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NURBS-based Representation of Urban Environments for Mobile Robots

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***Abstract*—Representation of the surrounding environment is a vital task for a mobile robot. Many applications for mobile robots in urban environments may be considered such as self-driving cars, delivery drones or assistive robots. In contrast to the conventional methods, in this paper a Non Uniform Rational B-Spline (NURBS) based technique is represented for 3D mapping of the surrounding environment. While in the state of the art techniques, the robot's environment is expressed in a discrete space, the proposed method is mainly developed for representation of environment in a continuous space. Exploiting the information theory, the generated representation has much lower complexity and more compression capability in relation to some state of the art techniques. In addition to representation in a lower dimensional space, the NURBS based representation is invariant against 3D geometric transformations. Furthermore, the NURBS based representation can be employed for obstacle avoidance and navigation. The applicability of the proposed algorithm is investigated in some urban environments through some publicly available data sets. It has been shown by some experiments that the proposed method has better visual representation and much better data compression compared to some state-of-the-art methods.**

I. Introduction

Mobile robots are used in diverse applications such as scientific research, surveillance or military applications. Exploration or navigation in an unknown environment requires the simultaneous localization and mapping (SLAM). The perception of the environment is accomplished by grabbing sensor observations. Microsoft Kinect camera, stereo camera and Lidar sensors are widely used for this purpose.

KinectFusion has been presented for real-time reconstruction of indoor environments using Kinect [1]. The RGB-D point clouds are merged into a truncated signed distance function (TSDF) where later is used for mesh construction by marching cubes algorithm. Later, Kintinuous developed for extending the spatial limits of KinectFusion and incrementally making a triangular mesh [2].

In contrast to the case of indoor environments where several surface reconstruction has been developed, few methods are available for representation of outdoor environments. OctoMap has been widely used for outdoor exploration and navigation [3]. The point cloud data are stored in an octree data structure probabilistically where occupied and free spaces can be determined. Surfel maps are another technique for representation of outdoor spaces [4]. Surfel maps are used for representation of indoor and outdoor spaces by a mixture Gaussian functions which can be used for loop closure detection.

Furthermore, the continuous 3D environment is represented in a discrete space by raw sensor observations or polygonal surfaces in most state-of-the-art techniques. Later, the environment representation shall be used for mobile robot navigation which is a vital task for either exploring an unknown environment or travelling towards a goal. Path planning in a discrete space can not be performed efficiently as it shall be applied to a continuous environment. Moreover, the conventional state-of-the-art environment representation methods have weak data compression capability and the huge amount of acquired data has to be kept in the memory. In this paper, a NURBS based representation is proposed for urban environments. In the proposed method, the discrete sensor observations are expressed by a continuous surface. Furthermore, the grabbed raw data are compressed by storing few control points of the NURBS surface.

The structure of the paper is as follows. The next section is devoted to some preliminaries required for development of the proposed method. The proposed method for representation of outdoor urban environments by NURBS surfaces is presented in Section III. The Section IV is dedicated to the experimental results, which is followed by the concluding remarks.

II. Preliminaries

In this paper, it is assumed that a mobile robot is performing either exploration or navigation towards a goal in an indoor or outdoor urban environment. The surrounding environment is perceived through a 3D sensor such as stereo camera, Microsoft Kinect or 3D laser scanner. Acquiring the point cloud data, a continuous representation shall be generated by NURBS, providing obstacle avoidance and navigation on a surface capabilities. It is supposed that the point cloud is organized and can be expressed by a matrix of p_h by p_w. In the case of unorganized point cloud, a uniform sampling is performed to extract an organized point cloud. In order to represent the acquired point clouds by NURBS, it is required to perform a classification step. The entropy maximization is employed for classification as it achieves a classification result with minimum bias [5], [6].

In Fig. 1 the overall process of generating a set of NURBS from point clouds is depicted. The acquired organized point cloud is mapped to a grid graph by representation of each

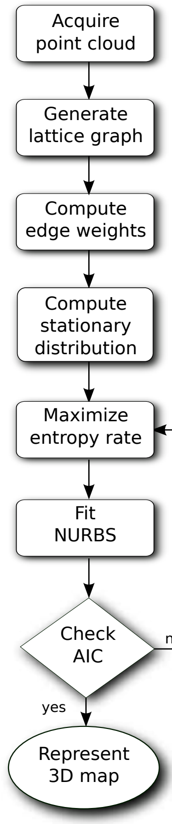


Fig. 1. The overall process of representation of point clouds by NURBS

3D point as a node of graph. Then the weights of the graph edges are computed based on the point-wise distances, in order to construct an undirected grid graph. In the next step, a random walk on the graph is performed to derive the stationary distribution. During this phase, the edges with low weights are removed iteratively, resulting in groups of nodes having stronger connection with each other. A greedy entropy maximization is developed for detection of groups in the graph according to the weights between nodes. Having the groups of nodes in hand, an efficient NURBS fitting technique based on the salient points is used for surface fitting [7]. In the following, the steps of the proposed method is explained in more detail.

A. Graph Based Representation of Organized Point cloud

In this section, the representation of an organized point cloud by an undirected grid graph is explained. In order to express the organized point cloud by a graph $G = (V, E)$, a set of nodes V and edges E shall be defined. Each 3D point in the acquired point cloud is expressed by a node v_i while the edge between nodes i and j is expressed by e_{ij}. Each edge has a non-negative weight represented by a real function $W : \mathcal{E} \rightarrow \mathcal{R}^{+} \cup \{0\}$. It is assumed that the graph is symmetric, the edges e_{ij} and e_{ji} have the same value.

$$W_{ij} = W_{ji} \tag{1}$$

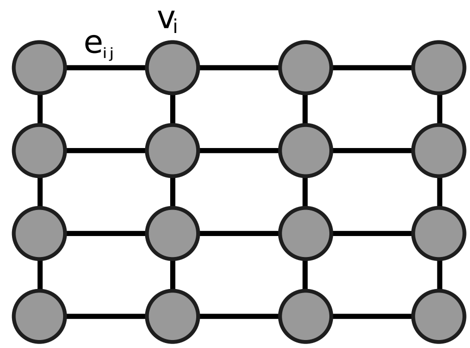


Fig. 2. Undirected lattice graph.

Since it is assumed that the grabbed point cloud is organized, it is expressed as a lattice graph such as the one depicted in Fig. 2.

The classification of point cloud is performed on the graph which results in $\mathcal{S} = \{\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_k\}$, where each $\mathcal{S}_i$ consists of nodes having stronger connection between each other in relation to nodes of other groups. The constructed groups have no intersection and their union is equal to the original graph.

The entropy of a random variable X is expressed by p_X which indicates the amount of uncertainty and is defined as

$$H(X) = -\sum_{x \in \mathcal{X}} p_X(x) \log p_X(x) \tag{2}$$

where $\mathcal{X}$ is the support of random variable X. The conditional entropy which indicates the uncertainty of X while having Y is expressed by

$$\begin{aligned} H(X|Y) &= \sum_{y \in \mathcal{Y}} p_Y(y) H(X|Y = y) \\ &= -\sum_{y \in \mathcal{Y}} p_Y(y) \sum_{x \in \mathcal{X}} p_{X|Y}(x|y) \log p_{X|Y}(x|y) \end{aligned} \tag{3}$$

where $\mathcal{Y}$ is the support set of Y and $p_{X|Y}$ is the conditional density function.

The entropy rate for a random process $\{X_i\}$ is defined as

$$\mathcal{H}(X) = \lim_{n \to \infty} H(X_1, X_2, \ldots, X_n) \tag{4}$$

which indicates the amount of remaining uncertainty after a random walk. For a stationary process, the limit of equation (4) can be computed, where for a Markov process we have

$$\mathcal{H}(X) = \lim_{n \to \infty} H(X_n|X_{n-1}) = \lim_{n \to \infty} H(X_2|X_1) \tag{5}$$

It has been shown that a random walk on an undirected graph has a close relation to the classification of graphs [8]. In the graph classification by random walk, a particle is moved from a node to another one probabilistically. The random walk of a particle can be expressed by $\{X_n\}$ which is a sequence of graph edges. The probability of moving from the ith node to the jth node, is expressed as

$$P_{ij} = \frac{W_{ij}}{\sum_k W_{ik}} \tag{6}$$

In this situation, the stationary distribution can be shown simply by $\mu_i = \frac{W_i}{2W}$. In this relation we have $W_i = \sum_j W_{ij}$ and $W = \sum_{i,j:j>i} W_{ij}$. Therefore, the entropy rate is

$$H(X) = -\sum_i \mu_i \sum_j P_{ij} \log P_{ij} \tag{7}$$

Now we show that the entropy rate can be expressed as difference of two convex functions and the optimization problem can be solved efficiently [9].

Theorem II.1 (The entropy rate of a random walk on an undirected graph can be expressed as difference of two convex functions.)**.**

Proof. Based on the equation(4) we have

$$\begin{aligned} H(X) &= -\sum_i \mu_i \sum_j P_{ij} \log P_{ij} \qquad (8)\\ &= -\sum_i \frac{W_i}{2W} \sum_j \frac{W_{ij}}{W_i} \log \frac{W_{ij}}{W_i} \\ &= -\sum_i \sum_j \frac{W_{ij}}{2W} \log \frac{W_{ij}}{W_i} \\ &= \sum_i \sum_j \frac{W_{ij}}{2W} \log \frac{W_i}{2W} - \sum_i \sum_j \frac{W_{ij}}{2W} \log \frac{W_{ij}}{2W} \\ &= \sum_i \frac{W_i}{2W} \log \frac{W_i}{2W} - \sum_i \sum_j \frac{W_{ij}}{2W} \log \frac{W_{ij}}{2W} \end{aligned}$$

Assuming $W_{ij} \neq 0$, the entropy rate can be expressed as the difference of two convex functions $\zeta \log \zeta$ over the domain of $\{\zeta \in \mathcal{R}^n | \zeta > 0\}$.

$$\begin{aligned} H(X) &= g(W) - h(W) \qquad (9)\\ g(W) &= \sum_i \frac{W_i}{2W} \log \frac{W_i}{2W} \\ h(W) &= \sum_i \sum_j \frac{W_{ij}}{2W} \log \frac{W_{ij}}{2W} \end{aligned}$$

□

Based on this theorem, in the next part, a greedy approach for classification of point clouds is presented. Finally, the classification result is used for NURBS fitting.

III. Proposed Method

A. Point Cloud Classification and NURBS Fitting

In order to fit NURBS surface patches to a point cloud, it is required to perform classification. Many techniques are developed for graph classification such as spectral clustering [11], modularity based clustering [12] and information theory based methods [13], [14] just to mention few. As the graph classification based on the entropy rate maximization has the minimum bias, is employed here. In order to classify the graph, the greedy algorithm (1) is used.

In the greedy algorithm (1), an undirected graph of n nodes is considered. In the initialization step, the sampling rate is set to 2. Then, the corresponding nodes are selected and a sub-graph is constructed. In the sub-graph, initially each node is assigned to a separate group. Then, the process of joining groups with maximum probability is performed until the AIC condition is met. Finally, the groups of nodes are used for NURBS fitting from salient points [7]. In the next iteration,

Algorithm 1 Greedy NURBS surface fitting approach

Input: An undirected graph with n nodes.
The sampling rate $sr = 2$ is initialized.
Main loop
Based on the sampling rate, the corresponding nodes are selected and a sub-graph is constructed.
For each node, a group is considered.
Local loop
Add the ith node to the jth node having the maximum probability P_{ij}.
Fit the NURBS surface on each group.
Repeat local loop until AIC condition is met.
Increase the sampling rate.
If the sampling rate is reach the maximum possible value, finish the process.
Repeat the main loop.

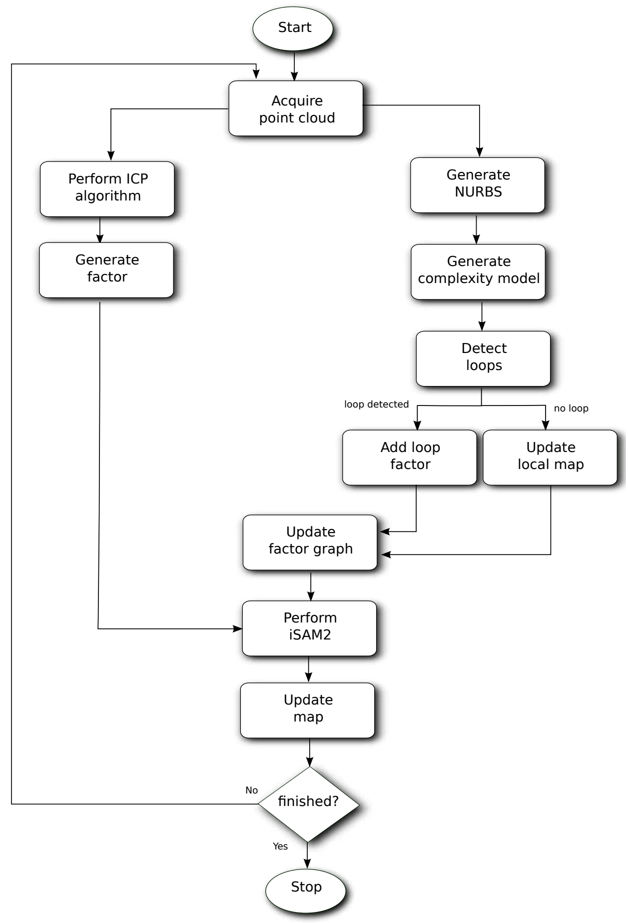


Fig. 3. The overall process of proposed method for 3D mapping.

TABLE I
DATASETS

Data Set	# Scans	Traveled Dist.(m)	Scene Type	Sensor Type
Freiburg room	2890	7.11	Indoor	Kinect
KITTI (00)	4541	3721.73	Urban-Dynamic	Lidar
KITTI (06)	1101	1231.22	Urban-Dynamic	Lidar
KITTI (07)	1101	694.38	Urban-Dynamic	Lidar

(a) (b)

Fig. 4. Experimental results of Freiburg room [10]. (a) The color image. (b) The NURBS surfaces.

the sampling rate is increased if it is lower than the maximum possible value. In each iteration, new nodes are added to the groups having maximum probability value, maximizing the entropy rate. The maximization process is continued until the AIC condition is met and a trade of between the number of model parameters and fitting error is achieved.

Having the NURBS surfaces in hand, a 3D map of the environment can be achieved by performing a Simultaneous Localization And Mapping (SLAM) algorithm. In this paper, iSAM2 algorithm [15] is employed for solving the SLAM problem and the proposed method is used for 3D map generation. In the following section, the process of 3D map generation is explained in more detail.

B. Localization And Mapping

After development of a NURBS based representation of urban environment, in this section the 3D mapping and localization approach is presented. The proposed approach is added to the iSAM2 algorithm and performs mapping and localization in real-time. The overall process of the proposed method for 3D mapping is shown in Fig. 3.

The grabbed point clouds are used for estimation of relative transformation by performing ICP algorithm. The output of this stage is fed to the factor graph generation phase. As the factor graph is being constructed, the 3D mapping is performed simultaneously and NURBS surfaces are constructed in real-time. A geometric transformation invariant representation is generated for NURBS surfaces as described in [16]. Sub-routes are compared by computing the information distance between NURBS surfaces and loop closure constraints are added to the factor graph in the case of low information distance [17]. The iSAM2 and 3D mapping algorithms are executed iteratively and the map of environment and robot's location are updated. This process is continued until the robot's task is accomplished.

TABLE II
THE ROOT MEAN SQUARED ERROR (RMSE) OF TRAVELLED PATH.

Data Set	Proposed method	ORB-SLAM
KITTI (00)	5.16	6.68
KITTI (06)	3.70	14.68
KITTI (07)	0.80	3.36

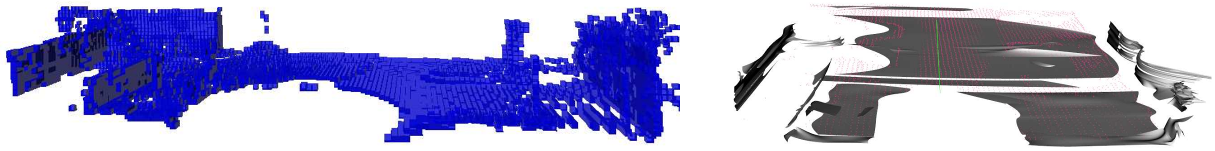
TABLE III
THE MAP SIZE OF NURBS, OCTOMAP AND SURFEL MAP.

Data Set	NURBS size	OctoMap size	Surfel map size
KITTI (00)	5.01 MB	111 MB	222 MB
KITTI (03)	1.37 MB	9.8 MB	19.6 MB
KITTI (07)	2.11 MB	28.6 MB	57.2 MB

In the next section the experimental results of the proposed approach in some urban environments are presented. it has been shown that the NURBS based representation of mobile robot's surroundings is applicable to the indoor and outdoor urban environments. The amount of data compression and the quality of representation of the proposed method is compared to that of some state of the art techniques.

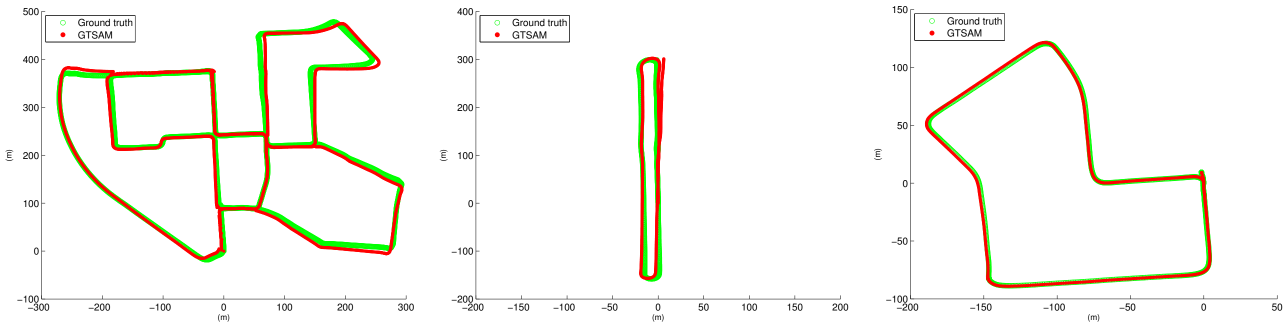
IV. EXPERIMENTAL RESULTS

As mentioned before, most of the outdoor representation techniques for mobile robots are mainly based on the raw point clouds, voxels or mixtures of Gaussians. For instance, in [19] a voxel based representation is presented. Discrete representation of environment, storing a huge amount of point cloud data and providing no suitable navigation method are the draw-backs of this method. Surfel maps [4] are a relatively new approach for representation of outdoor environments. This technique is widely used as it provides the location of the mobile robot in real-time. However, it only stores the position, orientation of surface and color data for limited portion of the 3D space. Finally, the OctoMap [20] is another widely used



(a) (b)

Fig. 5. Experimental results of KITTI dataset [18]. (a) The OctoMap representation. (b) The NURBS surface representation.



(a) KITTI(00) map.

(b) KITTI(06) map.

(c) KITTI(07) map.

Fig. 6. Experimental results of performing iSAM2 in KITTI dataset.

technique for expression of the robot's surroundings. In this method, the point cloud data are stored probabilistically in an octree structure. As it has been seen, the conventional methods have limitations such as discrete representation, storing huge amount of data, no suitable method for easy obstacle detection and navigation on a surface. As the real world is continuous in nature, representation of the acquired data in a continuous space is more convenient.

The proposed approach is implemented in Ubuntu linux and is written in C++ using the multi-threading technique. For each task of initial processing of data, estimation of relative geometric transformation by ICP, execution of the iSAM2 algorithm, NURBS surface fitting and representation of the map of the environment are implemented in a separate thread. The solving of SLAM problem in real-time has been made possible by simultaneous execution of these tasks.

In order to evaluate the applicability of the proposed method, one indoor [10] and some outdoor urban environments [18] are chosen from publicly available datasets. The Pioneer SLAM dataset is generated in an indoor environment using Kinect and the KITTI datasets are grabbed in a dynamic outdoor urban environment using a Lidar. The properties of these datasets such as the number of scans, the travelled distance and the type of environment are shown in Table I.

The first experiment is performed on a dataset of indoor environment which is captured by a Kinect sensor [10]. The results of this experiment can bee seen in Fig. 4. The color image of the Freiburg room is shown in Fig. 4(a) while the resulting NURBS surfaces are depicted in Fig. 4(b).

In the next experiment, the proposed method is applied to the KITTI dataset [18]. The visual appearance of the NURBS surfaces is compared to that of OctoMap method in Fig. 5 where Fig. 5(a) is generated by OctoMap method while the same scene is represented by NURBS surfaces which can be seen in Fig. 5(b).In the proposed method, the 3D scene is expressed by a set of continuous NURBS surfaces instead of using voxels.

Finally, the proposed method is used with iSAM2 in order to solve the SLAM problem. Various sequences from the KITTI dataset are selected for this experiment where the estimated path after performing iSAM2 algorithm with the ground truth data are depicted in Fig. 6. The root mean squared error (RMSE) of estimated path for the selected KITTI sequences is reported in Table II and compared to that of ORB-SLAM algorithm [21]. The RMSE of the proposed method, which is based on the iSAM2 algorithm, is lower than the RMSE of the ORB-SLAM algorithm.

The compression of the acquired data from sensors shall be considered as an important feature when long-term navigation or exploration is considered for a mobile robot. In this experiment, the map size of the proposed method is compared to that of OctoMap and Surfel map techniques. The result of this experiment can be seen in Table III where it can be seen that the map size of the proposed method is much lower than the OctoMap and Surfel map techniques as the NURBS surfaces can be expressed by few control points and cover a large area of scanned environment. This is made possible as in urban environments, many smooth surfaces are available.

Based on these experiments, it can be concluded that the proposed method can efficiently represent outdoor urban environments. The proposed method required no prior information about the surrounding environment or training. The proposed mapping approach can be integrated with iSAM2 algorithm in a SLAM framework for long-term exploration or navigation.

V. Conclusions

In this paper we have proposed a method for representation of urban environments in a continuous space for mobile robots. The proposed representation, can be used for obstacle detection and navigation on a surface. Furthermore, the acquired sensor observation are compressed by expression of data using NURBS surfaces. The proposed mapping technique is integrated in a SLAM framework and applied to some publicly available datasets. The experimental results indicates that the NURBS based mapping can be efficiently used for 3D mapping of indoor and outdoor urban environments.

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