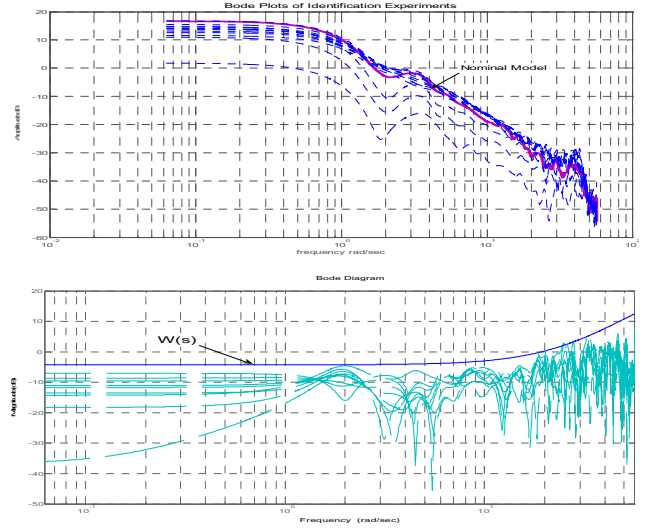


Figure 5: Frequency response estimate; Nominal plant and uncertainty profile.

The nominal model of the system is determined from average of the estimated models. Our aim is to achieve an unstructured uncertainty bound. The uncertainty weighting function is estimated as  $W(s) = 29(s+29)^3/(s+100)^3$ . Relatively small uncertainty at low frequencies is promising a suitable  $H_{\infty}$  controller design, but sharp increase of the uncertainty at  $\omega=29$  rad/sec warns about the limitations on achievable closed loop bandwidth for this system.

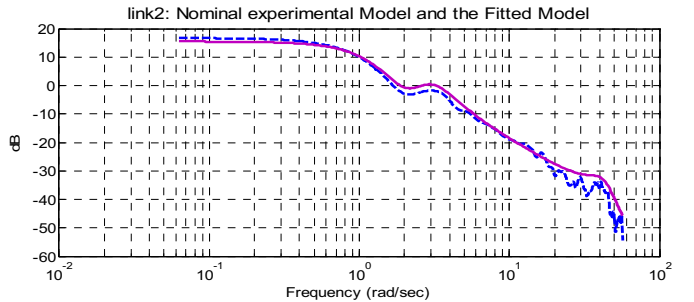


Figure 6: Nominal Model and the Fitted Model.

The next step is to adapt a linear model to this nominal model. The final  $P_o$ , after numerous iterations is illustrated in figure 6.

$$P_o = \frac{28000 (s+6)^2 (s+5) (s^2 + 1.2s + 4.36)}{(s+1.5) (s+10) (s+11) (s^2 + 1.8s + 1.3) (s^2 + 2s + 9.41) (s^2 + 16s + 1828)}$$

After designing some controllers using different performance weighting functions, we figured out that some shapes of  $W_s$

lead to stabilizing controllers whereas the others make the closed loop system unstable. However based on simulations, we expected the closed loop behaviour of all those cases to be the same.

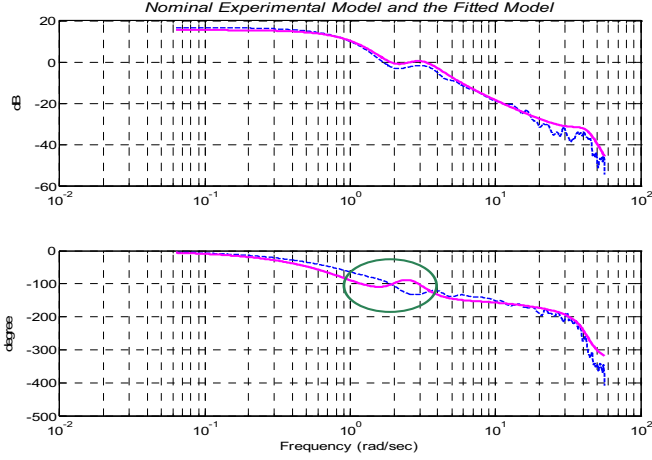


Figure 7: Difference between phase behaviour between Nominal Model and Fitted Model.

To analyse this problem, let us reconsider the nominal model and the fitted model. As it can be seen in figure 7, we can achieve an admissible estimation for amplitude which is not the case for phase. Specially in the frequency range [1~ 4] rad/s, which is depicted in figure 8, the fitted model differs from the true system.

Considering the point that this specific frequency is located at the middle of system's usual frequency range of work, we expect that the uncertainty in phase of nominal model has significant effects on the stability and performance of the closed loop system. Our crucial assumption is that there is a nonlinear element in true system that makes the modelling too complicated. As it is clear in figure 8, we can not model the behaviour of phase of system due to it's variation in different identification experiments. The problem of modelling in our case occurs in modelling of phase, but  $W(s)$  contains only the uncertainty of amplitude. So it is essential to find another solution to compensate ignoring of phase uncertainty.

#### IV. CASE STUDY AND IMPLEMENTATION

In order to verify the effect of phase uncertainty on closed loop behavior of system, consider the following performance weighting functions,

$$W_{s1} = \frac{10}{(s+1)(s^2+2s+2.44)} \quad (6)$$

$$W_{s2} = \frac{15}{(s^2+4s+4.5)} \quad (7)$$

Then the mixed sensitivity problem is solved for the obtained fitted model, with an upper bound for control effort corresponding to  $W_u = 0.1$ . In spite of  $W_{s2}$ ,  $W_{s1}$  leads to unstable closed loop system and the related control signal,

stimulates the vibration dynamical modes of FJR. Then the sensitivity function is calculated in two different ways: by using the fitted model and the nominal model.

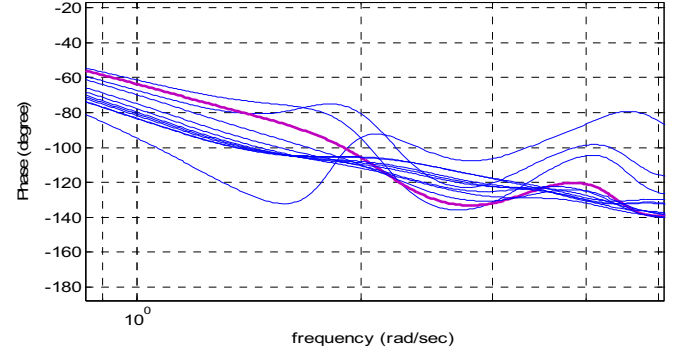


Figure 8: The close up, the phase behaviour in frequency range [1~ 4] rad/s

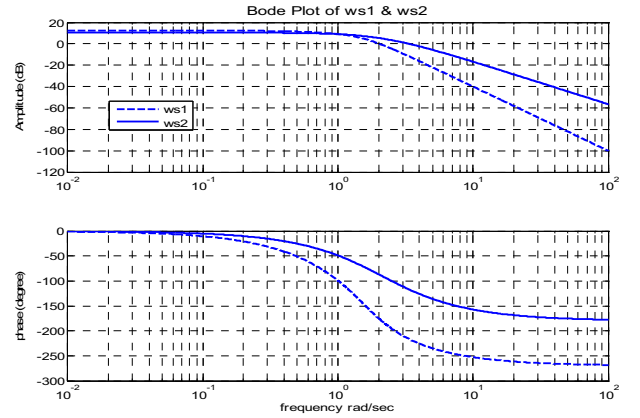


Figure 9: The two different performance weighting functions that theoretically lead to similar closed loop performance.

$$S_i = \frac{1}{1+k_i(s)P(s)}, \quad i=1,2 \quad (8)$$

Figure 10.a, illustrates the sensitivity function obtained by using fitted model for both controllers, while figure 10.b illustrates the sensitivity functions obtained by using nominal model. It is obvious that there is a big difference between these two sensitivity functions in frequency range [1 4] rad/s. hence it means that  $k_1$  and  $k_2$  have to result in approximately same behaviour in closed loop system theoretically. But in practice, they lead to different closed loop behaviour. The uncertainty of phase in frequency range [1 4] rad/s, leads the controller not to guarantee closed loop stability in practice. Hence, it is essential to add another stage to the procedure of the  $H_\infty$  controller design. After designing the  $H_\infty$  controller, the sensitivity function obtained by nominal model should be re-checked. If the sensitivity function is relatively small in frequency range of phase uncertainty, it results into a stabilizing controller while the robustness is preserved. The upper limit of acceptable closed loop sensitivity function can be obtained experimentally. The details of the performance weighting function design are elaborated in the next section.

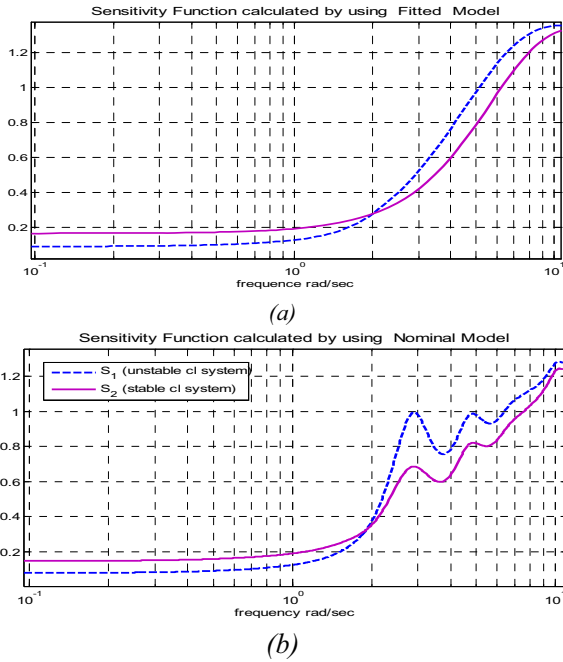


Figure 10: the two sensitivity function of  $k_1$  and  $k_2$  ; a: using Fitted Model, b: using Nominal Model.

Designing  $W_u(s)$  and  $W_s(s)$  are not independent procedures , hence, during each stage of designing  $W_s(s)$ , not only the sensitivity function should be checked, but also  $W_u(s)$  should be regulated. Note that the function  $W_u(s)$  can be chosen to be a constant level as  $W_u(s) = \alpha$  which may cause  $\|U(s)\| < 1/\alpha$ . This selection will limit the magnitude of  $U(s)$  in all frequencies which may limit the resultant bandwidth. As an alternative,  $W_u(s)$  can be shaped in the frequency domain. Increasing the function  $W_u(s)$  in high frequency region will result in decreasing the level of the control action fast transients or jumps. This can be seen empirically in simulation studies. The following is chosen for  $W_u(s)$ :

$$W_u = \frac{s + 50}{s + 500} \quad (9)$$

The performance weighting function is determined in order to have solution for the mixed sensitivity problem as well as to have maximum reachable bandwidth. After the procedure of checking the resultant sensitivity function, the resultant  $W_s(s)$  and controller are:

$$W_s = \frac{75}{(s + 1.5)(s^2 + 3.6s + 6.85)} \quad (10)$$

$$K = \frac{47626(s + 0.7)(s + 1 \pm j2.8)(s + 25)(s + 500)}{(s + 0.5 \pm j2.1)(s + 41 \pm j17)(s + 5.2)(s + 5.8)} \quad (11)$$

The related sensitivity function is shown in figure 11 and being relatively low in frequency range of phase uncertainty guaranties the closed loop system to be robustly stable. To analyse the performance of the closed loop system, the controller is implemented for the FJR. First a sinusoid reference trajectory with frequency 2rad/sec is considered and the closed loop response is illustrated in figure 12.

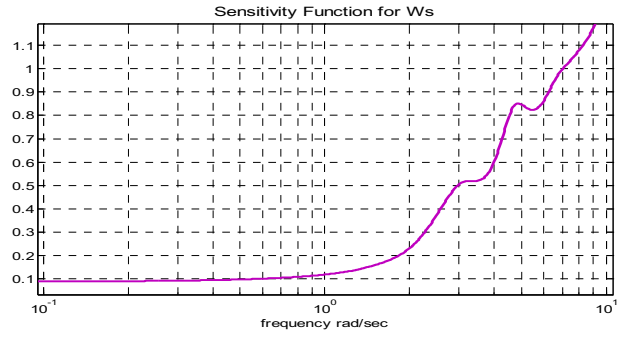


Figure 11: sensitivity function of the stable closed loop system

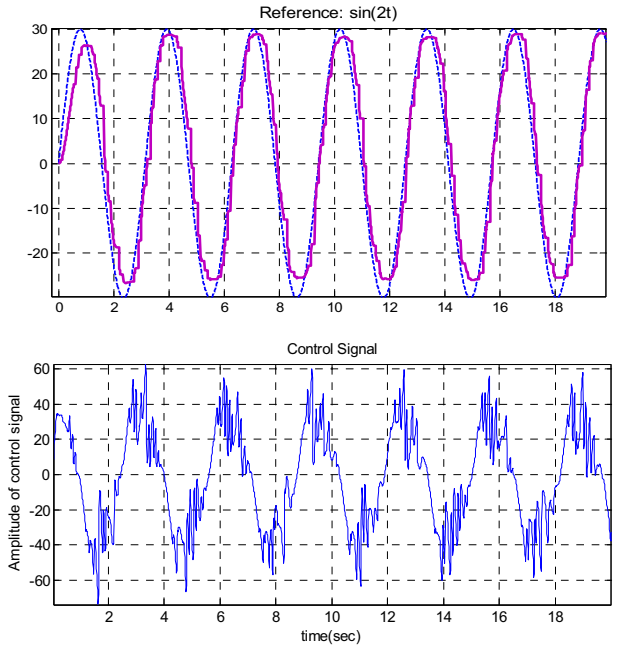


Figure 12: tracking of the 2 rad/s sinusoid reference and control signal

The tracking error is quite small despite the limited control effort. Considering the point that the saturation limit of control signal is  $\pm 128$ , the resulting control signal is acceptable. Figure 13 illustrates the closed loop tracking performance in which the reference signal is smoothed square wave in order to avoid saturation, which is usual in practical tests. As it can be seen, the settling time is quite suitable and the steady state error is about 10%. This amount of error comparing to simulations [7], which is usually about 5%, is quite well in practice.

The better steady state behaviour is reachable by regulating performance weighting function based on proposed procedure.

The steady state error of this system is straight result of dead zone of the motor which is about  $\pm 10$ . As it can be seen when the angle of the link reaches to steady state, the control signal enters to the dead zone. In order to modify the steady state error we should shape the performance weighting function to lead to a controller with higher dc gain.



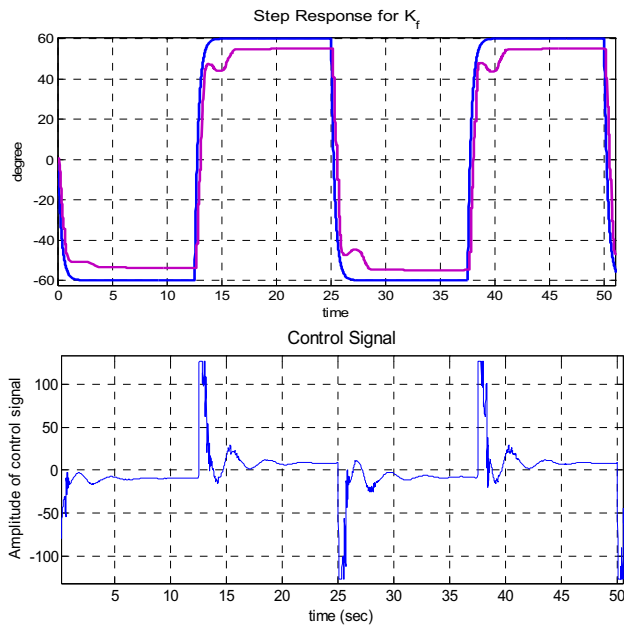


Figure 13: tracking of the smoothed square wave and control signal

The other solution is to improve the dc motors' characteristics. Finally, the above results indicate the capability of  $H_\infty$  controller to motion control of the flexible joint robot with relatively high flexibility characteristics.

## V. CONCLUSIONS

In this paper, the ability of  $H_\infty$  controller to design a suitable controller for an uncertain FJR is investigated in practice. The identification experiments demonstrate a type of phase behaviour that can not be modelled accurately. Ignoring this unmodeled behaviour may results in generating a control signal which stimulates the oscillatory modes of FJR. A new procedure of design is proposed to avoid the instability caused by this unmodeled phase behaviour that can not be encapsulated in multiplicative uncertainty. The effectiveness of the proposed procedure is thoroughly investigated by different tests. At the end, the experimental results reveal the capability of the controller to stabilize the closed loop system and to reduce the tracking error in the presence of the actuator limitation.

## REFERENCES

- [1] Spong M. W., "Modeling and Control of elastic joint robots," *Journal of Dynamic Systems, Measurement, and Control*, 1987
- [2] Ozgoli S., Taghirad H. D., "A Robust Composite Controller for Flexible Joint Robots with a Fuzzy Anti Saturation Strategy", *European Journal of Scientific Research*, Vol 2, No 1, pp 62-88, March 2005.
- [3] F. Ghorbel and M. W. Spong, *Integral manifolds of singularly perturbed systems with application to rigid link flexible joint multibody systems*, Int. J. Non-linear Mech. 34, 133-155(2000)
- [4] R. A. Al-Ashoor , R. V. Patel and K. Khorasani, "Robust adaptive controller design and stability analysis for flexible-joint manipulators", *IEEE Trans. Syst. Man Cybernet.* 23,589-602(1993)

[5] Y. H. Chen and M. C. Han, "Robust control design for uncertain flexible-joint manipulators: a singular perturbation approach," in: *Proc. 32<sup>nd</sup> Conf. on Decision and Control*, pp. 611-616

[6] Taghirad H. D., Khosravi M. A., "A Robust Linear Controller for Flexible Joint Manipulators," *IEEE/RSJ Conf. on Intelligent Robots and Systems, IROS'04*, 3: 2936-2941, Oct 2004, Japan

[7] Taghirad H. D., Bakhshi GH., "Composite- $H_\infty$  Controller Synthesis for Flexible Joint Robots", *IEEE/RSJ Conf. on Intelligent and Robotic Systems*, pp2073-2078, Lausanne, 2002

[8] Taghirad H. D., Rahimi H., "QFT Controller Synthesis for A Nonlinear Flexible Joint Robot," *ICEE05, 13<sup>th</sup> Iranian Conf. on Electrical Engineering, Zanjan University, Iran*, May 2005

[9] H.D. Taghirad and P.R. Belanger, " $H_\infty$ -based robust torque control of harmonic drive systems," *Journal of Dynamic Systems, Measurements, and Control, ASME Pub.*, 123(3) pp 338-345, Sept. 2001

[10] J.Doyle, B.Francis, A.Tannenbaum, *Feedback Control Theory*, Macmillan 1990

[11] Lennart Ljung, *System Identification:Theory For The User*, PRENTICE HALL 1987