DYNAMIC ANALYSIS AND CONTROL OF CABLE DRIVEN ROBOTS WITH ELASTIC CABLES

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Received July 2011, Accepted October 2011
No. 11-CSME-72, E.I.C. Accession 3312

Abstract

In this paper modeling and control of cable driven redundant parallel manipulators with flexible cables, are studied in detail. Based on new results, in fully constrained cable robots, cables can be modeled as axial linear springs. Considering this assumption the system dynamics formulation is developed using Lagrange approach. Since in this class of robots, all the cables should remain in tension for the whole workspace, the notion of internal forces are introduced and incorporated in the proposed control algorithm. The control algorithm is developed in cable coordinates in which the internal forces play an important role. Finally, asymptotic stability of the closed loop system is analyzed through Lyapunov theorem, and the performance of the proposed algorithm is studied by simulations.

Keywords: Cable driven robot; redundancy; elastic cables; flexibility; internal force; Lyapunov stability.

ANALYSE DYNAMIQUE ET COMMANDE DES ROBOTS PARALLÈLES ENTRAÎNÉS PAR CÂBLES

Résumé

Dans cette étude, la modélisation et la commande des mécanismes parallèles redondants entraînés par câbles sont étudiées en détail en tenant compte de la flexibilité dans les câbles. Dans cette classe de robots, les câbles doivent rester sous tension dans l'espace atteignable global. Basé sur des nouveaux travaux concernant les mécanismes à câbles complètement contraints, les câbles peuvent être modélisés comme des ressorts axiaux. Considérant cette hypothèse, la formulation dynamique du système est développée en utilisant l'approche de Lagrange. Les forces internes sont introduites et intégrées dans l'algorithme de contrôle proposé. Cet algorithme est formulé dans le système de coordonnées opérationnelles dans lequel les forces internes jouent un rôle primordiale. Enfin, il est démontré que le système en boucle fermé est asymptotiquement stable à travers l'analyse de Lyapunov, et la performance de l'algorithme proposée est vérifiée par des simulations.

Keywords: Robots parallèles entraînés par câbles; redondance; câbles élastiques; flexibilité; force intérieure; stabilité de Lyapunov.

1. INTRODUCTION

Increasing performance demands necessitates design of new types of robots with larger workspace, being capable to perform at higher accelerations. It is well known that parallel manipulators can generally perform better than serial manipulators in terms of stringent stiffness and acceleration requirements. However, their limited workspace and the existence of singular regions inside their workspace confines the applications of parallel manipulators for large workspace requirement. In a cable driven parallel manipulator the linear actuators of parallel manipulator are replaced by electrical powered cable drivers, which leads immediately to a larger workspace. A cable driven parallel robot consists of an end-effector and a number of active cables connected to the end-effector from one side, while the cable drivers are fixed to the base. By this means the position and orientation of the end-effector is forced toward the desired values by careful control of cables lengths. Cable driven robots have some advantages over conventional robots, and since cables are used in their structure, they are potentially suited for large workspace applications such as large adaptive reflector and SkyCam [1,2]. Since cables have negligible mass and inertia, this type of robots are also suitable for high acceleration applications. Characteristics such as transportability and ease of assembly/disassembly, reconfigurability by changing the location of motors and updating the control algorithm and economical structure and maintenance due to simple mechanical structure and low cost and simple mechanical components, insure their high potential in many applications such as handling of heavy materials [3], high speed manipulation [4], rapidly deployable rescue robots, cleanup of disaster areas [5], and access to remote locations and interaction to hazardous environment [6].

The most important distinction of cable driven robots to conventional parallel robot roots in the major property of cables in their actuation. Cables work only under tension and they can be used only to *pull* and *not to push* any object. Therefore, in this class of robots, the cables must remain in tension in the whole workspace. Based on this fact, cable driven robots can be classified into two types; under constrained and fully constrained robots [5]. This paper focuses on the control of fully constrained cable driven robots, in which a major challenge in mechanical and control design of them, is the nonlinear behavior of the cables. Cables are usually flexible and show elongation under tension. This flexibility may lead to position and orientation errors. Moreover, the system may encounter unavoidable vibrations which may cause uncontrollability of the robot. Cable induced vibration may be a major concern for applications which require high bandwidth or high stiffness [7].

Though, the cable characteristics have been studied from long time ago, especially in civil engineering, using cables in parallel robots demonstrates a quite new analysis horizon. Generally, in civil engineering applications cables are very heavy and bulky, and only static analysis is performed in the design of bridge type structures [8,9]. However, in cable driven robots the cables are very light, and their dynamic analysis is of utmost importance in order to carefully study the motion of the end-effector. Reported studies on the effect of cable flexibility on modeling, optimal design and control of such manipulators are very limited and these effects are usually neglected in the literature. Stiffness analysis of the cable robots with flexible cables, is reported in [10] and [11]. Behzadipour and Khajehpour introduce a four springs model for cable and achieve necessary and sufficient conditions for stability of system based on positive definiteness of the robot stiffness matrix [10]. Kozak et al. considers the mass of the cables and by using a static model of cable, they show how cable sagging affects the kinematics and stiffness of the system [11]. In [12], a static model of cable is proposed and static deformation of

cables is determined. The vibration problem of cable driven robots has received less attention though, and in order to model the vibration due to flexibility of cables, Zhang et al. has used wave equation to model the cable vibration, provided that the cables lengths are constant [13]. However, in practice this assumption is very limiting for cable driven robots. Kawamura et al. showed that the internal forces of cables can play a vital role in reducing the vibration of the system [14]. To investigate the vibration analysis of cable driven robots, linear and nonlinear springs are used to model the behavior of cables [12,14]. However, in most of the reported researches it is assumed that cable has only axial flexibility and the transversal flexibility is negligible.

In the dynamic modeling of cable driven robots, it should be noticed that a complete dynamic model for cable driven robots is very complicated. Furthermore, since the obtained model shall be used in controller design, further simplifications are needed. Thus, in practice it is proposed to only include the dominant effects in the dynamic analysis. For this reason in many robotic applications, cable mass is neglected and cable is considered as a rigid element [15,16]. With those assumptions the dynamics of cable driven robot is reduced to the end-effector dynamics. However, in practice using this assumption will mislead the results in control especially the stability of the manipulator. Ottaviano and Castelli have shown suitability of neglecting the cables mass, based on numerical and experimental results given in [17]. They have shown that this assumption is valid if the ratio of end-effector to cables masses is large or generally, the ratio of the end-effector wrenches to the tensions is small. Using natural frequencies of system, Diao and Ma have shown in [7], that in fully constrained cable driven robots the vibration of cable manipulator due to the transversal vibration of cables can be neglected compared to that of axial flexibility. It has been justified, therefore, to only model the cable as an axial spring in cable driven robots. By this means, this model will describe the dominant dynamic characteristics of the cables and can be used in the dynamic model of cable robot. Based on these observations, in this paper axial spring is used to model dominant dynamics of cable and a more precise model of the cable driven robot with flexible cable is derived and being used in the controller design and stability analysis.

In this paper, considering axial flexibility in cables, a new dynamic model for fully constrained cable driven robots is presented. In this structure the cables' lengths with and without tension are considered as the describing states in the model. Then the control of the system is studied in detail, while the stringent requirement of keeping the cables in tension is fully addressed. Next, the stability of the system is analyzed through Lyapunov second method, and it is proven that the closed loop system with the proposed control algorithm is asymptotically stable. Finally, the performance of the proposed algorithm is examined through simulations.

2. DYNAMIC ANALYSIS

Generally, four different models of cable have been considered for the dynamic analysis of cable driven robots. The simplest one is massless inextensible model. It is assumed in this model that cables have no mass and no flexibility. In elastic but massless models, only the elasticity of cable is considered and the mass of cable is ignored. In many applications especially in fully constrained applications, where the workspace is not very large and the cable mass is much lower than that of the end-effector, this model is found to be appropriate [7,17]. Other models are also reported in the literature, namely cable with continuous mass and elasticity [11,12], and cables with lumped mass and elasticity [18], which are mostly used in static analysis of cable robots.

2.1. Robot Dynamics with Ideal Cables

In this section let us first assume that the mass and flexibility of the cables can be ignored. The dynamics of cable driven robot under this assumption reduces to that of the end-effector, and therefore, it can be expressed by the following vector equation [15,16]:

$$\mathbf{M}(\mathbf{x})\ddot{\mathbf{x}}+\mathbf{N}(\mathbf{x},\dot{\mathbf{x}})=-\mathbf{J}^T\boldsymbol{\tau} \qquad \boldsymbol{\tau}\geq 0 \tag{1}$$

in which,

$$\mathbf{N}(\mathbf{x},\dot{\mathbf{x}})=\mathbf{C}(\mathbf{x},\dot{\mathbf{x}})\dot{\mathbf{x}}+\mathbf{G}(\mathbf{x}) \tag{2}$$

and,

$\mathbf{M}(\mathbf{x})$: Mass matrix of the robot, $\mathbf{C}(\mathbf{x},\dot{\mathbf{x}})\dot{\mathbf{x}}$: Coriolis and centrifugal terms, $\mathbf{G}(\mathbf{x})$: Vector of gravity terms, $\mathbf{J}$: Jacobian matrix of robot and $\mathbf{x}$: Vector of generalized coordinates for position and orientation of end-effector. On the other hand, the actuators dynamics is represented by

$$\mathbf{I}_m\ddot{\mathbf{q}}+\mathbf{D}\dot{\mathbf{q}}-r\boldsymbol{\tau}=\mathbf{u} \tag{3}$$

where,

$\mathbf{q}$: Angles vector of motors shaft, $\mathbf{I}_m$: Actuator moments of inertia matrix, $\mathbf{D}$: Actuator viscous friction matrix, r: The radius of pulleys, $\boldsymbol{\tau}$: Cable tension vector and $\mathbf{u}$: Motor torque vector.

As for the position reference, define all $\mathbf{q}$ to be zero when the end-effector centroid is located at the center of the frame; from this configuration positive angle $\mathbf{q}$ will cause a change $\Delta\mathbf{L}$ in cables lengths, therefore, we have:

$$\Delta\mathbf{L}=r\mathbf{q}=\mathbf{L}-\mathbf{L}_0\Rightarrow\mathbf{q}=r^{-1}(\mathbf{L}-\mathbf{L}_0) \tag{4}$$

Where $\mathbf{L}_0$ is the initial length vector at $\boldsymbol{x}=\mathbf{0}$. By differentiating and using manipulator Jacobian definition $\dot{\mathbf{L}}=\mathbf{J}\dot{\mathbf{x}}$:

$$\dot{\mathbf{q}}=r^{-1}\dot{\mathbf{L}}=r^{-1}\mathbf{J}\dot{\mathbf{x}}\ ,\ \ddot{\mathbf{q}}=r^{-1}\mathbf{J}\ddot{\mathbf{x}}+r^{-1}\dot{\mathbf{J}}\dot{\mathbf{x}} \tag{5}$$

Using Eqs. (5), (3) and (1) and some manipulations we can show that:

$$\mathbf{M}_{eq}(\mathbf{x})\ddot{\mathbf{x}}+\mathbf{N}_{eq}(\mathbf{x},\dot{\mathbf{x}})=\mathbf{J}^T\mathbf{u} \tag{6}$$

in which,

$$\mathbf{M}_{eq}=r\mathbf{M}(\mathbf{x})+r^{-1}\mathbf{J}^T\mathbf{I}_m\mathbf{J}$$

$$\mathbf{N}_{eq}=r\mathbf{N}(\mathbf{x},\dot{\mathbf{x}})+r^{-1}\mathbf{J}^T\mathbf{I}_m\dot{\mathbf{J}}\dot{\mathbf{x}}+r^{-1}\mathbf{J}^T\mathbf{D}\mathbf{J}\dot{\mathbf{x}} \tag{7}$$

It can be seen that actuator dynamics is transformed into task space by Jacobian matrix, which is a projection from cable length space to task space.

2.2. Robot Dynamics with Elastic Cables

In the parallel cable driven robots, each cable is driven by a pulley which is usually driven by an electric motor with gears. In order to observe the vibration phenomena, elasticity of cables must be considered in modeling and control [7,14]. In this type of robots when the flexibility in cables is considered, actuator position is not directly related to end-effector position. Thus for such models, positions of actuators and end-effector may be considered as vector of system states. In other words, both the cables lengths before tension and after tension, that are related to end-effector position by Jacobian matrix, may be used as independent variables in order to analyze the dynamics of the manipulator. New research results have shown that in fully constrained cable robots, dominant dynamics of cables are longitudinal vibration [7], therefore, axial spring model can suitably describe the effects of dominant dynamics of cable.

In order to model a general cable driven robot with n cables assume that: $L_{1_i} : i=1,2,...,n$ denotes the length of ith cable with tension and $L_{2_i} : i=1,2,...,n$ denotes the length of the ith cable without tension. If the system is rigid, then $L_{1_i}=L_{2_i}$, $\forall i$. Let us denote:

$$\mathbf{L}=(L_{1_1},L_{1_2},\cdots,L_{1_n},L_{2_1},L_{2_2},\cdots,L_{2_n})^T=(\mathbf{L}_1^T,\mathbf{L}_2^T)^T \tag{8}$$

Furthermore, if the flexibility is modeled with a linear axial spring with constant k_i, then the potential energy of system can be expressed by: $P=P_0+P_1$. In this equation P_0 is the potential energy of rigid robot and P_1 is the potential energy of cables. Using linear axial spring model for cable, the total potential energy of cables is

$$P_1=\frac{1}{2}(\mathbf{L}_1-\mathbf{L}_2)^T\mathbf{K}(\mathbf{L}_1-\mathbf{L}_2) \tag{9}$$

where, $\mathbf{K}$ is the stiffness matrix of cables. Furthermore, kinetic energy of system is

$$K=\frac{1}{2}\dot{\mathbf{x}}^T\mathbf{M}(\mathbf{x})\dot{\mathbf{x}}+\frac{1}{2}\dot{\mathbf{q}}^T\mathbf{I}_m\dot{\mathbf{q}} \tag{10}$$

in which, $\mathbf{x}$ is the generalized coordinate in task space, $\mathbf{q}$ is the motor shaft position vector, $\mathbf{M}(\mathbf{x})$ is the mass matrix and $\mathbf{I}_m$ is the actuator moments of inertia. The Lagrangian is expressed by

$$\mathcal{L}=K-P=\frac{1}{2}\dot{\mathbf{x}}^T\mathbf{M}(\mathbf{x})\dot{\mathbf{x}}+\frac{1}{2}\dot{\mathbf{q}}^T\mathbf{I}_m\dot{\mathbf{q}}-P_0-\frac{1}{2}(\mathbf{L}_1-\mathbf{L}_2)^T\mathbf{K}(\mathbf{L}_1-\mathbf{L}_2) \tag{11}$$

The equations of motion of the system can be written using Euler-Lagrange formulation:

$$\frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{x}_i}\right)-\frac{\partial\mathcal{L}}{\partial x_i}=Q_i \tag{12}$$

With some manipulation the final equations of motion may be derived by the following form:

$$\mathbf{M}(\mathbf{x})\ddot{\mathbf{x}}+\mathbf{N}(\mathbf{x},\dot{\mathbf{x}})=\mathbf{J}^T\mathbf{K}(\mathbf{L}_2-\mathbf{L}_1) \tag{13}$$

$$\mathbf{I}_m\ddot{\mathbf{q}}+r\mathbf{K}(\mathbf{L}_2-\mathbf{L}_1)+\mathbf{D}\dot{\mathbf{q}}=\mathbf{u} \tag{14}$$

in which,

$$\mathbf{N}(\mathbf{x},\dot{\mathbf{x}}) = \mathbf{C}(\mathbf{x},\dot{\mathbf{x}})\dot{\mathbf{x}} + \mathbf{G}(\mathbf{x}) \ , \ \mathbf{L}_2 - \mathbf{L}_0 = r\mathbf{q}$$

In these equations $\dot{\mathbf{L}}_1 = \mathbf{J}\dot{\mathbf{x}}$, and other parameters are defined as before. Equations (13) and (14) represent cable driven robot as a nonlinear and coupled system. This representation includes both rigid and flexible subsystems and their interactions.

3. CONTROLLER DESIGN

3.1. Internal Forces

In cable driven robot end-effector is supported by cables. However, since cables can provide only tension, actuator redundancy is necessary due to unidirectional characteristic of cable tension. Such redundant actuation for a cable driven robot is similar to the actuation of the multi-fingered robots, in which, contact between fingers and an object is regarded as frictionless points. Therefore, it is possible to apply the concept of vector closure which is introduced in the research of multi-finger robots to parallel cable driven robots. Generally, vector closure is expressed in the following way [19]:

In an n-dimensional space, a set of vector $\mathbf{J}^T$ is a vector closure if and only if $\mathbf{J}^T$has at least $n+1$ vectors $(\mathbf{j}_1, \ldots, \mathbf{j}_{n+1})$ satisfying the following two conditions ($\mathbf{j}_i$ is $n \times 1$):

1) Each set of n vectors in $n+1$ vectors in $\mathbf{J}^T$ is linearly independent.

2) A vector $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_{n+1})^T$ exists (β_i is scalar), that satisfies

$$\mathbf{J}^T\boldsymbol{\beta} = \sum_{i=1}^{n+1} \mathbf{j}_i \beta_i = \mathbf{0}_{\mathbf{n}\times 1} \tag{15}$$

in which, each element of vector $\boldsymbol{\beta}$ has the same sign (positive or negative), $\beta_i > 0$ (for any i) or $\beta_i < 0$ (for any i).

It is well known in parallel robots that Jacobian transpose relates the resultant forces applied on the end-effector to the cable tensions $\boldsymbol{\tau}$ [20]:

$$\mathbf{f} = \mathbf{J}^T\boldsymbol{\tau} \tag{16}$$

where, matrix $\mathbf{J}^T$ denotes Jacobian transpose and may be expressed by its columns: $\mathbf{J}^T = [\mathbf{j}_1, \mathbf{j}_2, \ldots, \mathbf{j}_m]$. The vector closure conditions mean that each cable tension remains positive while any resultant force vector can be generated. This result implies that at least $n+1$ cables are necessary to realize the motion with n degrees of freedom. Since in cable driven robots actuator redundancy is a requirement, the number of the cable actuators are greater than the degrees of freedom, and therefore, the Jacobian matrix is not square. The inverse relation to calculate the tension in cables from the resultant force using pseudo inverse may be given by:

$$\boldsymbol{\tau} = \boldsymbol{\tau}_o + (\mathbf{I} - \mathbf{J}^\dagger\mathbf{J}^T)\mathbf{c} \tag{17}$$

in which, $\mathbf{J}^\dagger$ denotes the pseudo inverse of the matrix $\mathbf{J}^T$, $\boldsymbol{\tau}_o$ denotes the base pseudo inverse solution given by $\boldsymbol{\tau}_o = \mathbf{J}^\dagger\mathbf{f}$, $\mathbf{I}$ denotes an $m \times m$ identity matrix, and $\mathbf{c}$ is any arbitrary vector in

$\mathbf{R}^m$. This general solution consists of two parts, the first part is the base solution $\boldsymbol{\tau}_o$, and the second part spans the null space of the matrix $\mathbf{J}^T$ and can be considered as internal forces among the cables. Notice that since the internal force spans the null space of $\mathbf{J}^T$ it does not contribute into the resulting force applied to the end-effector, and it only produces tension in cables in order to keep all the cables in tension. Internal force plays an important role in our proposed control algorithm.

3.2. Control Algorithm in Cable Length Space

In this section the proposed control algorithm in cable length space is discussed. This control algorithm consists of a simple PD control plus internal forces to ensure that all cables are in tension in the whole workspace, and a gravity compensation term in companion with a damping term. By using desired set point vector $\mathbf{q}_d$, the control input $\boldsymbol{u}$ is proposed to be:

$$\mathbf{u} = \mathbf{K}_p(\mathbf{q}_d - \mathbf{q}) - \mathbf{K}_v\dot{\mathbf{q}} + \mathbf{Q} + r\mathbf{Q}_G + \mathbf{K}_s(\dot{\mathbf{L}}_2 - \dot{\mathbf{L}}_1) \tag{18}$$

where, $\mathbf{K}_p(n \times n)$, in which n is the number of cables, $\mathbf{K}_v(n \times n)$ and $\mathbf{K}_s(n \times n)$ denote feedback gain matrices. The term $\mathbf{Q}(n \times n)$ denotes internal force vector and satisfies

$$\mathbf{J}^T\mathbf{Q} = 0 \tag{19}$$

It is important to note that the vector $\mathbf{Q}$ does not contribute into motion of the end-effector, and only causes internal forces in the cables. This term ensures that all cables remain in tension in the whole workspace. Moreover, this term increases the stiffness of the system, and as a result, minimizes the vibration in transversal direction of the cables. The term $\mathbf{Q}_G$ is added to compensate the gravitational force. This vector must satisfy

$$\mathbf{J}^T\mathbf{Q}_G = \mathbf{G}(\mathbf{x}) \tag{20}$$

Furthermore, $\mathbf{L}_2$ and $\mathbf{L}_1$ vectors are cables lengths without and with tension, respectively. $\mathbf{L}_2$ is relatively measured by shaft encoder at the motor side and is related to $\mathbf{q}$ by $r\mathbf{q} = \mathbf{L}_2 - \mathbf{L}_0$. $\mathbf{L}_1$ denotes cables lengths when cables are in tension and can be measured by pot strings. In the following section we discuss the stability of the closed loop system based on this proposed control algorithm.

3.3. Stability Analysis

To show that the control law given in Eq. (18) achieves set point tracking, consider the following Lyapunov function:

$$V = \frac{1}{2}\dot{\mathbf{q}}^T\mathbf{I}_m\dot{\mathbf{q}} + \frac{1}{2}\dot{\mathbf{x}}^T\mathbf{M}(\mathbf{x})\dot{\mathbf{x}} + \frac{1}{2}(\mathbf{L}_1 - \mathbf{L}_2)^T\mathbf{K}(\mathbf{L}_1 - \mathbf{L}_2) + \frac{1}{2}(\mathbf{q}_d - \mathbf{q})^T\mathbf{K}_p(\mathbf{q}_d - \mathbf{q}) \tag{21}$$

The Lyapunov function is generated using the total energy in the system. Since $\mathbf{q}_d$ is constant, the time derivative of the Lyapunov function V is given by:

$$\dot{V} = \dot{\mathbf{q}}^T \mathbf{I}_m \ddot{\mathbf{q}} + \dot{\mathbf{x}}^T \mathbf{M}(\mathbf{x})\ddot{\mathbf{x}} + \frac{1}{2}\dot{\mathbf{x}}^T \dot{\mathbf{M}}(\mathbf{x})\dot{\mathbf{x}} + (\mathbf{L}_1 - \mathbf{L}_2)^T \mathbf{K}(\dot{\mathbf{L}}_1 - \dot{\mathbf{L}}_2) + (\mathbf{q}_d - \mathbf{q})^T \mathbf{K}_p(-\dot{\mathbf{q}}) \tag{22}$$

Substitute Eqs. (13) and (14) and proposed control effort (18) in Eq. (22) :

$$\begin{aligned}\dot{V} = {} & \dot{\mathbf{q}}^T \left[\mathbf{K}_p(\mathbf{q}_d - \mathbf{q}) - \mathbf{K}_v\dot{\mathbf{q}} + \mathbf{Q} + r\mathbf{Q}_G + \mathbf{K}_s(\dot{\mathbf{L}}_2 - \dot{\mathbf{L}}_1)\right] - \dot{\mathbf{q}}^T \mathbf{D}\dot{\mathbf{q}} - \dot{\mathbf{q}}^T r\mathbf{K}(\mathbf{L}_2 - \mathbf{L}_1) + \\ & \dot{\mathbf{x}}^T \left[\mathbf{J}^T \mathbf{K}(\mathbf{L}_2 - \mathbf{L}_1) - \mathbf{C}(\mathbf{x},\dot{\mathbf{x}})\dot{\mathbf{x}} - \mathbf{G}(\mathbf{x})\right] + \frac{1}{2}\dot{\mathbf{x}}^T \dot{\mathbf{M}}(\mathbf{x})\dot{\mathbf{x}} \\ & + (\mathbf{L}_1 - \mathbf{L}_2)^T \mathbf{K}(\dot{\mathbf{L}}_1 - \dot{\mathbf{L}}_2) - (\mathbf{q}_d - \mathbf{q})^T \mathbf{K}_p \dot{\mathbf{q}}\end{aligned}$$

To simplify this equation, use the skew-symmetricity of the robot mass matrix [20,21]

$$\dot{\mathbf{x}}^T (\mathbf{M}(\mathbf{x}) - 2\mathbf{C}(\mathbf{x},\dot{\mathbf{x}}))\dot{\mathbf{x}} = 0 \tag{23}$$

Therefore,

$$\begin{aligned}\dot{V} = {} & -\dot{\mathbf{q}}^T \mathbf{K}_v \dot{\mathbf{q}} - \dot{\mathbf{q}}^T \mathbf{D}\dot{\mathbf{q}} + \dot{\mathbf{q}}^T (\mathbf{Q} + r\mathbf{Q}_G) - \dot{\mathbf{x}}^T \mathbf{G}(\mathbf{x}) + \dot{\mathbf{q}}^T \mathbf{K}_s(\dot{\mathbf{L}}_2 - \dot{\mathbf{L}}_1) + \\ & \left[\dot{\mathbf{x}}^T \mathbf{J}^T \mathbf{K}(\mathbf{L}_2 - \mathbf{L}_1) - \dot{\mathbf{q}}^{\mathbf{T}} r\mathbf{K}(\mathbf{L}_2 - \mathbf{L}_1) + (\mathbf{L}_1 - \mathbf{L}_2)^T \mathbf{K}(\dot{\mathbf{L}}_1 - \dot{\mathbf{L}}_2)\right]\end{aligned} \tag{24}$$

Notice the Jacobian mapping between $\dot{\mathbf{L}}_1$ and $\dot{\mathbf{x}}$ and the kinematics relation between $\dot{\mathbf{L}}_2$ and $\dot{\mathbf{q}}$,

$$\dot{\mathbf{L}}_1 = \mathbf{J}\dot{\mathbf{x}} \ , \ \dot{\mathbf{L}}_2 = r\dot{\mathbf{q}} \tag{25}$$

Substitute these relations into Eq. (24) and simplify:

$$\dot{V} = -\dot{\mathbf{q}}^T (\mathbf{K}_v + \mathbf{D} - r\mathbf{K}_s)\dot{\mathbf{q}} + \dot{\mathbf{q}}^T (\mathbf{Q} + r\mathbf{Q}_G) - \dot{\mathbf{x}}^T (\mathbf{G}(\mathbf{x}) + \mathbf{J}^T \mathbf{K}_s \dot{\mathbf{q}}) \tag{26}$$

By choosing large value for $\mathbf{K}_v$ and appropriate value for $\mathbf{K}_s$ such that $(\mathbf{K}_v + \mathbf{D} - r\mathbf{K}_s)$ becomes a large and positive definite matrix the above analysis shows that $\dot{V}$ may become negative definite as long as $\dot{\mathbf{q}}$ is not zero. This by itself is not sufficient to prove the desired asymptotic stability result, since manipulator can reach to a position where $\dot{\mathbf{q}} = 0$ but $\mathbf{q} \neq \mathbf{q}_d$. To show that this cannot happen we show that when $\dot{\mathbf{q}} = 0$ then the dynamics of system is degenerated to a reduced order system whose $\mathbf{G}(\mathbf{x})|_{\dot{\mathbf{q}}=0} = 0$, therefore $\dot{V} = 0$. Then LaSalle's theorem is used to show that $\mathbf{q}$ tends to $\mathbf{q}_d$. To show this suppose that $\dot{\mathbf{q}} = \mathbf{0}$. Then $\ddot{\mathbf{q}} = \mathbf{0}$ and from Eqs. (18) and (14) the following equations hold.

$$\begin{aligned}\mathbf{u} &= \mathbf{K}_p(\mathbf{q}_d - \mathbf{q}) + \mathbf{Q} + r\mathbf{Q}_G - \mathbf{K}_s\dot{\mathbf{L}}_1 \\ \mathbf{K}(\mathbf{L}_2 - \mathbf{L}_1) &= r^{-1}\mathbf{u} = r^{-1}[\mathbf{K}_p(\mathbf{q}_d - \mathbf{q}) + \mathbf{Q} + r\mathbf{Q}_G - \mathbf{K}_s\dot{\mathbf{L}}_1]\end{aligned}$$

Therefore, Eq. (13) can be rewritten in the form of:

$$\mathbf{M}(\mathbf{x})\ddot{\mathbf{x}}+\mathbf{C}(\mathbf{x},\dot{\mathbf{x}})\dot{\mathbf{x}}+\mathbf{G}(\mathbf{x})=r^{-1}\mathbf{J}^T[\mathbf{K}_p(\mathbf{q}_d-\mathbf{q})+\mathbf{Q}+r\mathbf{Q}_G-\mathbf{K}_s\dot{\mathbf{L}}_1]$$

or

$$\mathbf{M}(\mathbf{x})\ddot{\mathbf{x}}+\mathbf{C}(\mathbf{x},\dot{\mathbf{x}})\dot{\mathbf{x}}=r^{-1}\mathbf{J}^T[\mathbf{K}_p(\mathbf{q}_d-\mathbf{q})-\mathbf{K}_s\dot{\mathbf{L}}_1]+(\mathbf{J}^T\mathbf{Q}_G-\mathbf{G}(\mathbf{x}))+r^{-1}\mathbf{J}^T\mathbf{Q}$$

Use Eq. (19) and (20) and simplify the resulting equation to:

$$\mathbf{M}(\mathbf{x})\ddot{\mathbf{x}}+\mathbf{C}(\mathbf{x},\dot{\mathbf{x}})\dot{\mathbf{x}}=r^{-1}\mathbf{J}^T[\mathbf{K}_p(\mathbf{q}_d-\mathbf{q})-\mathbf{K}_s\dot{\mathbf{L}}_1] \tag{27}$$

This system is a reduced order subsystem of the original system (13) and (14). it is observed in Eq. (27) when $\dot{\mathbf{q}}=\mathbf{0}$, then $\mathbf{G}(\mathbf{x})|_{\dot{q}=0}=\mathbf{0}$, and thus $\dot{V}=0$. Based on LaSalle's theorem [22], it can be concluded that the motion converges to the largest invariant set which satisfies $\dot{V}=0$. In this case the maximum invariant set is the responses of system to Eq. (27). To show that $\dot{\mathbf{x}}$ will tend to origin, we should prove that the above system (27) is asymptotically stable. Consider another Lyapunov function for the degenerated system as follows:

$$V_1=\frac{1}{2}\dot{\mathbf{x}}^T\mathbf{M}(\mathbf{x})\dot{\mathbf{x}}+\frac{1}{2}(\mathbf{q}_d-\mathbf{q})^T\mathbf{K}_p(\mathbf{q}_d-\mathbf{q}) \tag{28}$$

The time derivation of V_1 is given by

$$\dot{V}_1=\dot{\mathbf{x}}^T\mathbf{M}(\mathbf{x})\ddot{\mathbf{x}}+\frac{1}{2}\dot{\mathbf{x}}^T\dot{\mathbf{M}}(\mathbf{x})\dot{\mathbf{x}}+(\mathbf{q}_d-\mathbf{q})\mathbf{K}_p(-\dot{\mathbf{q}}) \tag{29}$$

Substitute Eq. (27) and $\dot{\mathbf{q}}=\mathbf{0}$, and obtain

$$\dot{V}_1=\dot{\mathbf{x}}^T[r^{-1}\mathbf{J}^T\mathbf{K}_p(\mathbf{q}_d-\mathbf{q})-r^{-1}\mathbf{J}^T\mathbf{K}_s\dot{\mathbf{L}}_1-\mathbf{C}(\mathbf{x},\dot{\mathbf{x}})\dot{\mathbf{x}}]+\frac{1}{2}\dot{\mathbf{x}}^T\dot{\mathbf{M}}(\mathbf{x})\dot{\mathbf{x}} \tag{30}$$

Moreover, from relations (23) and (25) the above equation can be simplified to

$$\dot{V}_1=-r^{-1}\dot{\mathbf{x}}^T\mathbf{J}^T\mathbf{K}_s\mathbf{J}\dot{\mathbf{x}}+r^{-1}\dot{\mathbf{x}}^T\mathbf{J}^T\mathbf{K}_p(\mathbf{q}_d-\mathbf{q}) \tag{31}$$

Define $\mathbf{K}_s=k_s\mathbf{I}_{n\times n}$ with $k_s>0$, therefore matrix $\mathbf{J}^T\mathbf{K}_s\mathbf{J}$ is positive definite. By choosing large value for $\mathbf{K}_s$ and appropriate upper bound for $\mathbf{K}_p$ such that $\dot{V}_1\leq 0$, system (27) is stable. Thus, $\dot{\mathbf{x}}$ will tend to zero. In this case by choosing suitable gain matrices in the proposed controller, it is proved that $\dot{V}$ is zero when $\dot{\mathbf{q}}=\mathbf{0}$ and as a result $\dot{\mathbf{x}}=\mathbf{0}$, and hence, $\ddot{\mathbf{q}}=\ddot{\mathbf{x}}=\mathbf{0}$. Therefore, from equations of motion (13) and (14) with our proposed control (18) we must have

$$\mathbf{J}^T\mathbf{K}_p(\mathbf{q}_d-\mathbf{q})=\mathbf{0} \tag{32}$$

This equation can have both a trivial solution ($\mathbf{K}_p(\mathbf{q}_d-\mathbf{q})=\mathbf{0}$) and a non-trivial solution ($\mathbf{K}_p(\mathbf{q}_d-\mathbf{q})\neq\mathbf{0}$), since $\mathbf{J}^T$ is a non-square matrix. In the case of non-trivial solution, the motion

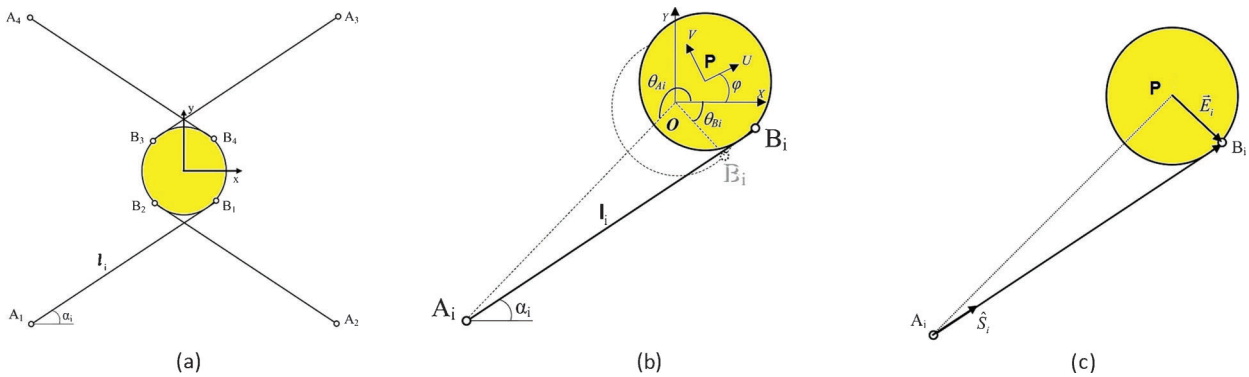

Fig. 1. (a): The schematics of planar cable mechanism (b): Kinematic configuration of mechanism (c): Vector definitions for Jacobian derivation.

stops at some point $\mathbf{q}_0$ which is not equal to $\mathbf{q}_d$. However, from result 1[1] and result 2[2] stated in [14] based on vector closure, it can be inferred that non-trivial solution cannot exist. Therefore, we conclude that $\mathbf{q} \rightarrow \mathbf{q}_d$ as time t tends to infinity as long as the motion is within the vector closure space.

4. SIMULATIONS

A simulation study has been performed in order to verify the effectiveness of the proposed control algorithm. In the following simulation study, the results of the closed loop performance of planar cable driven manipulator is examined. Our model of a planar cable robot [23] consists of a moving platform that is connected by four cables to the base platform shown in Fig. 1(a). As it is shown in Fig. 1(a), A_i denote the fixed base points of the cables, B_i denote point of connection of the cables on the moving platform, l_i denote the cable lengths, and α_i denote the cable angles. The position of the center of the moving platform $\mathbf{P}$, is denoted by $\mathbf{P} = [x_P, y_P]$, and the orientation of the manipulator moving platform is denoted by ϕ with respect to the fixed coordinate frame. Hence, the manipulator posses three degrees of freedom $\mathbf{x} = [x_P, y_P, \phi]$, with one degree of actuator redundancy.

4.1. Kinematics and Jacobian

For kinematic analysis, as it is shown in Fig. 1(b), a fixed frame $O : xy$ is attached to the fixed base at the point $\mathbf{O}$, the center of the base point circle which passes through A_i. Moreover, another moving coordinate frame $\mathbf{U} : UV$ is located on moving platform at its center of mass $\mathbf{P}$. Assume that the point A_i lies at the radial distance of R_A from point $\mathbf{O}$, and the point B_i lies at the radial distance of R_B from point $\mathbf{P}$ in the xy plane, when the manipulator is at central location. For inverse kinematics analysis, it is assumed that the position and orientation of the moving platform $\mathbf{x} = [x_P, y_P, \phi]^T$ is given and the problem is to

[1]Result 1: If $n+1$ vectors $\mathbf{w}_i (1 \leq i \leq n+1)$ in a n-dimensional space are vector closure, vectors $\mathbf{x} = [x_1, x_2 \cdots, x_{n+1}]^T$ which belong to the null space of the matrix $\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_{n+1}]$ are given by Case(1) $x_i = 0$ for any i or Case(2) $x_i > 0$ for any i or Case(3) $x_i < 0$ for any i.

[2]Result 2: If two vectors q_e and q_s are in a vector closure space, the difference vector $q_e - q_s$, must contain both positive and negative elements.

find the length variable of the manipulator $\mathbf{L}_1 = [l_1, l_2, l_3, l_4]^T$. Let's define the instantaneous orientation angle of $B_i^{\cdot}s$:

$$\phi_i = \phi + \theta_{B_i}$$

With some manipulation we can show that [23],

$$l_i = [(x_P - x_{A_i} + R_B \cos(\phi_i))^2 + (y_P - y_{A_i} + R_B \sin(\phi_i))^2]^{\frac{1}{2}}$$

Jacobian analysis plays a vital role in the study of robotic manipulators. Jacobian matrix not only reveals the relation between the length variable velocities $\dot{\mathbf{L}}_1$ and the moving platform velocities $\dot{\mathbf{x}}$, it constructs the transformation needed to find the actuator forces from the wrench acting on the moving platform. For the geometry of the manipulator as illustrated in Fig. 1(c), the manipulator Jacobian matrix $\mathbf{J}$ is,

$$\mathbf{J} = \begin{bmatrix} S_{1x} & S_{1y} & E_{1x}S_{1y} - E_{1y}S_{1x} \\ S_{2x} & S_{2y} & E_{2x}S_{2y} - E_{2y}S_{2x} \\ S_{3x} & S_{3y} & E_{3x}S_{3y} - E_{3y}S_{3x} \\ S_{4x} & S_{4y} & E_{4x}S_{4y} - E_{4y}S_{4x} \end{bmatrix} \tag{33}$$

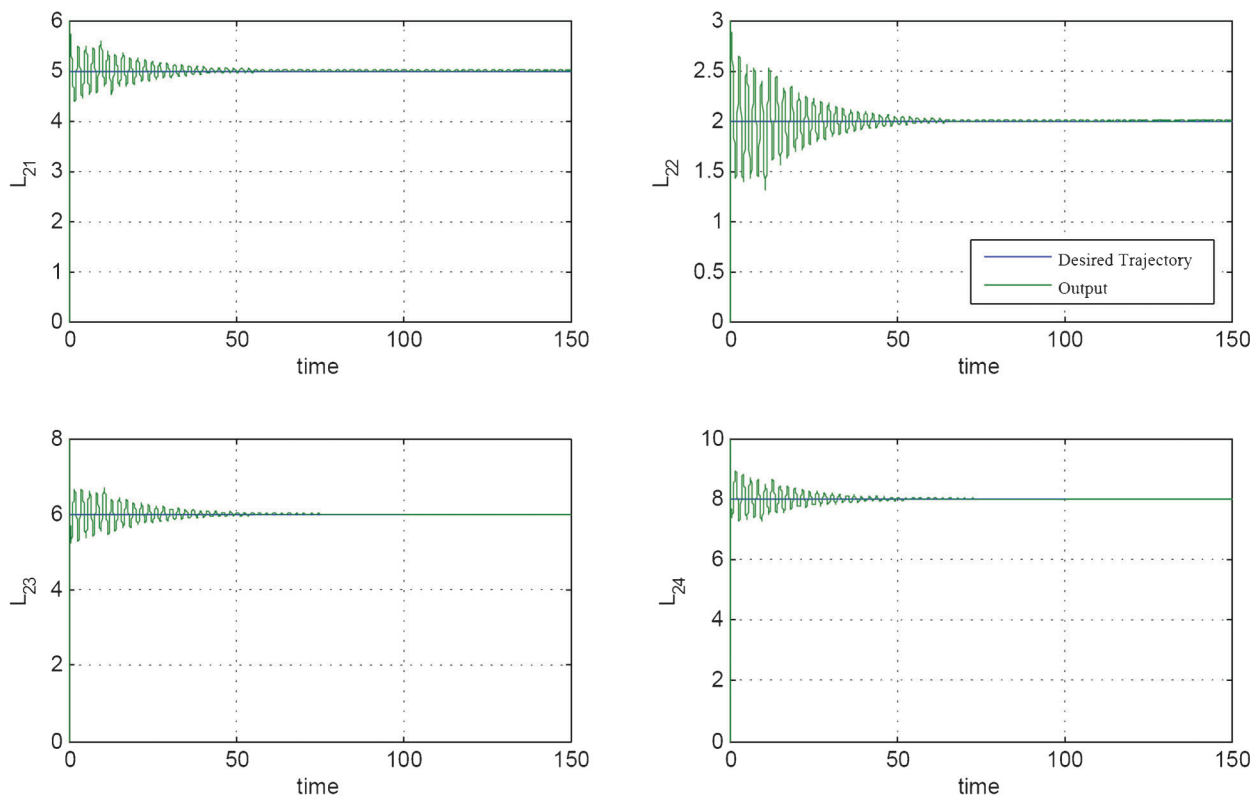

Fig. 2. Plots of desired and actual position of $L_2 = rq + L_0$.

in which, the subscripts x, and y denote the corresponding component of the S_i and E_i vectors. Note that the Jacobian matrix $\mathbf{J}$ is a non-square 4×3 matrix, since the manipulator is redundantly actuated.

4.2. Control

The equations of motion for the end-effector can be written in the following form [24],

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{G} = \mathbf{F}$$

in which, $\mathbf{x} = [x_P, y_P, \phi]$, and by considering flexibility in the cables,

$$\mathbf{F} = \mathbf{J}^T \mathbf{K}(\mathbf{L}_2 - \mathbf{L}_1)\,,\ \mathbf{L}_2 = r\mathbf{q} + \mathbf{L}_0$$

$$\mathbf{I}_m \ddot{q} + \mathbf{D}\dot{q} + r\mathbf{K}(\mathbf{L}_2 - \mathbf{L}_1) = \boldsymbol{\tau}$$

and,

$$\mathbf{M} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_z \end{bmatrix} \quad \text{and} \quad \mathbf{G} = \begin{bmatrix} 0 \\ mg \\ 0 \end{bmatrix}$$

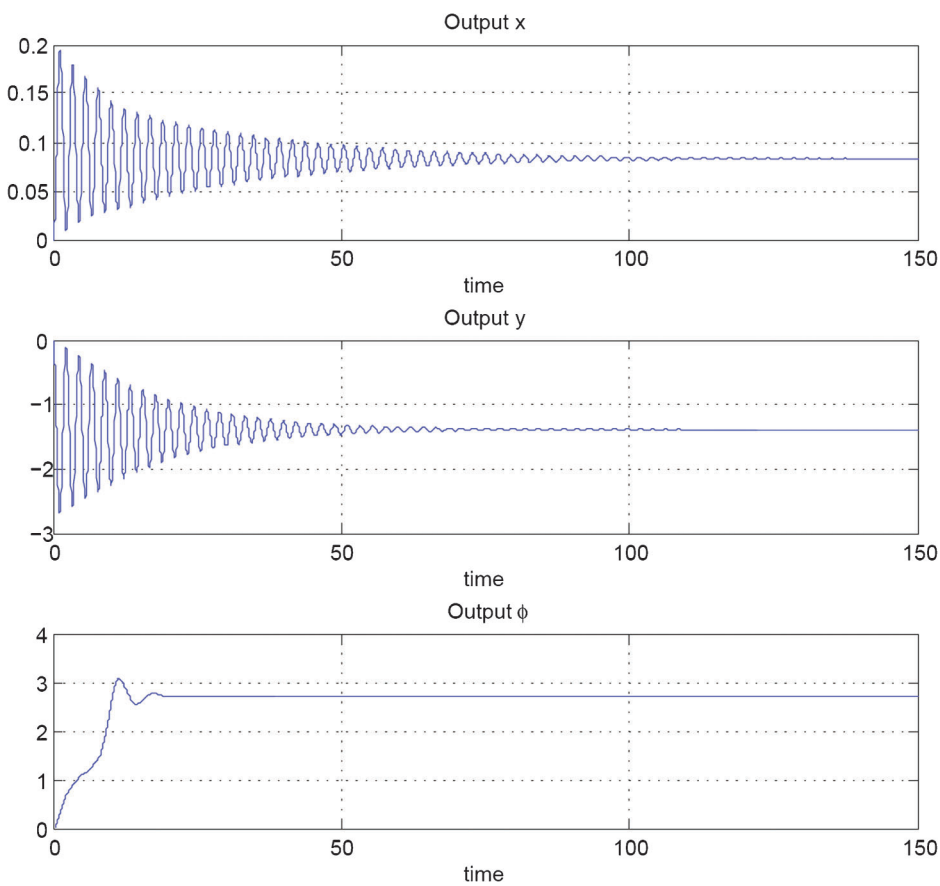

Fig. 3. Plots of position and orientation of end-effector in task space.

The parametric values are as follows: $\mathbf{I}_m = 40\ \mathbf{I}_{4\times4}$, $\mathbf{D} = 15\ \mathbf{I}_{4\times4}$, $r = 0.1$, $\mathbf{K} = 100\ \mathbf{I}_{4\times4}$, $m = 50$, $\mathrm{I}_z = 20$ all in SI units. In order to demonstrate a high flexible system, $\mathbf{K}$ is intentionally chosen very low. To show the effectiveness of the proposed control algorithm suppose that the system have to move from initial position $\mathbf{L}_2 = [2,1,3,4]$ to a fixed position $\mathbf{L}_2 = [5,2,6,8]$. The controller is based on Eq. (18) and consists of four components. Controller gain matrices are chosen as $\mathbf{K}_p = 3000\mathbf{I}$, $\mathbf{K}_v = 1500\mathbf{I}$ and $\mathbf{K}_s = 1000\mathbf{I}$ to satisfy the stability conditions. As illustrated in Fig. 2, the tracking outputs are all stable and the controller achieves the desired steady state values. The reason for the overshoot and existence of vibration in transient response is the exaggerated elasticity values for the cables, which is considered as $\mathbf{K} = 100\ \mathbf{I}_{4\times4}$, which is intentionally chosen to be much smaller than real values. Figure 3 shows the dynamic behavior of the closed-loop system in task space with the proposed control algorithm. Although, the system is very flexible the proposed control algorithm can suitably damp the vibrations. Figure 4 represents the stability of system in another point of view. The illustrated 3D phase portrait is plotted with respect to the derivatives of positions $\dot{x}_P, \dot{y}_P$ and the derivative of the orientation $\dot{\phi}$, of the end-effector, and it clearly shows asymptotic stability of the origin. These simulations illustrate the guaranteed stability of the proposed controller in presence of flexibility in the cables.

5. CONCLUSIONS

In this paper modeling and control of cable driven robots with elastic cables are examined in detail. In the modeling of this class of manipulators cables are modeled by linear axial springs, and the

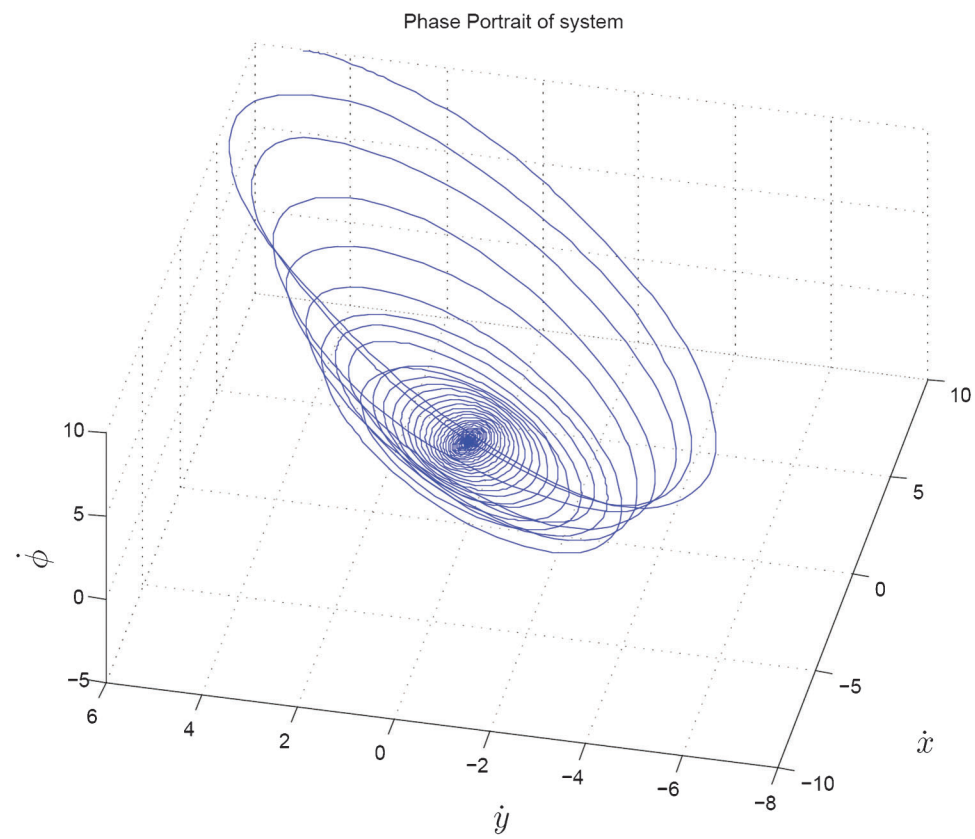

Fig. 4. Phase portrait of system in task space.

model of fully constrained cable driven robot is derived using Euler-Lagrange approach. Since in such robots cables must remain in tension in the whole workspace, the notion of internal force is introduced and directly used in the proposed control algorithm. The proposed control algorithm is designed in cable length space and consists of four components. A simple PD control on the tracking error, the internal force to ensures that all cables are in tension, a gravity compensation term and finally a damping term. The stability of the closed-loop system is analyzed through Lyapunov second method, and it is shown that the proposed controller is capable to stabilize the system in presence of flexible cables. Finally the performance of the proposed controller is examined through simulations.

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