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Delay-Dependent Stabilization of Linear Input-Delayed Systems with Composite State-Derivative Feedback:
Constant and Time-Varying Delays

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Abstract: *This paper considers stability problem for input-delayed systems for both constant and time-varying delay case. A new composite state-derivative control law is introduced, in which, a composition of the state variables and their derivatives appear in control law. By this means, the resulting closed-loop system becomes a particular time-delay system of neutral type. The significant specification of this neutral system is that its delayed term coefficients depend on the control law's parameters. This condition provides new challenging issues which has its own merits in theoretical as well as practical aspects. In the present paper, new delay-dependent sufficient conditions are derived in presence of both constant and varying time-delay in terms of matrix inequalities. The resulting controllers guarantee asymptotic stability of the closed-loop system. Simulation studies are presented to verify the stability conditions obtained within the theorems.*

Keywords: stability, state-derivative feedback, time-delay.

1. Introduction

Time-delay phenomena appear in many systems and processes, such as chemical and thermal processes, inferred grinding model, manual control, population dynamic model, rolling mill and systems with long transmission line [6, 2, 3]. The problem of stability and control of time-delay systems has been followed through two categories, namely: delay-dependent criteria and delay-independent criteria. Generally speaking, delay dependent case is less conservative than the delay independent case, but the later is also more useful when the effect of the time delay is small.

Stability and stabilization of time delay systems of retarded or neutral type is a problem of recurring interest in control research community. Recently, much attention has been drawn to improve delay-dependent criteria which are considered less conservative than delay-independent ones. The Lyapunov-Krasovskii functionals are usually used to derive the stability conditions mostly in terms of some matrix inequalities. Regarding to stabilization of neutral systems, asymptotic stability of these systems with multiple time delay has been considered in [1]. A delay-dependent sufficient condition has been proposed in this work in terms of linear matrix inequalities (LMIs). Chen et.al. has focused on the stability conditions of systems with discrete and distributed delays in [10]. Moreover, a new model transformation for this system has been proposed by Chen [4] which has been led to a less conservative delay-dependent sufficient condition compared to the previous work. According to robust stability, Yue et.al., have considered the robust stability of an uncertain neutral system with discrete and distributed delay and parametric uncertainty in [5]. A delay-dependent sufficient condition has been proposed by Yue et.al. in [7] which guarantees exponential stability for uncertain systems with parametric uncertainty and single delay. The problem of finding stability condition for systems with time-varying delays has been the center of attention in some research such as [11,8]. Lien proposed a delay-dependent sufficient condition for an uncertain neutral system with discrete and distributed time-varying delay in [11]. For uncertain systems with single time-varying delay, robust stability conditions have been derived in terms of linear matrix inequalities by Zhao et.al. [8], depending on the upper bound of delay and its derivative.

A general representation of a neutral system is shown as

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{n} A_{h_i} x(t-h_i) + \sum_{j=1}^{k} A_{d_j} \dot{x}(t-d_j) + Bu(t) \quad (1)$$

To the best knowledge of the authors, in all developed theories for neutral systems, no analysis and synthesis has been derived when both $x(t-h_i)$ and $\dot{x}(t-h_i)$ s' coefficients are dependent on the controller's parameter, whereas these condition has its own merits in practical application as well as leading to a new challenging theoretical development. Furthermore, no state-derivative feedback has been proposed in the literature for input-delay systems, whereas it is of great importance in many practical problems. The importance of the above conditions is observed in the control of active vibration suppression systems.

To benefit the advantages of the state feedback as well as acceleration feedback, or generally state derivative feedback, we introduce a new composite control law. In this composite controller the state feedback is added to the state-derivative feedback as it is shown by the following equation:

$$u = K_1 x + K_2 \dot{x} \tag{2}$$

Assume the general representation of an input time-delay system as follows:

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{n} B_i u(t - h_i) + Ew(t) \tag{3}$$

Applying the control law (2) to the input time-delay system (3) leads to a time delay closed system of neutral type which is represented as follows:

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{n} B_i K_1 x(t - h_i) + \sum_{i=1}^{n} B_i K_2 \dot{x}(t - h_i) + Ew(t) \tag{4}$$

As it is seen in the Equation (4), both $x(t-h_i)$ and $\dot{x}(t-h_i)$ s' coefficients are dependent on the controller parameters. Therefore, finding K_1 and K_2 is not only a new challenging problem as far as theoretical issues is concerned, but also it is an effective remedy to obtain desired performance in many applications. The main purpose of this paper is to elaborate this problem in detail and to design some stabilizing controller for the closed-loop system in presence of both constant and varying time-delays. For this aim, both constant and time-varying delay cases are analyzed and delay-dependent stability conditions are obtained in terms of some matrix inequalities.

This paper is organized as follows. Problem formulation is introduced in Section 2. In Section 3, for constant time delay system a stabilizing controller is designed in terms of some matrix inequalities. The sufficient conditions provided in section 3 are extended to systems with varying time-delay in section 4. Illustrative examples are provided in section 5, and finally, the conclusions are drawn in Section 6.

2. Problem Formulation

In this paper, we consider the following time-delay system with input delay:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t - \tau(t)) + Ew(t) \\ z(t) &= Cx(t) + Dw(t) \end{aligned} \tag{5}$$

where x is the state, $w \in \Re^p$ is the disturbance input of system that belongs to $L_2[0,\infty)$, $\tau(t)$ is the time-varying delay of the system and is assumed to satisfy $0 < \tau(t) \le \bar{\tau}$, $u \in \Re^m$ is the system input and $z \in \Re^q$ is the controlled system output. The matrices $A \in \Re^{n\times n}$, $B \in \Re^{n\times m}$, $E \in \Re^{n\times p}$, $C \in \Re^{q\times n}$, $D \in \Re^{q\times p}$ are assumed to be known. Considering control law (2), the state space equations of the closed-loop system is given by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + BK_1 x(t - \tau(t)) + BK_2 \dot{x}(t - \tau(t)) + Ew(t) \\ z(t) &= Cx(t) + Dw(t) \\ x(t_0 + \theta) &= \phi(\theta) \qquad \forall \theta \in [-\tau, 0] \end{aligned} \tag{6}$$

Therefore, the resulting closed-loop system (6) is a time-delay system of neutral type which both coefficients of $x(t-\tau)$ and $\dot{x}(t-\tau)$ depending on the controller parameters. Here, we state two well known lemmas which will be used further in the main result of the paper.

Lemma 2.1: Assume $a(.) \in \Re^{na}$, $b(.) \in \Re^{nb}$ and $N \in \Re^{na\times nb}$ are defined on the interval Ω then for any matrices $X \in \Re^{na\times nb}$, $Y \in \Re^{na\times nb}$ and $Z \in \Re^{na\times nb}$, the following holds:

$$-2\int_\Omega a^T(\alpha) N b(\alpha) d\alpha \le \int_\Omega \begin{bmatrix} a(\alpha) \\ b(\alpha) \end{bmatrix}^T \begin{bmatrix} X & Y - N \\ Y^T - N^T & Z \end{bmatrix} \begin{bmatrix} a(\alpha) \\ b(\alpha) \end{bmatrix} d\alpha,$$

where

$$\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} > 0$$

Lemma 2.2 [9]: (Schur Complement) The LMI

$$\begin{bmatrix} Q(y) & S(y) \\ S^T(y) & R(y) \end{bmatrix} < 0$$

is equivalent to

$$R(y) < 0, \quad Q(y) - S(y)R(y)^{-1}S(y)^T < 0,$$

3. Stabilization with Constant Time-Delay

In order to design a stabilizing controller for the closed-loop system (6) with $\tau(t) = h$, we state the following theorem which provides delay-dependent stability conditions in terms of some matrix inequalities. The details are discussed as follows:

Theorem 3.1: Consider the time-delay system (5) with $\tau(t) = h$. The closed-loop system with control law (2) and $w(t) \equiv 0$ is asymptotically stable for any constant time-delay $h > 0$, if there exist positive definite symmetric matrices P, Q, R_1, R_2, Z_1, $Z_2 \in \Re^{n\times n}$ and matrices X_1, X_2, Y_1, $Y_2 \in \Re^{n\times n}$, K_1, $K_2 \in \Re^{m\times n}$ satisfying matrix inequalities (7) ~ (9).

$$\begin{bmatrix} X_1 & Y_1 \\ Y_1^T & Z_1 \end{bmatrix} > 0 \tag{7}$$

and

$$\begin{bmatrix} 2X_2 & hY_2 \\ hY_2^T & 2Z_2 \end{bmatrix} > 0 \tag{8}$$

In this case a Lyapunov-krasovskii functional candidate for the system (6) has the form

$$V = V_1 + V_2 + V_3 + V_4$$

where

$$V_1 = x(t)^T P x(t) \tag{10}$$

$$V_2 = \int_{-h}^{0} \int_{t+\beta}^{t} \dot{x}^T(\alpha) Z_1 \dot{x}(\alpha) d\alpha d\beta, \tag{11}$$

$$\begin{bmatrix} \Phi & PBK_1-Y_1-Y_2 & PBK_2-hY_2 & 0 & hA^TZ_1 & A^TR_1 & hA^TA^TZ_2 & A^TA^TR_2 \\ * & -Q & 0 & 0 & h(BK_1)^TZ_1 & (BK_1)^TR_1 & h(ABK_1)^TZ_2 & (ABK_1)^TR_2 \\ * & * & -R_1 & 0 & h(BK_2)^TZ_1 & (BK_2)^TR_1 & h(BK_1+ABK_2)^TZ_2 & (BK_1+ABK_2)^TR_2 \\ * & * & * & -R_2 & 0 & 0 & h(BK_2)^TZ_2 & (BK_2)^TR_2 \\ * & * & * & * & -hZ_1 & 0 & 0 & 0 \\ * & * & * & * & * & -R_1 & 0 & 0 \\ * & * & * & * & * & * & -hZ_2 & 0 \\ * & * & * & * & * & * & * & -R_2 \end{bmatrix} < 0 \quad (9)$$

$$V_3 = \int_{t-h}^{t} x^T(\alpha)Qx(\alpha)d\alpha + \int_{t-h}^{t} \dot{x}^T(\alpha)R_1\dot{x}(\alpha)d\alpha \quad (12)$$

$$V_4 = h^{-1}\int_{-h}^{0}\int_{-h}^{\beta}\int_{t+\eta}^{t} \ddot{x}^T(\alpha)Z'\ddot{x}(\alpha)d\alpha d\eta d\beta$$

$$+(1/2)\int_{t-h}^{t} \ddot{x}^T(\alpha)R'\ddot{x}(\alpha)d\alpha, \quad (13)$$

where $P=P^T>0$, $Q=Q^T>0$, $R_1=R_1^T>0$, $R'=R'^T>0$, $Z_1=Z_1^T>0$ and $Z'=Z'^T>0$.

Proof: Differentiating V_1 with respect to t gives us

$$\dot{V}_1 = 2x^T(t)P\dot{x}(t) = 2x^T(t)P\{Ax(t)+BK_1x(t-h)+BK_2\dot{x}(t-h)\}$$

It is possible to write

$$x(t-h) = x(t) - \int_{t-h}^{t}\dot{x}(\alpha)d\alpha \quad (14)$$

We introduce the following relation for the delayed derivative of the state:

$$\dot{x}(t-h) = h^{-1}\left[x(t)-x(t-h)-\int_{-h}^{0}\int_{t-h}^{t+\beta}\ddot{x}(\alpha)d\alpha d\beta\right] \quad (15)$$

Therefore

$$\dot{V}_1 = 2x^TP(A+BK_1+h^{-1}BK_2)x(t)$$

$$-2h^{-1}x^TPBK_2x(t-h) - 2x^TPBK_1\int_{t-h}^{t}\dot{x}(\alpha)d\alpha$$

$$-2h^{-1}x^TPBK_2\int_{-h}^{0}\int_{t-h}^{t+\beta}\ddot{x}(\alpha)d\alpha d\beta$$

Applying Lemma 2.1, the following inequalities will be obtained:

$$-2x(t)PBK_1\int_{t-h}^{t}\dot{x}(\alpha)d\alpha$$

$$\le \int_{t-h}^{t}\begin{bmatrix} x(t) \\ \dot{x}(\alpha)\end{bmatrix}^T \begin{bmatrix} X_1 & Y_1-PBK_1 \\ Y_1^T-(PBK_1)^T & Z_1\end{bmatrix}\begin{bmatrix} x(t) \\ \dot{x}(\alpha)\end{bmatrix}d\alpha \quad (16)$$

$$\begin{bmatrix} X_1 & Y_1 \\ Y_1^T & Z_1\end{bmatrix} > 0 \quad (17)$$

and

$$-2x(t)PBK_2\int_{-h}^{0}\int_{t-h}^{t+\beta}\ddot{x}(\alpha)d\alpha d\beta$$

$$\le \int_{-h}^{0}\int_{t-h}^{t+\beta}\begin{bmatrix} x(t) \\ \ddot{x}(\alpha)\end{bmatrix}^T \begin{bmatrix} X' & Y'-PBK_2 \\ Y'^T-(PBK_2)^T & Z'\end{bmatrix}\begin{bmatrix} x(t) \\ \ddot{x}(\alpha)\end{bmatrix}d\alpha d\beta \quad (18)$$

$$\begin{bmatrix} X' & Y' \\ Y'^T & Z'\end{bmatrix} > 0 \quad (19)$$

Therefore, with above conditions we obtain

$$\dot{V}_1 \le 2x^TP\left(A+BK_1+h^{-1}BK_2\right)x(t) - 2h^{-1}x^TPBK_2x(t-h)$$

$$+\int_{t-h}^{t}\begin{bmatrix} x(t) \\ \dot{x}(\alpha)\end{bmatrix}^T \begin{bmatrix} X_1 & Y_1-PBK_1 \\ Y_1^T-(PBK_1)^T & Z_1\end{bmatrix}\begin{bmatrix} x(t) \\ \dot{x}(\alpha)\end{bmatrix}d\alpha +$$

$$h^{-1}\int_{-h}^{t}\int_{t-h}^{t+\beta}\begin{bmatrix} x(t) \\ \ddot{x}(\alpha)\end{bmatrix}^T \begin{bmatrix} X' & Y'-PBK_2 \\ Y'^T-(PBK_2)^T & Z'\end{bmatrix}\begin{bmatrix} x(t) \\ \ddot{x}(\alpha)\end{bmatrix}d\alpha d\beta$$

$$= x^T\{A^TP+PA+h\left(X_1+X'/2\right)+Y_1+Y_1^T+h^{-1}\left(Y'+Y'^T\right)\}x$$

$$+x^T(t)(PBK_1-Y_1)x(t-h)+x^T(t-h)(PBK_1-Y_1)^Tx(t)$$

$$+x^T(t)(PBK_2-Y')\dot{x}(t-h)+\dot{x}^T(t-h)(PBK_2-Y')^Tx(t)$$

$$-h^{-1}x^T(t)Y'x(t-h)-h^{-1}x^T(t-h)Y'x(t)$$

$$+h^{-1}\int_{-h}^{0}\int_{t-h}^{t+\beta}\ddot{x}^T(\alpha)Z'\ddot{x}(\alpha)d\alpha d\beta + \int_{t-h}^{t}\dot{x}^T(\alpha)Z_1\dot{x}(\alpha)d\alpha \quad (20)$$

Also, the time derivative of V_4 can be represented as follows:

$$\dot{V}_4 = -h^{-1}\int_{-h}^{0}\int_{t-h}^{t+\beta}\ddot{x}^T(\alpha)Z'\ddot{x}(\alpha)d\alpha d\beta + (h/2)\ddot{x}^T(t)Z'\ddot{x}(t)$$

$$+(1/2)\ddot{x}^T(t)R'\ddot{x}(t)-(1/2)\ddot{x}^T(t-h)R'\ddot{x}(t-h)$$

$$= -h^{-1}\int_{-h}^{0}\int_{t-h}^{t+\beta}\ddot{x}^T(\alpha)Z'\ddot{x}(\alpha)d\alpha d\beta$$

$$+\left(Ax(t)+BK_1x(t-h)+BK_2\dot{x}(t-h)\right)^T.$$

$$A^T\left(R'/2+(h/2)Z'\right)A\left(Ax(t)+BK_1x(t-h)+BK_2\dot{x}(t-h)\right)$$

$$+\dot{x}^T(t-h)(BK_1)^T\left(R'/2+(h/2)Z'\right).$$

$$A\left(Ax(t)+BK_1x(t-h)+BK_2\dot{x}(t-h)\right)$$

$$+\left(Ax(t)+BK_1x(t-h)+BK_2\dot{x}(t-h)\right)^TA^T\left(R'/2+(h/2)Z'\right).$$

$$BK_1\dot{x}(t-h)+\ddot{x}^T(t-h)(BK_2)^T\left(R'/2+(h/2)Z'\right).$$

$$A\left(Ax(t)+BK_1x(t-h)+BK_2\dot{x}(t-h)\right)$$

$$+\left(Ax(t)+BK_1x(t-h)+BK_2\dot{x}(t-h)\right)^TA^T\left(R'/2+(h/2)Z'\right).$$

$$BK_2\ddot{x}(t-h)+\dot{x}^T(t-h)(BK_1)^T\left(R'/2+(h/2)Z'\right)BK_2\ddot{x}(t-h)$$

$$+\ddot{x}^T(t-h)(BK_2)^T\left(R'/2+(h/2)Z'\right)BK_1\dot{x}(t-h)$$

$$+\dot{x}^T(t-h)(BK_1)^T\left(R'/2+(h/2)Z'\right)BK_1\dot{x}(t-h)$$

$$+\ddot{x}^T(t-h)(BK_2)^T\left(R'/2+(h/2)Z'\right)BK_2\ddot{x}(t-h) \quad (21)$$

$$-(1/2)\ddot{x}^T(t-h)R'\ddot{x}(t-h)$$

It can be shown that the time derivative of V_2 and V_3 are

$$
\begin{aligned}
\dot{V}_2 = h\{ & x^T A^T Z_1 A x + 2x^T A^T Z_1 BK_1 x(t-h) \\
& + 2x^T A^T Z_1 BK_2 \dot{x}(t-h) \\
& + x^T(t-h)(BK_1)^T Z_1 BK_1 x(t-h) \\
& + 2x^T(t-h)(BK_1)^T Z_1 BK_2 \dot{x}(t-h) \\
& + \dot{x}^T(t-h)(BK_2)^T Z_1 BK_2 \dot{x}(t-h)\} \\
& - \int_{t-h}^{t} \dot{x}^T(\alpha) Z_1 \dot{x}(\alpha) d\alpha
\end{aligned} \tag{22}
$$

$$
\begin{aligned}
\dot{V}_3 = & x^T(t)Qx(t) - x^T(t-h)Qx(t-h) \\
& + \dot{x}^T(t)R_1\dot{x}(t) - \dot{x}^T(t-h)R_1\dot{x}(t-h)
\end{aligned} \tag{23}
$$

Then a new bound of $\dot{V}$ is as follows:

$$
\dot{V}(t) = \sum_{i=1}^{4} \dot{V}_i \le \xi^T \Psi \xi \tag{24}
$$

where $\xi = \begin{bmatrix} x(t) & x(t-h) & \dot{x}(t-h) & \ddot{x}(t-h) \end{bmatrix}$ and

$$
\Psi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} \\ * & \Phi_{22} & \Phi_{23} & \Phi_{24} \\ * & * & \Phi_{33} & \Phi_{34} \\ * & * & * & \Phi_{44} \end{bmatrix}
$$

where

$$
\begin{aligned}
\Phi_{11} = & A^T P + PA + Y_1 + Y_1^T + h(X_1 + X'/2) + h^{-1}(Y' + Y'^T) \\
& + A^T(R_1 + hZ_1)A + A^T A^T(R'/2 + (h/2)Z')AA + Q
\end{aligned}
$$

$$
\begin{aligned}
\Phi_{12} = & PBK_1 - Y - h^{-1}Y' + A^T(R_1 + hZ_1)BK_1 \\
& + A^T A^T(R'/2 + (h/2)Z')ABK_1
\end{aligned}
$$

$$
\begin{aligned}
\Phi_{13} = & PBK_2 - Y' + A^T(R_1 + hZ_1)BK_2 \\
& + A^T A^T(R'/2 + (h/2)Z')(BK_1 + ABK_2)
\end{aligned}
$$

$$
\Phi_{14} = A^T A^T(R'/2 + (h/2)Z')BK_2
$$

$$
\begin{aligned}
\Phi_{22} = & -Q + (BK_1)^T(R_1 + hZ_1)BK_1 \\
& + (ABK_1)^T(R'/2 + (h/2)Z')ABK_1
\end{aligned}
$$

$$
\begin{aligned}
\Phi_{23} = & (BK_1)^T(R_1 + hZ_1)BK_2 \\
& + (BK_1)^T A^T(R'/2 + (h/2)Z')(BK_1 + ABK_2)
\end{aligned}
$$

$$
\Phi_{24} = (BK_1)^T A^T(R'/2 + (h/2)Z')BK_2
$$

$$
\begin{aligned}
\Phi_{33} = & -R_1 + (BK_2)^T(R_1 + hZ_1)BK_2 \\
& + (BK_1 + ABK_2)^T(R'/2 + (h/2)Z')(BK_1 + ABK_2)
\end{aligned}
$$

$$
\Phi_{34} = (BK_1 + ABK_2)^T(R'/2 + (h/2)Z')BK_2
$$

$$
\Phi_{44} = -R'/2 + (BK_2)^T(R'/2 + (h/2)Z')BK_2
$$

If Ψ<0, i.e. $\dot{V} < 0$, then the asymptotic stability of the closed-loop system (6) is established. Ψ<0 is equivalent to

$$
\begin{bmatrix} \Phi_2 & PBK_1 - Y_1 - h^{-1}Y' & PBK_2 - Y' & 0 \\ * & -Q_1 & 0 & 0 \\ * & * & -R_1 & 0 \\ * & * & * & -R'/2 \end{bmatrix} + NM^{-1}N^T < 0
$$

where

$$
\Phi_2 = A^T P + PA + Y_1 + Y_1^T + h(X_1 + X'/2) + h^{-1}(Y' + Y'^T) + Q
$$

$$
M = diag\begin{pmatrix} hZ_1 & R_1 & (h/2)Z' & R'/2 \end{pmatrix}
$$

$$
N = \begin{bmatrix} hA^T Z_1 & A^T R_1 & (h/2)A^T A^T Z' & A^T A^T R'/2 \\ h(BK_1)^T Z_1 & (BK_1)^T R_1 & (h/2)(ABK_1)^T Z' & (ABK_1)^T R'/2 \\ h(BK_2)^T Z_1 & (BK_2)^T R_1 & (h/2)(BK_1 + ABK_2)^T Z' & (BK_1 + ABK_2)^T R'/2 \\ 0 & 0 & (h/2)(BK_2)^T Z' & (BK_2)^T R'/2 \end{bmatrix}
$$

Defining Z_2=$Z'/2$, R_2= $R'/2$, X_2= $X'/2$, $h^{-1}Y'$=Y_2, the above matrix inequality can be rewritten as

$$
\begin{aligned}
& \begin{bmatrix} \Phi & PBK_1 - Y_1 - Y_2 & PBK_2 - hY_2 & 0 \\ * & -Q_1 & 0 & 0 \\ * & * & -R_1 & 0 \\ * & * & * & -R_2 \end{bmatrix} + \\
& \begin{bmatrix} hA^T Z_1 & A^T R_1 & A^T A^T Z_2 & A^T A^T R_2 \\ h(BK_1)^T Z_1 & (BK_1)^T R_1 & (ABK_1)^T Z_2 & (ABK_1)^T R_2 \\ h(BK_2)^T Z_1 & (BK_2)^T R_1 & (BK_1 + ABK_2)^T Z_2 & (BK_1 + ABK_2)^T R_2 \\ 0 & 0 & (BK_2)^T Z_2 & (BK_2)^T R_2 \end{bmatrix} . M^{-1} \\
& \cdot \begin{bmatrix} hA^T Z_1 & A^T R_1 & A^T A^T Z_2 & A^T A^T R_2 \\ h(BK_1)^T Z_1 & (BK_1)^T R_1 & (ABK_1)^T Z_2 & (ABK_1)^T R_2 \\ h(BK_2)^T Z_1 & (BK_2)^T R_1 & (BK_1 + ABK_2)^T Z_2 & (BK_1 + ABK_2)^T R_2 \\ 0 & 0 & (BK_2)^T Z_2 & (BK_2)^T R_2 \end{bmatrix}^T < 0
\end{aligned} \tag{25}
$$

where

$$
\Phi = A^T P + PA + Y_1 + Y_1^T + h(X_1 + X_2) + Y_2 + Y_2^T + Q
$$

Similarly, the LMIs (17) and (19) are represented as

$$
\begin{bmatrix} X_1 & Y_1 \\ Y_1^T & Z_1 \end{bmatrix} > 0 \tag{26}
$$

and

$$
\begin{bmatrix} 2X_2 & hY_2 \\ hY_2^T & 2Z_2 \end{bmatrix} > 0 \tag{27}
$$

By the above representation, the LMIs (7) ~ (8) are provided. Moreover, using Schur complement for (25), the matrix inequality (9) is obtained. This completes the proof. □

4. Stabilization with Time-varying Delay

In this section, we are concerned with providing delay-dependent conditions for stability systems with time-varying input delay. The stability conditions are provided in terms of delay-dependent matrix inequalities. This subject is discussed in the following theorem in more details.

$$\begin{bmatrix} \Phi & PBK_1-Y_1-Y_2 & PBK_2-\bar{\tau}Y_2 & 0 & \bar{\tau}A^TZ_1 & A^TR_1 & \bar{\tau}A^TA^TZ_2 & A^TA^TR_2 \\ * & -Q & 0 & 0 & \bar{\tau}(BK_1)^TZ_1 & (BK_1)^TR_1 & \bar{\tau}(ABK_1)^TZ_2 & (ABK_1)^TR_2 \\ * & * & -R_1-\bar{\tau}Y_2 & 0 & \bar{\tau}(BK_2)^TZ_1 & (BK_2)^TR_1 & \bar{\tau}(BK_1+ABK_2)^TZ_2 & (BK_1+ABK_2)^TR_2 \\ * & * & * & -R_2 & 0 & 0 & \bar{\tau}(BK_2)^TZ_2 & (BK_2)^TR_2 \\ * & * & * & * & -\bar{\tau}Z_1 & 0 & 0 & 0 \\ * & * & * & * & * & -R_1 & 0 & 0 \\ * & * & * & * & * & * & -\bar{\tau}Z_2 & 0 \\ * & * & * & * & * & * & * & -R_2 \end{bmatrix} < 0 \quad (28)$$

Theorem 4.1: Consider the time-delay system (5). Given scalar $\bar{\tau} > 0$, the closed-loop system with control law (2) and $w(t)\equiv 0$ is asymptotically stable for any time-delay $\tau(t)$ satisfying $0 < \tau(t) \le \bar{\tau}$, if there exist positive definite symmetric matrices P, Q, R_1, R_2, Z_1, $Z_2 \in \Re^{n\times n}$, negative definite symmetric matrix Y_2 and matrices Y_1, X_1, $X_2 \in \Re^{n\times n}$, K_1, $K_2 \in \Re^{m\times n}$ satisfying matrix inequalities (28), (29) and (30).

$$\begin{bmatrix} X_1 & Y_1 \\ Y_1^T & Z_1 \end{bmatrix} > 0 \quad (29)$$

and

$$\begin{bmatrix} 2X_2 & \bar{\tau}Y_2 \\ \bar{\tau}Y_2^T & 2Z_2 \end{bmatrix} > 0 \quad (30)$$

Proof: Defining $\tau^{-1}Y' = Y_2$ by the assumption of $Y' = Y'^T < 0$ and then adding and subtracting the terms $x^T(t)(\tau Y_2)x(t)$ and $\dot{x}^T(t-\tau)(\tau Y_2)^T\dot{x}(t-\tau)$ in (24), a new upper bound for $\dot{V}$ can be rewritten as follows:

$$\begin{aligned} \dot{V}(t) = \sum_{i=1}^{4}\dot{V}_i \le\ & x^T\{A^TP+PA+\bar{\tau}\left(X_1+(1/2)X'\right)+Y_1+Y_1^T \\ & +\left(Y_2+Y_2^T\right)\}x+2x^T(t)\left(PBK_1-Y_1\right)x(t-\tau(t)) \\ & +2x^T(t)PBK_2\dot{x}(t-\tau(t))+\left(x^T(t)+\dot{x}(t-\tau(t))\right)^T. \\ & (-\bar{\tau}Y_2)\left(x^T(t)+\dot{x}(t-\tau(t))\right)-x^T(t)Y_2x(t-\tau(t)) \\ & -x^T(t-\tau(t))Y_2x(t)+(\bar{\tau}/2)\ddot{x}^T(t)Z'\ddot{x}(t) \\ & +(1/2)\ddot{x}^T(t)R'\ddot{x}(t)-(1/2)\ddot{x}^T(t-\tau(t))R'\ddot{x}(t-\tau(t)) \\ & +\left(Ax+BK_1x(t-\tau(t))+BK_2\dot{x}(t-\tau(t))\right)^T\bar{\tau}Z_1. \\ & \left(Ax+BK_1x(t-\tau(t))+BK_2\dot{x}(t-\tau(t))\right)+x^T(t)Qx(t) \\ & -x^T(t-\tau(t))Qx(t-\tau(t))+\dot{x}^T(t)R_1\dot{x}(t) \\ & -\dot{x}^T(t-\tau(t))R_1\dot{x}(t-\tau(t))+x^T(t)\left(\tau(t)Y_2\right)x(t) \\ & +\dot{x}^T(t-\tau)\left(\tau(t)Y_2\right)^T\dot{x}(t-\tau) \\ & =\Omega_1+x^T(t)\left(\tau(t)Y_2\right)x(t)+\dot{x}^T(t-\tau)\left(\tau(t)Y_2\right)^T\dot{x}(t-\tau) \\ & =\Omega \end{aligned}$$

If $\Omega<0$, i.e. $\dot{V}<0$, then the asymptotic stability of the closed-loop system is guaranteed for any constant time-delay $\tau(t)$ satisfying $0<\tau(t)\le\bar{\tau}$. For this aim, considering the constraint $Y_2 = Y_2^T < 0$, it suffices for $\Omega_1<0$ to be satisfied. Substituting

$$\ddot{x}(t) = (d/dt)\left(Ax(t)+BK_1x(t-\tau(t))+BK_2\dot{x}(t-\tau(t))\right),$$

and

$$\dot{x}(t) = Ax(t)+BK_1x(t-\tau(t))+BK_2\dot{x}(t-\tau(t)),$$

the inequality $\Omega_1<0$ is replaced by

$$\xi^T\Theta\xi<0 \quad (31)$$

where $\xi = \begin{bmatrix} x(t) & x(t-h) & \dot{x}(t-h) & \ddot{x}(t-h) \end{bmatrix}$ and

$$\Theta = \begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} & \theta_{14} \\ * & \theta_{22} & \theta_{23} & \theta_{24} \\ * & * & \theta_{33} & \theta_{34} \\ * & * & * & \theta_{44} \end{bmatrix}$$

where

$$\begin{aligned} \theta_{11} =\ & A^TP+PA+Y_1+Y_1^T+\bar{\tau}\left(X_1+X'/2\right)+A^T\left(R_1+\bar{\tau}Z_1\right)A \\ & +A^TA^T\left(R'/2+(\bar{\tau}/2)Z'\right)AA+(2-\bar{\tau})Y_2+Q \\ \theta_{33} =\ & -R_1-\bar{\tau}Y_2+\left(BK_2\right)^T\left(R_1+\bar{\tau}Z_1\right)BK_2 \\ & +\left(BK_1+ABK_2\right)^T\left(R'/2+(\bar{\tau}/2)Z'\right)\left(BK_1+ABK_2\right) \\ \theta_{ij} =\ & \Phi_{ij}\big|_{\tau=\bar{\tau}} \quad for\ i,j=1,2,3,4 \quad i\neq j \\ & and\ i,j=2,4 \quad i=j \end{aligned} \quad (32)$$

If Eq. Θ is negative definite, then inequality (31) is satisfied, i.e. $\dot{V}<0$, and the asymptotic stability of the closed-loop system (6) is established. Similar to the proof of Theorem 3.1, by Schur complement and defining the same change of variables, the matrix inequality (28) is obtained. Also, the condition (19) can be written as

$$\begin{bmatrix} X' & \tau(t)Y_2 \\ \tau(t)Y_2^T & Z' \end{bmatrix} > 0 \quad (33)$$

The above condition enforces X' to be positive definite. Using Schur complement and the same change of variables as the previous sections, we obtain

$$2X_2-\left(\tau(t)Y_2\right)\left(2Z_2\right)^{-1}\left(\tau(t)Y_2\right)^T>0 \quad (34)$$

On the other hand we have

$$2X_2-\tau(t)Y_2\left(2Z_2\right)^{-1}\left(\tau Y_2\right)^T>2X_2-\bar{\tau}Y_2\left(2Z_2\right)^{-1}\left(\bar{\tau}Y_2\right)^T \quad (35)$$

Therefore, satisfying the following inequality guarantees the inequality (34) to be satisfied.

$$2X_2-\left(\bar{\tau}Y_2\right)\left(2Z_2\right)^{-1}\left(\bar{\tau}Y_2\right)^T>0 \quad (36)$$

Applying Schur complement, the matrix inequality (30) is obtained. Moreover, similar to Theorem 3.1, the condition (29) is considered through the proof of this Theorem. This completes the proof. □

5. Numerical Examples

In this section we provide two examples regarding the stabilizing controller design to demonstrate the effectiveness of the proposed method.

Example 1: We consider the following input-delay system:

$$A=\begin{bmatrix}0 & 1\\ 5 & -0.8\end{bmatrix},\ B=\begin{bmatrix}0\\ 1\end{bmatrix},$$

$$C=\begin{bmatrix}0 & 1\end{bmatrix},\ D=0,$$

and $h=50ms$. Using Theorem 3.1, resulting controller gains are given by

$$K_1=\begin{bmatrix}-5.46 & -0.87\end{bmatrix},$$

$$K_2=\begin{bmatrix}-0.1 & -0.0108\end{bmatrix}$$

Figure (1) shows the state variables of the closed-loop system. This result illustrates the effectiveness of the proposed method which guarantees the stability of the closed-loop system for a prescribed constant time-delay h.

Example 2: In order to illustrate Theorem 4.1, we consider an unstable time-delay system with state-space equation (5) where

$$A=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 10 & -8\end{bmatrix},\ B=\begin{bmatrix}0\\ 0\\ 1\end{bmatrix},$$

$$C=\begin{bmatrix}1 & 0 & 0\end{bmatrix},\ D=0,$$

and $\overline{\tau}=40ms$. Applying Theorem 4.1, a pair of stabilizing controller gain in the feasibility region can be expressed as follows:

$$K_1=\begin{bmatrix}-6.18 & 0.524 & 1.75\end{bmatrix},$$

$$K_2=\begin{bmatrix}-5.703 & 1.37 & 0.214\end{bmatrix}$$

Figure (2) represents the state variable converged to zero. Therefore, the above controller gains guarantee the stability of the closed loop system for all varying time-delay $\tau(t)\le 40ms$.

6. Conclusions

Stabilizing of a time-delay system with input delay has been elaborated in this paper. The resulting closed-loop system with the proposed composite control law is a particular system of neutral type. In this system, the coefficients of delayed terms depend on the control law's parameters. The Lyapunov theory has been used to derive some new delay-dependent sufficient conditions for the stability of the closed loop system. For this aim, a new model transformation has been introduced for the time delay system of neutral type. Matrix inequalities have been derived as the stability conditions for both constant and varying time-delay. Moreover, two numerical examples have been presented in this paper. Simulation results demonstrate the validity of our method.

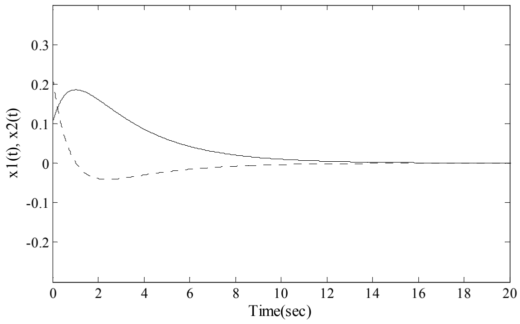

Fig. 1

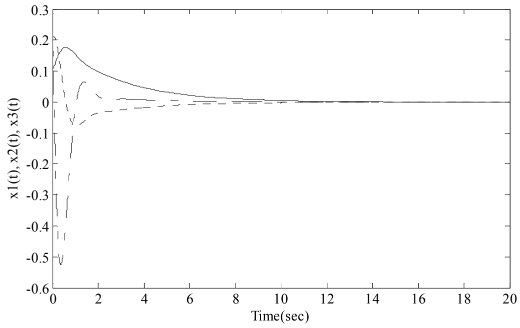

Fig. 2

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