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ANALYSIS OF FIRST ORDER SYSTEMS WITH VARIABLE TIME DELAY USING PI CONTROLLER AND SMITH PREDICTOR

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ABSTRACT
Smith predictor is one of the first structures used to compensate the time delay in closed loop systems. One of the simplest controllers which are used to control the industrial systems is PID controller. Although, we know that the derivative term is rarely used for controlling the time delay systems. Subtle identification of time delay is usually impossible in reality; on the other hand, PI controller design with smith structure is practiced upon the open loop delay free system. Therefore the structural uncertainty will be existed in open loop systems which make the designing of the PI controller very difficult. In this paper, at first, we analyze the effect of difference between modeled time delay and real time delay on the PI parameters, then the performance of the closed loop system compared with a second order delay free system is investigated; and finally, we compare the tracking of variable time delay system with fixed time delay system which both of them have the same parameters. In this analysis we assume that the fixed time delay is max of variable time delay.

KEY WORDS
PI Controller, Smith Predictor, Stability, Performance, Variable Time Delay.

1. Introduction

It's evident that all industrial systems, owing to their distributed nature, have some time delay features. In many practical cases delay is negligible small. Smallness of delay is related to the signals processed and hence to specification of closed loop and open loop system. In these cases we guess the system is delay free and we can design controller for delay free system, therefore a satisfactory performance will be occurred in real closed loop system. But in other time delay systems which delay is large in comparison with the largest time constant of the closed loop and open loop system, we can not design controller in such manner. If time delay isn't compensated, the closed loop system may be unstable and we have to put strong limits on open loop gain. Smith predictor is one of the first structures that used to control of industrial processes [1, 7]. Identification of system's dynamics, including open loop gain, time constant and pure time delay in open loop system are required. Block diagram of closed loop system has shown in fig (1),

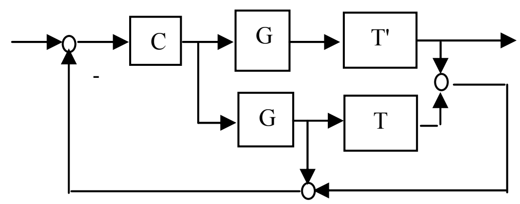

Figure 1. Closed loop System with variable time delay (T')

In the structure of smith predictor the feed forward pass is used to compensate the time delay, such that the delay in characteristic equation of closed loop system is omitted. This is occurred when the modeled time delay in smith predictor is equal to real time delay in open loop system [2, 3, 4]. Time delay usually varies in the processes with variation of process inlets and their parameters. Also the accurate method for online identification of delay does not exist. Therefore, it is required to analysis the effect of difference between modeled delay and real delay on controller parameters to receive the suitable response [5, 6]. In this paper we develop the separation principle for analyzing of stability and performance of closed loop system with variable time delay in open loop dynamics. We try to design PI controller until the closed loop system be stable , then we try to calculate the PI parameters for achieving suitable performance in transient and steady state response.

2. Analysis of Problem

In fig.1 G(s) and C(s) are respectively open loop free delay system and PI controller as follow,

$$G(s) = \frac{K_p}{s+p} \quad (1)$$

$$C(s) = \frac{K_c(s+q)}{s} \quad (2)$$

We can write closed loop transfer function as equation (3),

$$F(s)=\frac{CGe^{-\tau's}}{1+CG+CG(e^{-\tau's}-e^{-\tau s})} \tag{3}$$

Which τ and τ' are respectively a modeled pure time delay and real pure time delay of system.

G and $T'=e^{-\tau's}$ don't exist separately, these are shown separately just for analysis of the problem by the separation principle. So, there is not any difference between T'G and GT'.

We assume that G doesn't have any uncertainty and the system's model is completely specified, also parameters of G are completely determined.

Our purpose is obtaining the poles of F(s) in the left half of the complex plane. With assuming $A=\frac{1+CG}{CG}$, denominator of F(s) can be written as follow,

$$e^{-\tau s}=e^{-\tau's}+A(s) \tag{4}$$

Which, we assume $\tau<\tau'$. Therefore obtaining of stability condition of closed loop system is investigated in two cases: the first case is when the zero of PI will be omitted with the stable pole of the open loop system, the second case is when the pole zero elimination won't be happened. At last, two cases will be compared.

2.1 Pole Zero Cancellation

If one of the stable poles is eliminated with one stable zero, CG will be simplified as follow,

$$CG=\frac{K_c(s+q)}{s}\frac{K_p}{s+p}\overset{p=q}{=}\frac{K_pK_c}{s} \tag{5}$$

In the other side we have $s=\sigma+j\omega$ and $k=K_p\,K_c$ so eq.4 can be written as follow,

$$e^{-\tau(\sigma+j\omega)}=e^{-\tau'(\sigma+j\omega)}+1+\frac{1}{k}(\sigma+j\omega) \tag{6}$$

By separation of real and imaginary part of eq.6 we can write,

$$e^{-\tau\sigma}\cos\tau\omega=e^{-\tau'\sigma}\cos\tau'\omega+1+\frac{\sigma}{k} \tag{7}$$

$$e^{-\tau\sigma}\sin\tau\omega=e^{-\tau'\sigma}\sin\tau'\omega-\frac{\omega}{k} \tag{8}$$

In some of the real systems τ' is variable and we don't have exact information about it; therefore, to analyze the stability we assumed ω=0. It means that the system response doesn't have any oscillation. Therefore we can write eq.7 as follow,

$$e^{-\tau\sigma}=e^{-\tau'\sigma}+1+\frac{\sigma}{k} \tag{9}$$

Now we find k in eq.8 by converging ω to zero. So we have,

$$\lim_{\omega_i\to 0}\frac{e^{-\tau\sigma}\sin\tau\omega_i}{\omega_i}=\lim_{\omega_i\to 0}\frac{e^{-\tau'\sigma}\sin\tau'\omega_i}{\omega_i}-\frac{1}{k} \tag{10}$$

$$\Rightarrow k=\frac{1}{\tau' e^{-\tau'\sigma}-\tau e^{-\tau\sigma}} \tag{11}$$

Now we show that eq.8 has not any local maximum or minimum through variation of k,

$$H=e^{-\tau\sigma}\sin\tau\omega_i-e^{-\tau'\sigma}\sin\tau'\omega_i+\frac{\omega_i}{k}$$
$$\Rightarrow\frac{\partial H}{\partial k}=-\frac{\omega_i}{k^2} \tag{12}$$

Eq.12 shows that eq.8 doesn't have any max or min. thus if we have $k\le\frac{1}{\tau' e^{-\tau'\sigma}-\tau e^{-\tau\sigma}}$ so the only minimum of H will be occurred in ω=0.

Therefore the range of k will be found as follow,

$$0<k\le\frac{1}{\tau' e^{-\tau'\sigma}-\tau e^{-\tau\sigma}} \tag{13}$$

We show that the closed loop system will be stable for the k's in the above interval. If ω=0 then eq.6 will be as follow,

$$e^{-\tau\sigma}=e^{-\tau'\sigma}+1+\frac{\sigma}{k} \tag{14}$$

By substituting $k=\frac{1}{\tau' e^{-\tau'\sigma}-\tau e^{-\tau\sigma}}$ in eq.14, it follows that,

$$e^{-\tau'\sigma}(\tau'\sigma+1)-e^{-\tau\sigma}(\tau\sigma+1)+1=0 \tag{15}$$

So σ in eq.15 should be less than zero. If σ>0 then $e^{-\tau\sigma}<1$ and $e^{-\tau'\sigma}+\frac{\sigma}{k}\ge 0$ therefore $e^{-\tau\sigma}$ from eq.14 should be greater than 1 and it's impossible, because if σ>0 then $e^{-\tau\sigma}<1$.

2.2 Pole Zero Not Cancellation

Now we assume that pole zero cancellation in CG isn't occurred. Using eqs.1, 2 and 4 CG and A(s) will be as follow,

$$CG=\frac{k_c(s+q)}{s}\cdot\frac{k_p}{s+p}=\frac{k_pk_c(s+q)}{s(s+p)}=\frac{k(s+q)}{s(s+p)} \tag{16}$$

$$A(s)=1+\frac{s(s+p)}{k(s+q)} \quad (17)$$

with s=σ+jω A(s) will be as follow,

$$A(\sigma+j\omega)=1+\frac{(\sigma+j\omega)((\sigma+p)+j\omega)}{k(\sigma+q)+j\omega)} \quad (18)$$

$$\mathrm{Re}\{A\}=1+\frac{\sigma\left[(\sigma+p)(\sigma+q)+\omega^2\right]-\omega^2(q-p)}{k\left[(\sigma+q)^2+\omega^2\right]} \quad (19)$$

$$\mathrm{Im}\{A\}=\omega\frac{\sigma(q-p)+(\sigma+p)(\sigma+q)+\omega^2}{k\left[(\sigma+q)^2+\omega^2\right]} \quad (20)$$

Also we can write eq.4 as follow,

$$A(\sigma+j\omega)+e^{-(\sigma+j\omega)\tau'}-e^{-(\sigma+j\omega)\tau}=0 \quad (21)$$

If ω=0 then,

$$1+\frac{\sigma(\sigma+p)}{k(\sigma+q)}+e^{-\sigma\tau'}=e^{-\sigma\tau} \quad (22)$$

`In eq.22 if σ>0 then $e^{-\tau\sigma}<1$ so,

$$1+\frac{\sigma(\sigma+p)}{k(\sigma+q)}+e^{-\sigma\tau'}<1 \quad (23)$$

$$\Rightarrow\frac{\sigma(\sigma+p)}{k(\sigma+q)}<0 \quad (24)$$

For stable open loop system eq.24 isn't satisfied. By separation of real and imaginary part of eq.22 we can write,

$$\lim_{\substack{\omega_i\to 0\\ \omega_i\neq 0}}\mathrm{Im}\{A\}-e^{-\sigma\tau'}\sin\tau'\omega_i=e^{-\tau\sigma}\sin\tau\omega_i \quad (25)$$

$$\Rightarrow\frac{k(\sigma+q)^2}{\sigma^2+2\sigma q+pq}=\frac{1}{\tau' e^{-\sigma\tau'}+\tau e^{-\sigma\tau}} \quad (26)$$

As description of eq.12 we can write about eq.26,

$$0<\frac{k(\sigma+q)^2}{\sigma^2+2\sigma q+pq}\leq\frac{1}{\tau' e^{-\sigma\tau'}+\tau e^{-\sigma\tau}} \quad (27)$$

This equation is like eq.12, so it doesn't have any max or min. we use the following equation to find the closed loop pole for the largest amount of k,

$$\max_k\left(\frac{k(\sigma+q)^2}{\sigma^2+2\sigma q+pq}-\frac{1}{\tau' e^{-\sigma\tau'}+\tau e^{-\sigma\tau}}\right)=0 \quad (28)$$

The eqs.22, 28 have three variables q, k and σ which they will be solved assuming $\sigma<0$. Now, let us look at the conditions that closed loop system will have the fast response. If pole zero cancellation doesn't occurred, solving the above problem with variables q and k has more degree of freedom. What should be answered about that problem is that for which q and k we have the fast response. Using eqs.22, 24 we have:

$$\begin{cases}\sigma<-p \quad or \quad \sigma>-q \quad for \quad q<p\\ \sigma<-q \quad or \quad \sigma>-p \quad for \quad p<q\end{cases} \quad (29)$$

Now it's necessary to define the relationship between p and q. The left side of eq.28 should be positive, therefore $\sigma^2+2\sigma q+pq>0$ and q<p.

It means that the zero of PI controller can not be any where and it should be closer to origin than the pole of the open loop system. If p=q then $\sigma^2+2\sigma q+pq>0$ except for σ=-p. In other side the rise time of step response in closed loop system will be decreased by selecting σ=-p. this subject is shown in simulation results.

In above analysis we assume ω=0. It means that the system doesn't have oscillation in the response and this assumption will be conservative.

2.3 Analysis of Closed Loop Performance With Cancellation of Stable Pole and Zero

Previously, we showed a case that ξ=1. Now, we discus the case which ξ=0. This case defines the beginning of instability. In eqs.7, 8 if σ=0 we can write,

$$\cos\tau\omega=\cos\tau'\omega+1 \quad (30)$$

$$\sin\tau\omega=\sin\tau'\omega-\frac{\omega}{k} \quad (31)$$

From the above equations we have,

$$k=\frac{\omega}{\sin\tau'\omega-\sin\tau\omega} \quad (32)$$

We can choose k from eq.11 and eq.32 to obtain a suitable performance ($0\leq\xi\leq 1$),

$$\frac{1}{\tau' e^{-\sigma\tau'}+\tau e^{-\sigma\tau}}\leq k\leq\frac{\omega}{\sin\tau'\omega-\sin\tau\omega} \quad (33)$$

It's noticeable that k should be positive and because of eq.30 ω can not be zero. Therefore, in comparison with second order systems, we can find the poles of system as follow,

$$s=-\zeta\omega_n\pm j\omega_n\sqrt{1-\zeta^2} \quad (34)$$

Then we can write the relationship between σ, ω and ω_n as follow,

$$\Rightarrow \begin{cases} \sigma = -\zeta\omega_n \\ \omega = \omega_n\sqrt{1-\zeta^2} \end{cases} \tag{35}$$

In the above relationship we can find σ, ω_n to have suitable performance and the amount of k will be defined in 33. Therefore we have suitable response close to second order systems. In this analysis we tried to use analytical tools which are using in linear systems. Finally we can find the relationship between response of first order system with variable time delay and response of common second order systems. Continually we show the simulation result to approve our analysis.

Example: In this example we want to design the PI controller for an open loop system $G(s) = \frac{1}{s+1}$ with max time delay t'=10sec. Smith predictor with modeled time delay t=7sec has designed. The zero of PI controller is in q=-1 and this zero will be eliminated with the pole of open loop system in p=-1. We can find σ by solving eq.15 and put it in to eq.11 to find k.

The critical damping response will be occurred at k=0.06.

Also we can find variables σ and k from eqs.22 and 28 related to the case that pole zero cancellation isn't occurred.

Fig.2 shows comparison of step response when zero of PI controller is in q=-0.8 and when q=-1. Fig.2 shows that step response in the case of pole zero cancellation is fast.

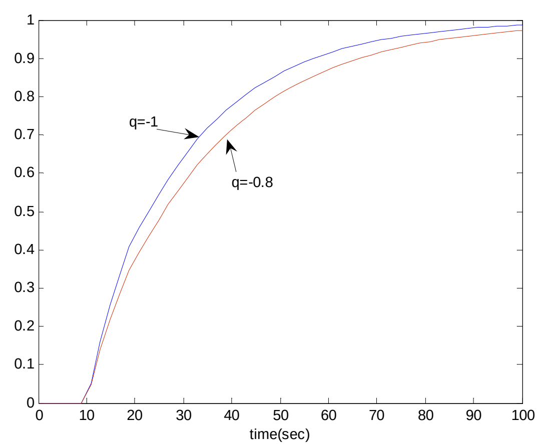

Figure 2. Step Response for $\omega = 0$

In eq.32, If $\omega \to 0$ we have $k = \frac{1}{\tau' - \tau} = 0.33$. We simulate closed loop system and its results are shown in fig.3 for PI zero in -1 and PI zero in -0.8.

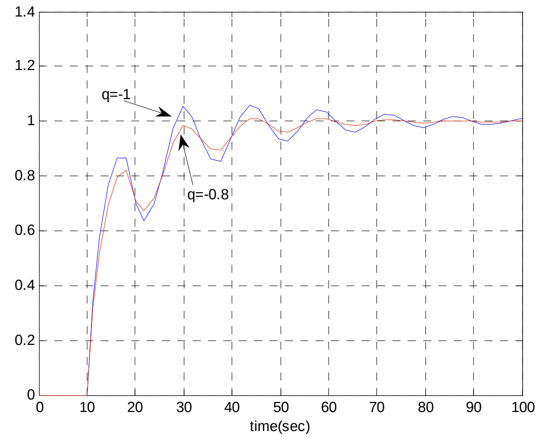

Figure 3. Step Response for $\omega \to 0$

If we choose k greater than 0.33 the step response of closed loop system will be oscillatory response. In fig.4 we show the response of closed loop system with k=0.5, q=-1 and q=-0.8. It's obvious that by increasing k the step response will be more oscillatory.

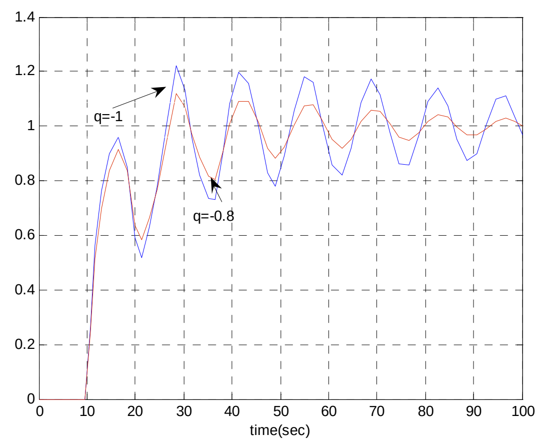

Figure 4. Step Response for k=0.5

Now we show the effect of difference between modeled time delay and real time delay on transient time response.

In fig.5 we have k=0.06 and we repeat previous simulation for q=-1, modeled time delays t=7sec and t=9sec. we can see step response for modeled time delay t=9sec is fast.

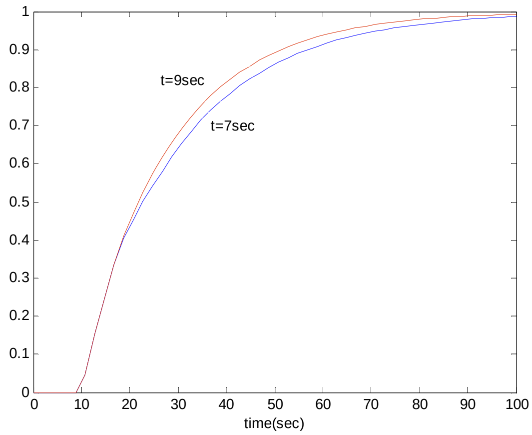

Figure 5. Step Response for k=0.06, effect of difference between tand t'

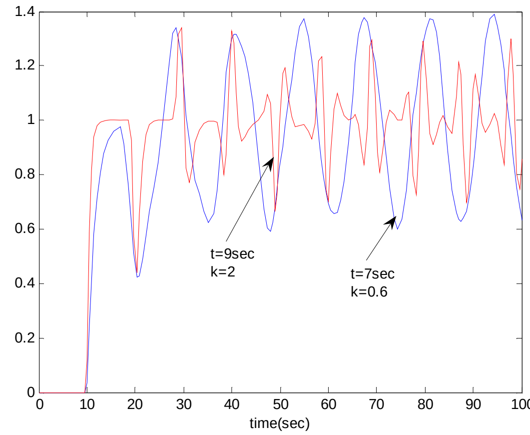

Figure 7. Comparison of oscillation response for t=9sec and t=7sec

If the modeled time delay is t=9sec by calculation of k in eq.32 we have k=1 and the step response will be similar to that in fig.3.

In fig.6 we compare step response of closed loop system that has modeled time delay t=9sec and the open loop gain k=1 with step response of closed loop system which has modeled time delay t=7sec and the open loop gain k=0.33.

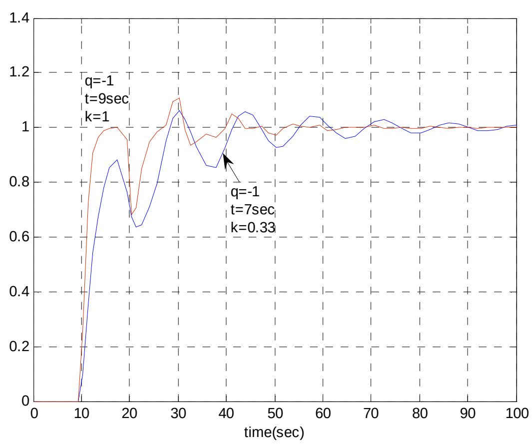

Figure 6. Comparison of step response for t=9sec and t=7sec

Obviously we can see whatever the modeled delay is closer to real delay; the open loop gain can be increased. Also for the closer modeled time delay to real time delay, step response of closed loop system is fast.

Fig.7 shows the comparison between step response of two cases t=9sec and t=7sec. In the case of t=9sec the gain can be greater than another one. In the both of cases the gain for starting of un damped oscillation is about two times of the gain that gives from eq.32.

In fig.8 we compare two cases t=7sec and t=13sec (the modeled time delay is greater than real time delay). In this simulation we select q=-1 and k=0.33 (from eq.32).

Fig.8 shows that if t>t' overshoot of step response will be large; however we select the open loop gain without any change.

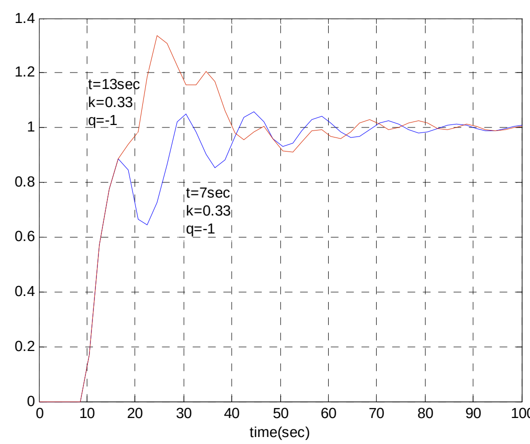

Figure 8. Comparison of step response for t=7 sec and t=13sec

Fig.9 shows the comparison between step response of two cases, t=9sec and t=11sec. In these cases we can select the gain greater than previous case. When t is closer to t' the step response of system is fast and the amount of gain can be increased.

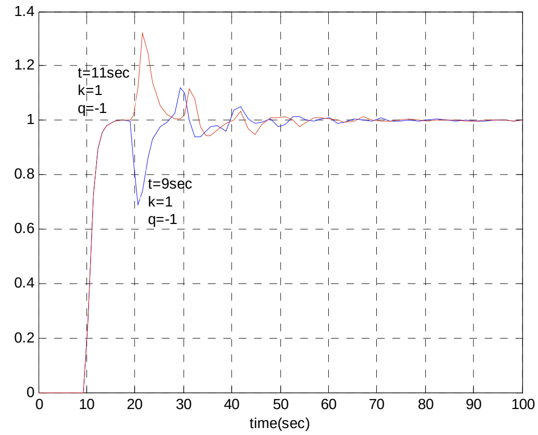

Figure 9. Comparison of step response for t=9 sec and t=11sec

In these simulations we tried to show effect of closeness of modeled time delay to real time delay on tuning of PI parameters. We showed whatever the modeled time delay get closer to real time delay, the gain of closed loop system can be increased and the settling time will be decreased. Also the zero of PI controller should be closer to origin than the pole of open loop system. And if pole zero cancellation occurs the closed loop system will have the fast response.

In the next simulations we show that when time delay is variable, the closed loop system has more satisfactory step response than the case of max fixed time delay system.

In fig.10 we compare responses of systems with variable time delay through function $delay = 7 + \sin(\pi t / 40)$ and fixed time delay t'=10sec. Obviously we can see that variable time delay system has a fast step response. In this simulation q=-1, k=0.33 and modeled time delay is t=7sec.

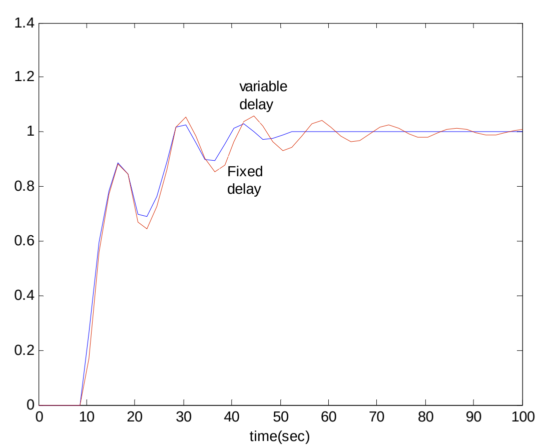

Figure 10. Comparison of step response for variable time delay and fixed time delay systems t'=10sec in the case t=7sec

In fig.11 we select the modeled time delay t=9sec and open loop gain is k=1. In this fig. the oscillation and settling time of response is less than results of fig.10.

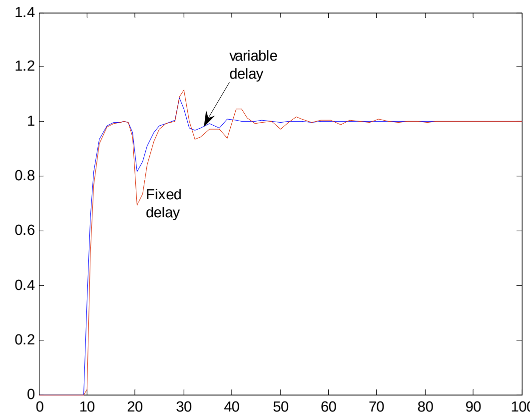

Figure 11. Comparison of step response for variable time delay and fixed time delay systems t'=10sec in the case t=9sec

In fig.12 we show the tracking of set points for variable time delay and fixed delay systems. In this case we select the zero of PI controller in -1 and the modeled time delay is chosen t=9sec and open loop gain is k=1.

Obviously, we see that the variable time delay system has less overshoot and fast step response. Consequently the variable time delay system has more satisfactory tracking than max fixed time delay system.

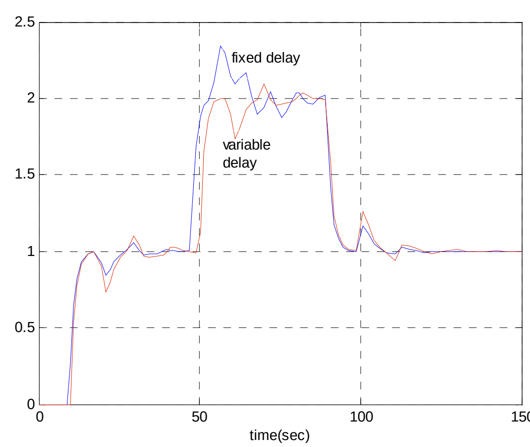

Figure 12. Comparison of step response tracking for variable time delay and fixed time delay systems t'=10sec in the case t=9sec

3. Conclusion

Since analyzing of time delay systems with variable time delay is complicated, in this paper, effective tools using smith predictor and PI controller are introduced for analyzing the closed loop systems and designing the controller.

In this analysis, the maximum difference between modeled time delay and real time delay has used as a principal parameter and the results of simulation shows the power of this method on system's analysis and controller's design. Thus we have developed separation principle and closed loop characteristic equation to define the transient step response in comparison with step response of second order systems. We could get a criterion for designing PI controller with structure of smith predictor. The simulation results show that whatever the modeled time delay is closer to max real time delay, the open loop gain can be increased and step response will be fast. Also the step response of variable time delay system have a more satisfactory response than max fixed time delay with the same amount of open loop gain and PI parameters.

Moreover, the simulation results show that when the open loop pole omitted by zero of PI controller, the closed loop system will present fast step response.

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