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Position Based Sliding Mode Control for Visual Servoing System

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***Abstract*— This paper presents a nonlinear controller for visual servoing system. Pose estimation is one of the fundamental issues in position-based visual servoing (PBVS) approach. A few researches have focused on controller synthesis under modeling uncertainty and measurement noise of estimated position. In this research, PD-type sliding surface is designed for tracking target. The control signal is obtained from the sliding surface and the stability of the algorithm is verified by Lyapunov theory. Moreover, a recent designed robust estimator based on unscented Kalman observer (UKO) cascading with Kalman filter (KF) is used to estimate the pose, velocity and acceleration of the target. The combination of the implemented estimator and the proposed controller provide a stable and robust structure in PBVS. The reported experimental results, verify the effectiveness of the proposed method in an industrial visual servoing system.**

***Index Terms*— Visual servoing system, Nonlinear controller, Sliding surface, Lyapunov theory.**

I.INTRODUCTION

Control and path planning in robot visual servoing (VS) system have been received great attention in the field of robotics [1,2]. To enhance the servoing performance of this system different nonlinear controllers have been applied on these systems owing to their complex nonlinear nature. One of the most celebrated controller schemes used for such systems is sliding mode control (SMC). In SMC, trajectories are converged to a sliding manifold in finite time and stay on the manifold for all future time. Variations of this method have been widely used for VS system [3,4,5]. Traditionally, a visual servoing system consists of path planning process and feedback control as two separating tasks. This separation limits direct tuning of the whole control system together. Owing to the characteristics of sliding mode control visual servoing can be performed without separation of path planning and control, and therefore, a unified method to enhance the performance of overall system may be achieved.

Position based visual servoing (PBVS) is one of the most accepted applied method in VS in practice [6]. In PBVS, first the estimation of the position and orientation (pose) of target with respect to the camera is performed, and this information is used to produce the appropriate control signal to track the target. The accuracy of pose estimation in this method is directly related to the camera calibration error and measurements noise [7]. Approaches based on extended Kalman filter (EKF) and unscented Kalman filter (UKF) has been developed to reduce these limitations, and have shown to be quite effective in practice [8,9,10]. Moreover, these filters can suitably estimate velocity and acceleration of the target if appropriate motion model is used in such implementations.

In [1], the sliding mode approach has been applied to PBVS for a 6 DOF problem. It is shown that there exists a desired path which the target visibility is guaranteed and the errors are bounded. However, this algorithm is not used in practical implementation. In [2], a sliding surface with time delay compensation is constructed by state pre-estimates and a sliding mode structure for robot joint controller is designed while considering uncertainties in the nonlinear dynamics of the robot. Although, this method is pertained to the dynamic of the robots, it is too complex to be implemented on industrial robots. In most of reported research, SMC is designed based on a nonlinear model of system dynamics [11]. Very limited researches have been focused on the stability analysis of the control and pose estimator combination with respect to the noisy and uncertain environment and constraints of the robot movement.

In this paper, a new approach is presented to synthesize control and path planning tasks of robot with SMC. A UKF cascade structure is used in the visual servoing system, which is recently proposed by the authors in [10]. Based on the estimated position, velocity and acceleration of the target, PD-type sliding mode surface is used as an appropriate manifold to generate a desired path. The stability and performance is guaranteed through the SMC control output, while the stability of the closed-loop control system is analyzed by Lyapunov theory. In the proposed controller structure, system uncertainties existing in the estimation model and observation noise are directly considered in the controller synthesis, and as a result the accuracy of the VS is

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enhanced. Furthermore, despite of the previous experimental work on this subject, the trajectory path is dynamically changed, and hence, owing to the movement of the target the sliding surface is adapted to the varying positions.

To reduce the chattering of the sliding mode controller, a modified exponential reaching law is employed to the design of visual controller [12,13]. It is shown that there exist a desired path for which all the trajectories can converge to the target, while the tracking errors are bounded. In what follows the PD-type sliding surface is developed for the five degrees of freedom motion of the robot end-effector, and it is used in the structure of PBVS system. Experiments performed on the industrial robot verify the tracking performance of the visual servoing control scheme.

II. Theoretical Background

In this section, first the formulation of UKF+KF pose estimators is reviewed, then practical implementation issues of applying control outputs to an industrial manipulator is described.

A. Experimental Setup

The experimental hardware setup, which is shown in Fig. 1, is composed of a 5-DOF RV-2AJ robot manipulator produced by Mitsubishi Co. augmented with a one degree of freedom linear gantry. This configuration provides the robot five degrees of freedom motion, with one degree of redundancy, and orientation of the wrist about the tool axis is not present in the structure. Additionally a PC equipped with a Pentium IV (1.84 GHz) processor and a 1 GB of RAM is utilized as the processor. A camera is attached on the end effector of the manipulator, which is a Unibrain Co. product with 30fs frame rate and a wide lens with 2.1 mm focal length. The proposed technique is performed in Visual Studio utilizing OPENCV Library.

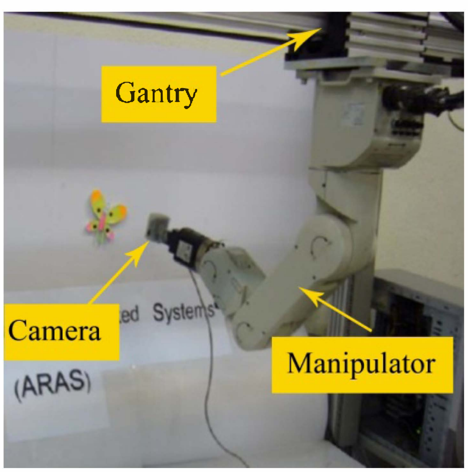


Fig. 1- Experimental setup

B. Feature Extraction and Pose Estimation

In pose estimation methods, the projected 2D coordinates of feature point of target, are employed to calculate the 3D coordinates of the target represented in the reference frame. There are several methods that apply EKF and UKF for pose estimation [14,15]. Recently [8] proposed EKF in addition to KF that guarantees its convergence, however, in [10] it is shown that performance of this method is directly dependent to the initial condition, and therefore, employment of UKF instead of EKF leads to better performance. Due to the promising features of this estimator, this technique is used as the pose estimation engine in this paper. In this estimator, the nonlinear-uncertain estimation problem is decomposed into a nonlinear-certain observation in addition to a linear-uncertain estimation problem. The first part is handled using the unscented Kalman observer (UKO) and the second part is accomplished by a Kalman filter (KF). The pose estimator is fed to a robust and fast modified principal component analysis (PCA) based feature extractor proposed in [10]. This robust method is used in this article to estimate target pose in an uncertain and noisy environment.

C. Singularity Avoidance

After estimating the desired dynamic parameter of target an online controller is required to generate the required motion to the manipulator. Since there is no prior knowledge on the desired trajectory, and the robot is a five degrees-of-freedom manipulator, it may encounter singular configurations within its prescribed trajectory. To remedy this problem, inverse kinematic of the manipulator is solved in an online routine to generate a singular free motion considering the joint limits for the robot. When there is no prior information about the via-points and final destination of desired trajectory, a practical motion planner is proposed by the authors, which is fully elaborated in [16].

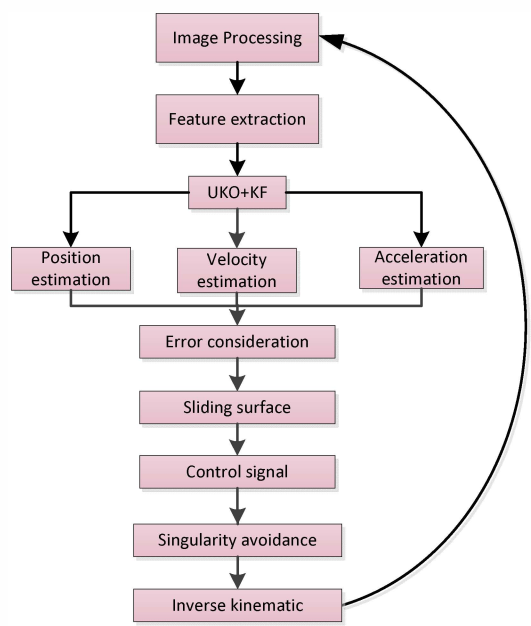


Fig. 2- Block diagram of the overall process

Fig. 3 represents the complete loop of the visual servoing system. An image is captured by the camera applied as the input of the “Feature Extractor” block. After estimating the target pose by UKO+KF, the relative pose is command to the SMC as a desired position.

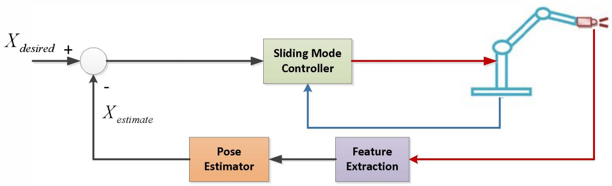


Fig. 3- The complete VS loop

III. Design of Position-Based SMC For Visual Servoing System

The whole process of proposed VS algorithm is demonstrated in Fig. 2. In first step, an image is captured by a camera. Then the target features are extracted from this image. After that position, velocity and acceleration of the object is estimated by UKO+KF estimator. The relative pose is considered as error and is applied as the input to the sliding surface. Then SMC produce velocity signal proportional to each degree-of-freedom of the robot. Such signals are employed to solve inverse kinematics problem. As a result, joint velocities according to singularity avoidance can be obtained. These joint velocities are used for angle calculation of each joint. Finally, these values are commanded the internal controller of our industrial robot. It is common in industrial robot to use position of each joint instead of its velocity, because of their internal controller. The whole process is repeated until the target is tracked perfectly.

Traditionally, a visual servoing system consists of feature tracking and feedback control as two separating processes. This separation limits tuning of the whole system performance. In this paper it is proposed to use sliding mode control to perform visual servoing without separating the tracking and control tasks. The path planning is accomplished using sliding mode surface and the performance is preserved by sliding mode control output. The stability of the closed-loop control system is analyzed by Lyapunov theory. Moreover, the robust behavior of SMC against estimation noise provides suitable tracking performance for the whole visual servoing systems.

A. Controller Design

In this approach, instead of using the dynamics of the system, we have used the estimated values from UKO+KF and their corresponding desired values to define the desired sliding surface and consequently the control effort. The main process is tracking a specific object by this technique, whether the object is moving or not SMC is in action.

First of all, let us define the desired sliding surface. Based on the position of the robot end-effector and the positon of the object, the state vector is converged toward the sliding surface and then the state shall smoothly slide on the surface to reach the target state. Because of movement of the target, the trajectory path is dynamically changing. Therefore, the position of current state may diverge from the sliding surface. Hence, the process has to be executed iteratively with a short refreshing period.

Several sliding mode controller has been applied in robotics applications. But these methods are based on a dynamic model of the system which produces the sliding surface [11]. Let us define the following PD-type sliding surface

$$S = \left(\frac{d}{dt} + \lambda\right) e = \dot{e} + \lambda e \tag{1}$$

in which, e is the error between the real position of the end-effector and the estimated position of the target. The surface vector in (1) is a five tuple $S = [s_x \ s_y \ s_z \ s_A \ s_B]^T$ which contains the manifold motion variable in x, y, z directions and orientation about the x and y axis, called in here A, and B orientations, respectively. There are two approaches to define the required control signal. The first design consists of only a switching term while in the second design a linearizing term is added to the switching term to ensure the sliding condition. It should be noted that this control signal is used to converge the trajectories toward the sliding surface, while the trajectories remain on the surface when $S = 0$.

The first control design is proposed as follows:

$$v_{e-e}^1 = -\beta(t) sgn(S) \tag{2}$$

in which, $\beta(t) \geq \varphi(t) + \beta_0$, $\beta_0 > 0$ and $sgn(\cdot)$ denotes the signum function. The second control signal design is proposed as follows:

$$v_{e-e}^2 = -\lambda^{-1}\ddot{e} + v_{des} - k sgn(S) \tag{3}$$

in which, k and λ are the positive constants. $\ddot{e}$ is the acceleration error between the end-effector and the goal. And, v_{des} is the desired velocity of the object.

B. Stability Analysis

In the design of the control laws, the sliding condition of the manifold is equal to $\dot{e} + \lambda e = 0$, constrains the motion of the system. Choosing $\lambda > 0$ guarantees that all the states of the system tends to zero as time tends to infinity. The rate of convergence can be controlled by the choice of λ. To specify the desired control signal, we need to define the desired velocity which is derived by the estimator. The variable S shall satisfy the following derivative equation:

$$\dot{S} = \ddot{e} + \lambda\dot{e} = \ddot{e} + \lambda(v_{e-e}^1 - v_{des}) \tag{4}$$

where $\dot{e}$ represents the difference between the velocity of the end-effector and the target. Furthermore, v_{e-e} is the end effector velocity which has been used as the control signal in the practical implementation.

To show the stability of this algorithm, various constraints have to be considered. To provide these circumstances, the control signal has to be limited by a specified function. This function is subject to robot velocity constraints and amplitude of estimation noise. The following inequality can be specified from (4). Such function ($\varphi(t)$) will satisfy the following inequality [13].

$$\left\| \frac{\ddot{e} - \lambda v_{des}}{\lambda} \right\|_\infty \leq \varphi(t) \tag{5}$$

The stability analysis of this algorithm is based on Lyapunov direct method. Consider the following positive definite Lyapunov function candidate $V = \frac{1}{2}S^T S$. Based on the upper bound of control signal, the derivative of Lyapunov function candidate $\dot{S}$ is determined by

$$\dot{V} = S\dot{S} = S(\ddot{e} - \lambda v_{des}) + S\lambda v_{e-e}^1 \le \lambda|S|\varphi(t) + \lambda S v_{e-e}^1 \quad (6)$$

Considering the first controller design the derivative of Lyapunov function will be certainty negative definite. Therefore, the trajectory reaches the manifold $S = 0$ in finite time and, once on the manifold, it cannot leave it, as forced by the condition on $\dot{V} \le -\lambda\beta_0|S|$.

For the second controller design v_{e-e}^2 the same analysis can be performed. In this approach, asymptotic stability is verified by the following derivative of Lyapunov function $\dot{V} = -k\lambda|S|$.

There is a difference between the first and second design based on the linearization term. In the presence of uncertainties, the analysis for second design could be failed owing to the accuracy estimations in observations. As it mentioned before, the first design permits to presume uncertainties in more suitable condition for VS problem. Therefore, the first control design is selected for practical implementation.

C. Modified Controller Design

SMC has encountered some limitations in practice, especially the chattering [6]. The most popular modification used to control limit the chattering is the boundary layer approach [6, 9]. A modified exponential reaching law is applied to the design of visual controller to attenuate the result of chattering in SMC. To eliminate chattering, the signum function can be replaced by a high-slope saturation function. The control law is altered as

$$v_{e-e}^1 = -\beta(t)sat(s/\varepsilon) \quad (7)$$

where $sat(\cdot)$ is the saturation function and ε is a positive constant. The slope of the linear portion of $sat(s/\varepsilon)$ is $1/\varepsilon$. Suitable approximation requires the use of small ε. For stability analysis a similar analysis can be performed when $|S| \ge \varepsilon$, and the derivatives of Lyapunov function satisfies the inequality $\dot{V} \le -\lambda\beta_0|S|$. Therefore, whenever $|S| \ge \varepsilon$, $|S(t)|$ will be strictly decreasing, until it reaches the set $\{|S| \le \varepsilon\}$ in a finite time and remains inside thereafter. Inside the boundary layer where $|S| \le \varepsilon$, we cannot have asymptotic stability and only uniformly ultimately bounded (UUB) stability with an ultimate bound can be obtained. The ultimate bound can be reduced by decreasing the depth of boundary layer [13].

IV. EXPERIMENTAL RESULTS

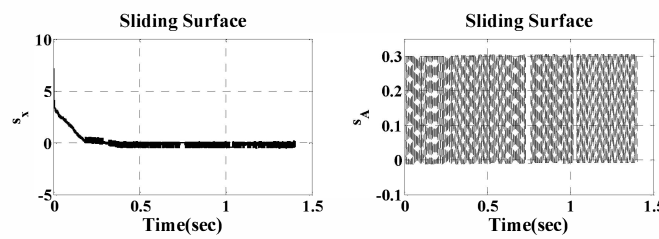
In this section, the efficiency and the performance of the proposed algorithm are verified through some experiments. In experiment I, performance of visual regulation with two different type of switching functions in SMC technique is analyzed. In experiment II, the overall performance of the servoing system is verified. According to the constraints of our problem, the controller parameters are tuned to the selections shown in table I. First, based on the tradeoff between $\dot{e}$ and e from eq. (1) the rate of convergence (λ) is selected. Then from eq. (5), the maximum acceleration of robot and maximum velocity of goal object function $\varphi(t)$ is chosen. According to the accuracy of tracking error β_0 is selected. The width of boundary layer (ε) is tuned in order to reduce the oscillations of control signal. Note that in these experiments it is desired to keep 18 cm distance from target in Z direction. Besides, to reach the relative positions of the object with respect to camera frame in X, Y directions and in A, B orientations these values have to be less than allowed tolerance.

A. Experiment I:

The aim of this experiment is to perform a visual regulation. For this purpose, the object has no motion and the position and orientation should be regulated to the desired pose. By using the error between the end effector and the estimated value of relative pose of object a desired manifold is produced. The main behavior of control signal is affiliated to the type of switching function. As a matter of fact, two different type, signum and saturation, functions are employed. Note that to analyze the behavior of these methods the initial pose of robot is set to the same condition.

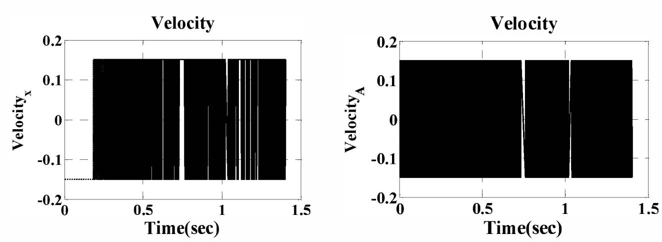
TABLE I. THE CONTROLLER PARAMETERS

Type of switching function	φ	β_0	λ	ε
Signum	0.1	0.05	1	-
Saturation	0.1	0.13	1	$\varepsilon_X = \varepsilon_Y = \varepsilon_Z = 0.5$ $\varepsilon_A = \varepsilon_B = 0.38$



(a) (b)

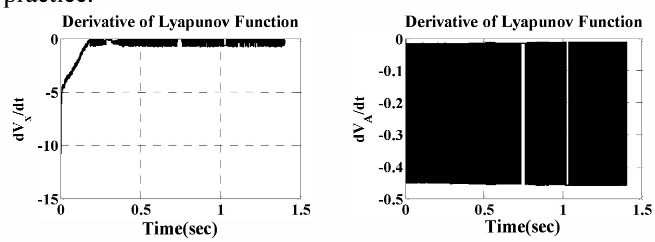
Fig. 4- Sliding surface with sing function in a: X direction, b: A orientation



(a) (b)

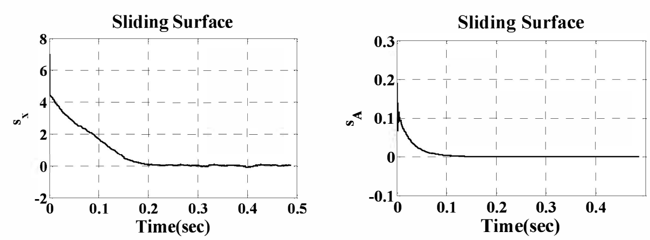
Fig. 5- Velocity with sign function in a: X direction, b: A orientation

The results of sliding surface, the velocity control command and derivative of Lyapunov function in X direction and A orientation are shown in Fig. 4, 5 and 6 for signum function respectively. X direction and A orientation are selected as the representatives of translation and orientation, respectively, to keep the number of illustrated figures at a managing level. As it can be seen in Fig. 4 (a), trajectory lies on the sliding surface in X direction and converges toward zero (in 0.2 second). The trajectories and the resulting sliding variables reach to zero which indicates that the error of velocity and position of end effector with respect to that of the target tend to zero in 0.8 second and lie on it after the object has been tracked. This issue can be seen in Fig. 5, too. The velocity in X direction is produced as -0.15 cm per second during the first 0.2 second and after the tracking error tends to zero. The oscillations are due to the chattering, however, the variations remains around zero. This indicates that the controllers have suitably commanded the robot to stay on the sliding surface. As the object has no rotational motion, the signal in A orientation of velocity and sliding surface is dithering around zero as it is shown in Fig. 4 and Fig. 5 (b). Fig.6 shows that derivative of Lyapunov function is always negative which verifies stability of this algorithm in practice.



(a) (b)

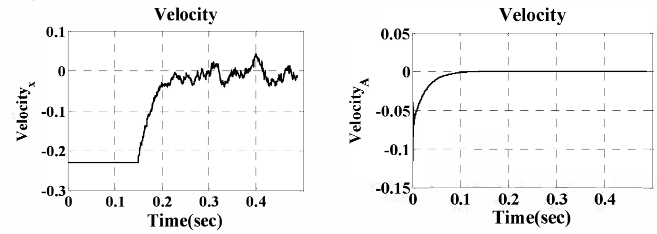
Fig. 6- Derivative of Lyapunov function with sign function in a: X direction, b: A orientation



(a) (b)

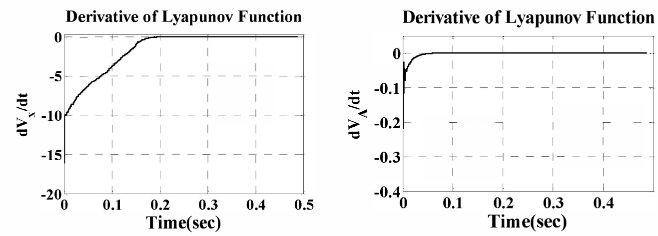
Fig. 7- Sliding surface with sat function in a: X direction, b: A orientation

The oscillations are significantly attenuated through the implemented filters in the inner structure of the robot. As a matter of fact, this drawback cannot significantly harm the mechanical parts of robot. It is of crucial importance to note that the chattering phenomenon is not desirable and better performance to control the robot is vital, hence the signum function was replaced by saturation function. The results of this case for sliding variable, velocity and derivative of Lyapunov function in X direction and A orientation are shown in Fig. 7, 8 and 9 respectively. The trajectories and the resulting sliding variable reach to the desired condition about 0.2 second, however, as it is seen in these figures between 0.2 and 0.5 seconds the tracking errors is not exactly zero and they are restricted in a bounded region. This is in complete agreement of the stability analysis that guarantees only UUB condition on tracking errors. In Fig. 8, the same behavior can be seen in velocity too, the signal is produced in X direction as -0.22 cm per second and after that this signal reaches to zero in 0.15 second. This signal is not smooth, but the oscillations are significantly reduces. Since the object has no rotational motion, the signal of velocity and sliding surface is reached to zero in 0.15 second. Fig. 9, illustrates derivative of Lyapunov function which is negative during the end effector movement. This verifies the overall stability of this algorithm.



(a) (b)

Fig. 8- Velocity with sat function in a: X direction, b: A orientation



(a) (b)

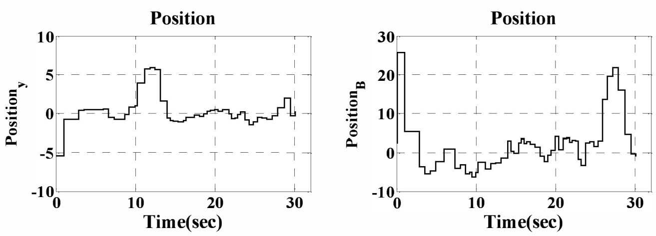
Fig. 9- Derivative of Lyapunov function with sat function in a: X direction, b: A orientation

B. Experiment II:

In this experiment, the overall performance of the proposed visual servoing technique is evaluated. In this experiment, some independent motion for object is designed. For this purpose the object is moved towards X, Y, Z, A and B directions. According to the aforementioned results, a saturation type SMC with the width of ε is considered for the experiment.

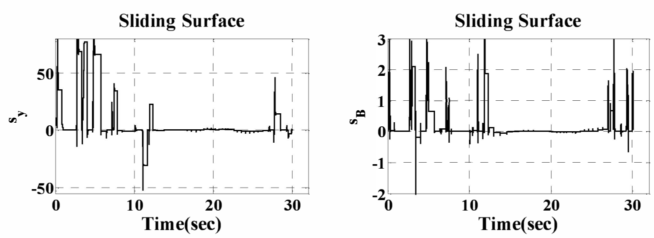
In order to have a better comprehension of the experiment, the estimated values of the object with respect to camera frame has been shown in the Fig. 10. While Fig. 11, demonstrates the sliding variables to show the tracking errors performance. As it can be seen in Fig. 10 and 11 when the object moves in each direction the errors with respect to this motion will rapidly grow, and therefore, the corresponding sliding variable will grow. Then the controller forces the robot to move toward the target. For example, focus on estimated relative poses in Y direction of Fig. 10 after about 8 seconds, the object started to move to approximately 6cm. Proportional to this movement the sliding variable in Y direction is generated and the controller compensate these errors after about 4 seconds, which will yield to zero. Similar analysis can be observed for B orientation, as well. As it can be seen in Fig. 10 and Fig. 11 at 25 second the object started to rotate around Y axis by about 22 degree, the estimator detects this rotation. Based on this movement the controller rotate end effector and the errors yield to zero.

To analyze the stability of visual servoing for the moving object, consider Fig. 12 that depicts the derivatives of Lyapunov function which is a suitable measure for the proposed controller strategy. The sliding surfaces in Fig. 11 and derivative of Lyapunov functions in Fig. 12 show non smooth movement. At first glance, the jitters in the plots seem to be non smooth movement, but if the plot is suitably magnified at the vicinity of jitters, it can be seen that the signal is really smooth. This fact was shown in experiment I and the performance is smooth and proper. Note that as the target is moved by human, the recorded trajectory in every manner includes minute oscillations and vibrations which stems in human disabilities. For detail examination of the overall performance of the VS system a video clip is given in:http://saba.kntu.ac.ir/eecd/aras/ movies/SMC-Vservo.mpg.



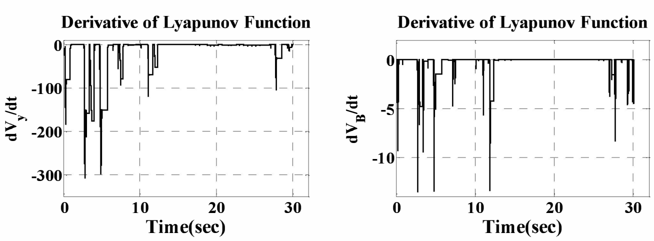
(a) (b)

Fig. 10- UKO+KF estimation of motion in a: Y direction, b: B orientation



(a) (b)

Fig. 11- Sliding surface in a: Y direction, b: B orientation



(a) (b)

Fig. 12. Derivative of Lyapunov function in a: Y direction, b: B orientation

V. CONCLUSIONS

In this paper, a new approach to control and planning of robot in PBVS has been proposed. In this method first the position, velocity and acceleration of target is estimated by robust estimator then a stable and robust SMC is employed to move toward the target. By means of this structure, path planning and control have been combined to have a unified approach. Several experiments have been executed to verify the accuracy and integrity of the proposed method. It has been observed in these experiments, that the method is well performing in a noisy background and in a practical environment. It may be concluded that the combination of robust pose estimation and proposed structure of SMC presents a robust technique for industrial implementation of a visual servoing system.

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