Motion Control of an Underactuated Parallel Robot with First Order Nonholonomic Constraint

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***Abstract*—Adding nonholonomic constraints in parallel manipulators, allows reduction of the actuated-joint number without affecting the reachable workspace. This principle applies to wrist robot in some underactuated designs. This paper studies steady state motion control for an nS-2SPU underactuated parallel wrist robot. First, a suitable Euler angles representation is selected and a new method for forward kinematic problem without extra sensor is proposed. Next, differential kinematics of the robot is analyzed considering first order nonholonomic constraint on angular velocity of the robot. By some manipulations, the derived equations are transformed into chain form, and a hierarchical sliding mode controller is designed for the system. Closed-loop performance of the proposed controller is compared to that of a traditional controller reported in the literature through simulations.**

***Index Terms*—underactuated wrist, parallel robot, nonholonomic constraint, motion control, sliding mode control.**

I. Introduction

An Underactuated Mechanical System (UMS) is a nonlinear system with fewer inputs than its Degrees Of Freedom (DOF) [1]. Because of lower number of inputs than DOF, control of this systems is challenging. There are many underactuated mechanical systems brought into real life application such as unmanned aerial vehicles (UAV), unmanned ground vehicles(UGV), underwater vehicles, mobile robots [2].

Two main situations lead to undeactuated mechanical systems [3], [4]. First case arise from dynamical property of the systems like aircraft, helicopters, mobile robots. However, in the second case the designer intentionally reduces the unwanted number of actuators to reduce the cost. As an example, the underactuated wrist proposed in [4] and satellites with two thrusters could be mentioned as a representative of such case. In general, UMS has nonholonomic constraints. Nonholonomic constraints are categorized as first and second order types, which are also called as velocity and acceleration constraint, respectively [5]. UGVs and UAVs are examples of first and second order nonholonomic systems, respectively.

Study on first order nonholonomic (FON) systems has been extensively reported in the last decade. As a representative of these researches, one may consider [6], in which a general modelling method for FON systems is proposed. It is shown that by satisfaction of the controllability rank condition, these systems can be moved to any desired configuration. However, because of velocity constraint, only special trajectories can be followed between two desired configuration points.

While most of underactuated robots are serial or mobile, underactuation is recently seen in parallel robots as well. Since, parallel robot consist of one or more closed-loop kinematic chain in its structure, It can provide higher precision, faster motion, and lower mass compared to that of serial structures. However, The closed-loop chains in parallel robots make their design more challenging, and their workspace much limited [12]. These design limitations may be removed fully or partially, if unwanted actuators are omitted, and an underactuated structure is used in applications where these types of robot can be proposed. By this means the growing application of parallel robots may be further extended [14]–[16].

Only few studies have been reported for underactuated parallel robots in recent years. In [17] motion planing for a planar underactuated cable-suspended robot is introduced. Grasp planning of an underactuated arm structure is proposed in [18]. Both the robots mentioned in [17], [18] have second order nonholonomic constraints. One parallel robot with first order nonholonomic constraint is introduced in [4]. Inverse kinematic and jacobian analysis of this robot is proposed in [18], while no control strategy is given for such mechanism.

While limited control studies have been reported for underactuated parallel robots, some control methods are developed for FON systems in general. In [6] a general algorithm to transform the equation of motion of such systems to chain form is introduced. Following this transformation, regulation problem is investigated and some controllers are proposed for these systems [7], [9], [10]. Furthermore, a discontinuous control for nonholonomic constrained system is proposed in [11]. While these general control algorithms have their own merits, significant modifications shall be considered to apply them to underactuated parallel manipulators. The motion trajectory constraints, necessitate appropriate motion representation for such robots, in order for these control structures to be applicable.

In this paper motion control for the nS-2SPU underactuated parallel robot wrist is reconsidered [18]. First, a suitable Euler angles representation is considered and then forward kinematic analysis is performed without any extra sensors. Then Differential kinematics of the robot is analyzed in this framework, considering first order nonholonomic constraint on angular velocity of the robot. By some manipulations, the derived equations are transformed into chain form [9], and a hierarchical sliding mode controller is designed for the system. Finally the simulation studies verifies the promising behavior of the proposed method being used in real implementations.

II. Structure Of The Robot

Wrist robot is a fully parallel robot with three rotational degrees of freedom, and the final position of its moving platform is a function of pure orientation. A general case of this structure is proposed in [19], while the proposed structure has three prismatic actuator as inputs in a S-3S$\underline{P}$U kinematic structure. Hence, the robot moving platform is connected to the base by spherical and universal joints in three similar limbs. Furthermore, a passive limb connects the moving platform to the fixed base by a spherical joint, by which the translations of the moving platform are suppressed.

Underactuated manipulator may be constructed from an ordinary robot based on some rules [20]. A three degrees of freedom rotation can be generated in a spherical joint where three axis of rotations are perpendicular. In order to suppress one degrees of rotation and construct an underactuated robot, one may constraint instantaneous rotation about any axes which are parallel to the base. By this means, a fully parallel wrist may be converted to a constrained one, if instantaneous rotation about an axis parallel the base is constrained. This may be accomplished by adding a roller to the spherical joint, by which the robot cannot rotate about the roller axis. This constraint does not limit the other two rotational degrees of freedom parallel to the base plane. By this means a nonholonomic spherical(nS) joint is constructed. In a three DoF manipulator, only two limbs are sufficient to provide the motion. Such structure which is schematically illustrated in Fig. 1 generates a nS-2S$\underline{P}$U wrist robot. Refer to [4] for more details about sphere roller contact and nonholonomic spherical joint in wrist robots.

III. Kinematics And Jacobian

Fig1 illustrates the underactuated parallel wrist robot, and the notations that are used in the kinematic analysis. As explained before, this robot has two actuated limbs and an unactuated one which connect base to the platform. An nS joint is used to stabilized the constrained motion of the robot. It will be shown in the following analysis, without these joints forward kinematic has infinite number of solutions.

To perform kinematic analysis of the robot, two frames are attached to robot. Frame B is fix and is attached to the base while frame P is movable and is attached to the moving platform, while both these frames coincide at point O. An appropriate choice for center of frame might be considered at the geometrical center of nS joint. A_i and B_i are the center of the S and U joint of the i^{th} S$\underline{P}$U limb respectively. Let us use b index for the base and p index for that of the platform. Consider the x_b-axis as the bisector of angle $\angle A_1OA_2$ while z_b-axis perpendicular to the base plane, and y_b-axis perpendicular to these two axes as illustrated in Fig. 1, and i_b, j_b denote unit vector of coordinate axes. Axes of movable frame is determined similar to that of the fixed frame, in which x_p, y_p are located in the moving platform plane. Consider, a_1, a_2 as the unit vectors along the line passes through O and A_1, A_2, respectively. Furthermore, consider b_1, b_2 as the unit vector along the line passing through O and B_1, B_2, respectively. c_i, d_i and e_i are the length of OA_i, OB_i and A_iB_i, respectively.

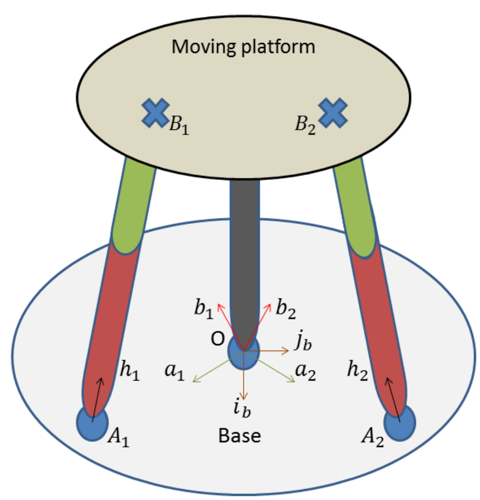

Fig. 1. The nS-2S$\underline{P}$U underactuated wrist robot. The center of fixed frame B and moving frame P coincide at O.

A. Kinematics Analysis

A rotation matrix is used to define the orientation of the moving platform with respect to the base frame. For this means, we use fixed XYZ Euler angles. The importance of this selection will be explained later. Considering this representation, the rotation matrix is determined as:

$$\begin{aligned} {}^bR_p &= R_z(\gamma)R_y(\beta)R_x(\alpha) \\ &= \begin{bmatrix} c\beta c\gamma & s\alpha s\beta c\gamma - c\alpha s\gamma & c\alpha s\beta c\gamma + s\alpha s\gamma \\ c\beta s\gamma & s\alpha s\beta s\gamma + c\alpha c\gamma & c\alpha s\beta s\gamma - s\alpha c\gamma \\ -s\beta & s\alpha c\beta & c\alpha c\beta \end{bmatrix} \end{aligned} \quad (1)$$

in which, $c\theta$ denotes $\cos(\theta)$ and $s\theta$ denotes $\sin(\theta)$. α is rotation angle about x_b. β is rotation angle about y_b axis and γ is rotation angle about z_b. In the triangle A_iOB_i we have:

$$d_ih_i = e_ib_i - c_ia_i \quad (2)$$

All three vector in 2 must be represented in one frame.

1) Inverse kinematic: In inverse kinematic problem, we suppose that Euler angles are known, and therefore, rotation matrix is specified, and length of two limbs are to be extracted. It is clear that ${}^pb_i, {}^ba_i$ are constant vectors. Note that Left superscript b(p) denote, the corresponding vector is represented in the base frame (platform frame). Hence, equation 2 may be written as [18]:

$$d_i\,{}^bh_i = e_i\,{}^bR_p\,{}^pb_i - c_i\,{}^ba_i \quad (3)$$

Dot multiply each side of 3 to itself:

$$d_i^2 = (e_i\,{}^bR_p\,{}^pb_i - c_i\,{}^ba_i)^T(e_i\,{}^bR_p\,{}^pb_i - c_i\,{}^ba_i) \quad (4)$$

d_i will be obtained:

$$d_i = \| e_i \, {}^bR_p \, {}^pb_i - c_i \, {}^ba_i \| \tag{5}$$

And therefore, inverse kinematic problem has a unique solution.

2) Forward kinematic: In forward kinematic problem, lengths of two limbs is known and rotation matrix (Euler angles) must be specified. In parallel robots, its hard to find an analytic solution for this problem. To solve this problem we have two equation with three unknowns, and therefore, in general it has infinite number of solutions. However, in practice, robot has nS joint that keep it in a constant orientation. It is shown that with two extra sensor, a unique forward kinematic solution exists [18].

In what follows a unique solution is obtained without need of any extra sensors. Suppose that initial orientation of the robot $\big(R(0)\big)$ is known. for an infinitesimal motion and in the next sampling time, the rotation matrix of robot may be calculated bt $R(1)R(0)$, in which R(1) is unknown. Because of nS joint, robot can instantaneously rotate only about any axis parallel to the base plane. If we use screw representation, it is clear that the z component of the screw coordinate is zero, i.e. $s_z = 0$. Hence,

$$\begin{aligned} &d_i(1) = \| e_i \, R(1)R(0) \, {}^bb_i - c_i \, {}^aa_i \| \\ &\text{subject to} \qquad s_x^2 + s_y^2 = 1 \end{aligned} \tag{6}$$

Therefore, by this means two equations with three unknowns and one constraint may be solved uniquely either analytically or numerically. In k^{th} sample, equations are in the following form:

$$\begin{aligned} &d_i(k) = \| e_i \, R(k)R^{k-1} \, {}^bb_i - c_i \, {}^aa_i \| \\ &\text{subject to} \qquad s_x^2 + s_y^2 = 1 \end{aligned} \tag{7}$$

in which, $R^{k-1} = R(k-1)R(k-2)\dots R(0)$ is known.

After finding rotation matrix, it is easy to find Euler angles:

$$\begin{aligned} \beta &= Atan2(-r_{31}, \pm\sqrt{r_{11}^2 + r_{21}^2}) \\ \gamma &= Atan2(r_{21}/c\beta, r_{11}/c\beta) \\ \alpha &= Atan2(r_{32}/c\beta, r_{33}/c\beta) \end{aligned} \tag{8}$$

Equation 8 is valid when $\beta \neq \pm\pi/2$. This is one of the reason why we select this Euler angles. It is clear that the robot cannot rotate about X and Y axis more than $\pi/2$ because of its mechanical structure. Therefore, α, β shall be bounded by $-\pi/2 < \alpha, \beta < \pi/2$.

B. Jacobian Analysis

The Jacobian matrix relates angular velocity ω to $\dot{d}_1$ and $\dot{d}_2$. Therefore, dimension of Jacobian matrix is 2×3. However, the robot has a nonholonomic constraint and cannot rotate instantaneously about z_b axis. Hence, Jacobian matrix is 3×3.

Time derivative of equation 4 yields to:

$$d_i \dot{d}_i = c_i e_i (a_i \times b_i)^T \omega \tag{9}$$

Furthermore, the nonholonomic constraint may be represented by $k_b^T \omega = 0$. These equations are collected as:

$$N\omega = D\dot{\lambda} \tag{10}$$

in which,

$$N = \begin{bmatrix} c_1 e_1 (a_1 \times b_1)^T \\ c_2 e_2 (a_2 \times b_2)^T \\ k_b^T \end{bmatrix} \tag{11}$$

and $D = diag(d_1, d_2, 1), \dot{\lambda} = (\dot{d}_1, \dot{d}_2, 0)^T$. Further details of Jacobian analysis is reported in [18]. The only important fact worth to ne noted is that singular point in underactuated robot is fewer than ordinary robot.

IV. Differential Kinematics

In robots with first order nonholonomic constraint, differential kinematic equations are derived from constraint on velocity. Constraint is written in a matrix form and from null space of matrix, kinematic equation may be derived [21]. Let us drive the differential kinematic equations of motion of nS-2S$\underline{\text{P}}$U robot.

We know that time derivative of Euler angles is not equal to the angular velocity of the moving platform. At first, we drive this relation by using $\dot{R} = \omega_\times R$m where:

$$\omega = E \begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix} \tag{12}$$

and,

$$,E = \begin{bmatrix} c\beta c\gamma & -s\gamma & 0 \\ c\beta s\gamma & c\gamma & 0 \\ -s\beta & 0 & 1 \end{bmatrix}. \tag{13}$$

The nonholonomic constraint $k_b^T \omega = 0$ is equal to :

$$\begin{bmatrix} -s\beta & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix} = 0 \tag{14}$$

The solution of equation 14 is equal to the null space of third row of E which is determined by:

$$G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \sin(\beta) & 0 \end{bmatrix} \tag{15}$$

Hence, the final differential kinematics relations are as follows.

$$\begin{cases} \dot{\alpha} = v_1 \\ \dot{\beta} = v_2 \\ \dot{\gamma} = \sin(\beta) v_1 \end{cases} \tag{16}$$

v_1, v_2 are arbitrary value and control inputs of system. The second reason why we select this Euler angles representation is now clear. By usung this representation which is found from the physical properties of the constrained motion, the null space specification is much simplified.

V. CONTROLLER DESIGN

Controller design at this stage ensures the motion control of the robot from an initial configuration to its final destination. Notice that because of the nonholonomic constrained this motion cannot be performed at any desired trajectories, and only trajectories satisfying the motion constraints can be tracked. At this stage we present regulation control of the robot, while trajectory tracking is being worked out currently in the research group.

Equation 16 is in driftless form. We propose to transform them into a chain form. Consider the following changes of coordinates:

$$\begin{array}{c} z_1 = \alpha, \quad z_2 = \sin(\beta), \quad z_3 = \gamma, \\ u_1 = v_1, \quad u_2 = \cos(\beta) v_2. \end{array} \tag{17}$$

By this means the transformed equation in chain form may be derived by

$$\begin{cases} \dot{z}_1 = u_1 \\ \dot{z}_2 = u_2 \\ \dot{z}_3 = z_2 u_1 \end{cases} \tag{18}$$

The final reason of preferable motion representation by Euler angles 1 is that equation 17 is valid when $-\pi/2 < \beta < \pi/2$, which is satisfied by the motion constrains of the robot in this representation.

The controller objective is to converge all the states in 18 to zero. Here for a system in chain form, two controller is designed. First, by using discontinues transformation proposed in [11], all the states asymptotically converge to zero. Second, by designing a sliding mode control law, a robust controller is presented.

A. Discontinues Transformation

Use the following transformation:

$$w_1 = z_1, \quad w_2 = \frac{z_3}{z_1}, \quad w_3 = z_2. \tag{19}$$

Furthermore, use $u_1 = -kz_1 = -kw_1$ in which k denotes the controller gain. The transformed equations are divided in two parts, in which the first equation is inherently stable, and the remaining equations are controllable.

$$\begin{cases} \dot{w}_1 = -kw_1 \\ \dot{w}_2 = kw_1 - kw_2 \\ \dot{w}_3 = u_2 \end{cases} \tag{20}$$

By this means w_2, w_3 state representations are linear and controllable, and can be easily controlled by u_2. The advantage of using linear controller is that control law is simple and smooth, but it isn't robust against uncertainty.

B. Sliding mode

In order to robustify the proposed controller proposed in last section, a sliding mode control is proposed for the system represented in 18. One of the main advantages of sliding mode control is its robustness against modeling uncertainties.

The sliding mode control which is proposed here is hierarchical. Because of the structure of robot, α, β must be bounded by:

$$-\pi/2 < -c \leq \alpha, \beta \leq c < \pi/2 \tag{21}$$

which c is a positive constant. In what follows the proposed controller is stated in terms of a theorem, by which it is proven that asymptotically convergence to zero is guaranteed while the above mentioned bounds are satisfied.

Theorem. *Consider system 18 with the following control law:*

$$u_1 = \begin{cases} -k_1 z_2^* z_3 & if \quad z \in \Omega_2 \backslash \Omega_3 \\ -k_2 z_1 & if \quad z \in \Omega_3 \\ 0 & otherwise \end{cases} \tag{22}$$

$$u_2 = \begin{cases} -k_4 sign(z_2) & if \quad z \in \Omega_1 \\ -k_3 sign(z_2 - z_2^*) & otherwise \end{cases} \tag{23}$$

in which, $\Omega_1 = \{z : z_3 < \varepsilon\}$*,* $\Omega_2 = \{z : z_2 - z_2^* = 0\}$*,* $\Omega_3 = \{z : z_2 = 0, z_3 < \varepsilon\}$*, while* z_2^* *is a constant that satisfies:*

$$-c \leq -\frac{z_3(0)}{z_2^*} + z_1(0) \leq c \tag{24}$$

where, $z_1(0), z_3(0)$ *are initial values, and* k_i *are the positive controller gains.*

Proof. First let us show asymptotically convergence of all states to zero in four stages. Next boundedness of z_1, z_2 will be shown.

- **S1** If $z_2 \neq z_2^*$, then set $u_1 = 0, u_2 = -k_3 sign(z_2 - z_2^*)$. It is easy to show that z_2 converges to z_2^* in finite time while z_1, z_3 are constant. If it's not, go to next stage.
- **S2** If $z \in \Omega_2$, set $u_1 = -k_1 z_2^* z_3$ and $u_2 = -k_3 sign(z_2 - z_2^*)$. By this means z_2 remains constant and equal to z_2^*, while z_3 will converge to zero exponentially and $z_1 = \alpha$ satisfies the required bound of 21.
- **S3** If $z_3 < \varepsilon$, $z \in \Omega_1$ then set $u_1 = 0, u_2 = -k_4 sign(z_2)$. Therefore, z_1, z_3 remain constant and z_2 converges to zero in finite time.
- **S4** If $z_2 = 0, z_3 \leq \varepsilon$, set $u_1 = -k_2 z_1$, then $z \in \Omega_3$ and z_1 converges to zero asymptotically.

According to the above control law, it is clear that z_2 is bounded with upper bound $z_2^* < c$, therefore β satisfy 21. Furthermore, $|z_1| \leq c$ since in stage 2 when $u_1 = -k_1 z_2^* z_3$:

$$z_3 = z_3(0) e^{-k_1 z_2^{*2} t}$$

Therefore:

$$z_1 = \frac{z_3(0)}{z_2^*} e^{-k_1 z_2^{*2} t} - \frac{z_3(0)}{z_2^*} + z_1(0)$$

The final value of z_1 is $-\frac{z_3(0)}{z_2^*} + z_1(0)$ which is lower than c. Also in stage 1 and stage 3, z_1 is constant and in stage 4, z_1 converge to zero exponentially. By this the proof is complete. □

Remark 1. *In this proof all states converge to zero. For regulation to a desired nonzero values, the technique proposed*

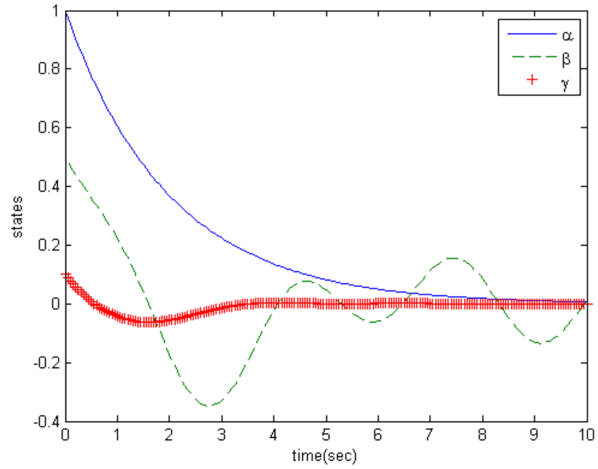

Fig. 2. Closed-loop performance of discontinues controller: State convergence

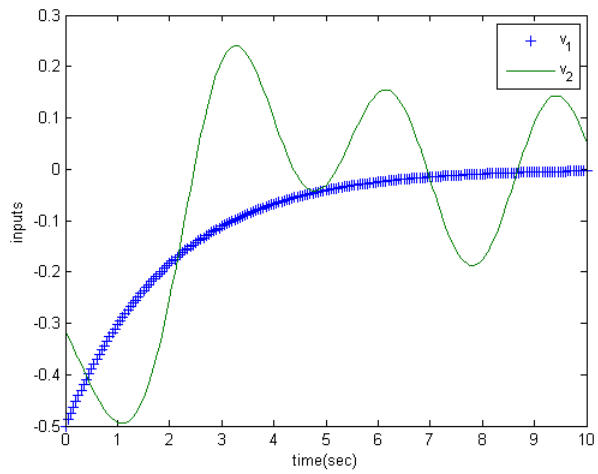

Fig. 3. Closed-loop performance of discontinues controller: Control effort

in [9] is recommended, by which after a change of coordinate, new equations are in chain form, and the above controller works well.

Remark 2. *In sliding mode controller, u_1 is piecewise continues but u_2 has a bang-bang form. It's possible to drive a control law without chattering but modeling uncertainties, and external disturbance will be not totally rejected.*

Remark 3. *The sliding mode controller is robust against disturbance that affect only z_2. It is sufficient to choose k_3, k_4 larger than the upper bound of disturbance to reject it.*

VI. Simulation Results

In order to verify the performance of the proposed controllers for underactuated wrist robot, a simulation is performed. The initial values in this simulation is set to:

$$\begin{bmatrix}\alpha(0)\\ \beta(0)\\ \gamma(0)\end{bmatrix} = \begin{bmatrix}1\\ 0.5\\ 0.1\end{bmatrix} \tag{25}$$

The final value is set to $[0,0,0]^T$. A sinusoidal disturbance with upper bound of 0.2 is added to v_2 in equation 16. First we apply discontinues controller which is proposed in subsectionV-A. The gain of controller is set to $k = 0.5$. u_2 is determined with state feedback so as the linear part of equation20 becomes stable. Closed-loop simulation results are shown in figures. 2 and 3. As it is seen in Fig2 it is evident that β do not converge to zero while the other two states converge to zero. This is because the disturbance is applied in this channel and the controller can not reject it. However, the advantage of this controller is its smooth control effort which is shown in Fig3.

Next, we applied sliding mode controller proposed in subsectionV-B. The value of c, the upper bound of α, β is set to one. A suitable choice for z_2^* that satisfy inequality 24 is 0.2. Furthermore, the controller gains are set to $k_1 = 20, k_2 = 2$ and k_3, k_4 that shall be greater than the upper bound of disturbance, are set to one. Simulation results with this controller are shown in figures 4, and 5. As it can be seen in Fig. 4 at first β reach to $\arcsin(z_2^*) \simeq 0.2$ in a short time while other states are constant. After this time period, z_3 converge to zero while z_1 is decreasing. Then z_2 reach to zero very fast and at last z_1 converges to zero exponentially. It's clear that with this controller, all states converge to zero while an bounded external disturbance is applied to robot. This is the main advantage of sliding mode controller. Fig5 shows the inputs of wrist robot. it is seen in this figure that v_1 is piecewise smooth while v_2 has chattering. in order to avoid chattering notice Remark 2.

VII. Conclusions

In this paper a study on Kinematic analysis and control of an underactuated wrist mechanism is performed. It is shown that background literature on the analysis of this type of robot in addition to the proposed controller in the area of underactuated system can be well merge together to propose a suitable analysis and control for this system. As reported in this paper, a suitable Euler angles representation is proposed in the study of inverse and forward kinematics of the system. Then a new method to obtain unique forward kinematic solution of the robot without need to any extra sensor is presented. Next Differential kinematics of the robot is analyzed and through some manipulations, these equations are transformed into chain form. Finally and a hierarchical sliding mode controller is designed for the system, and the closed-loop performance of the proposed controller is compared to that of a traditional controller reported in the literature through.

The study shall be completed by generalization of the proposed controller for trajectory tracking, as well as adding some effort to reduce chattering. This issues are currently under investigation in our research group.

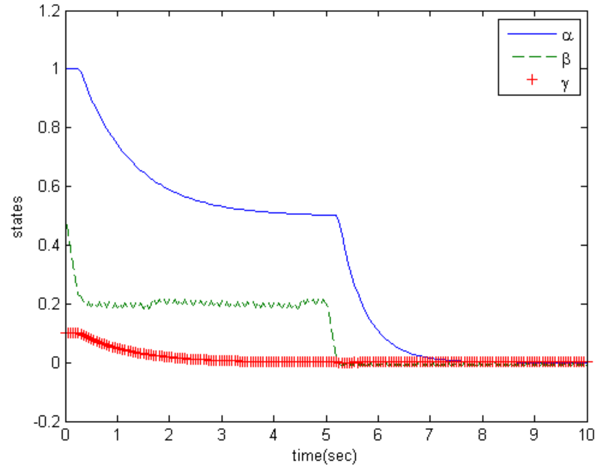

Fig. 4. Closed-loop performance of hieratical sliding mode controller: State convergence

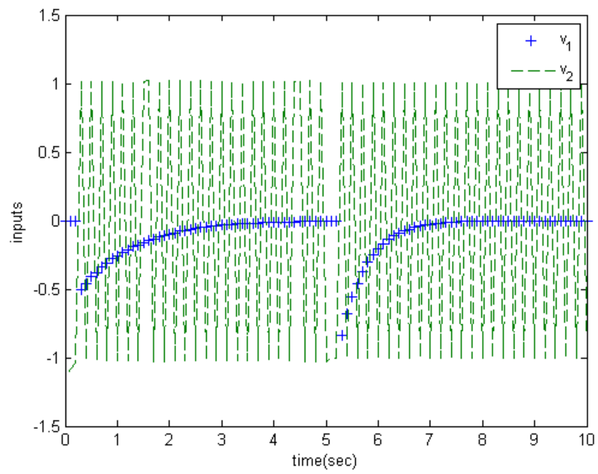

Fig. 5. Closed-loop performance of hieratical sliding mode controller: Control effort

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