A Force Reflection Impedance Control Scheme for Dual User Haptic Training System

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Abstract—In this paper, an impedance control based training scheme for a dual user haptic surgery training system is introduced. The training scheme allows the novice surgeon (trainee) to perform a surgery operation while an expert surgeon (trainer) supervises the procedure. Through the proposed impedance control structure, the trainer receives trainee's position to detect his (her) wrong movements. Besides, a novel force reflection term is proposed in order to efficiently utilize trainer's skill in the training loop. Indeed, the trainer can interfere into the procedure whenever needed either to guide the trainee or suppress his (her) authority due to his (her) supposedly lack of skill to continue the operation. Each haptic device is stabilized and the closed loop stability of the nonlinear system is investigated. Simulation results show the appropriate performance of the proposed control scheme.

Index Terms—Haptic, Surgery Training, Dual-user, Impedance control, Stability Analysis.

I. INTRODUCTION

Research on haptic systems has received a great deal of attention lately as it is utilized in many applications such as space missions, undersea explorations, handling sensitive chemicals, surgery, etc [1]. The main objectives of controlling a teleoperated system are stability and transparency. Several researches may be found in the literature regarding these two objectives since improving one of them usually degrades the other.

A recently developed class of haptic system application is based on dual user haptic consoles. Dual user haptic systems allow two human operators to perform a task cooperatively. This configuration are often used when there is a meaningful skill level difference between operators and one of them (the trainee) is supposed to learn from the other (the trainer). The challenge here is to propose a proper cooperative scheme to ensure the correct performing of the task as well as improving the trainee’s skill level. There are many control structures proposed such as PD+D control [2], H∞ control [3], adaptive control [4], robust control [5], virtual fixture based control [6], [7], six channel shared control architecture [8], etc. A thorough overview on the subject may be found in [9].

It is evident that modern surgeries are bending towards minimal invasiveness. The idea is to perform a surgery through small incisions. There are several challenges involved with performing such surgical operations due to indirect access to the organ and the absence of tactile feedback. The recent advances in video imaging, endoscopic technology, instrumentation and robotics made it possible to perform Minimally Invasive Surgery (MIS) in practical cases [10]. Nonetheless, performing MIS requires relatively higher skill level in comparison to open surgery. Previously, in the surgery training programs, practicing on animals, artificial tissues or dead organs were the only possible ways. These methods are often expensive, unethical or not always possible. The haptic systems, on the other hand, have proved to provide an efficient training framework for the surgical procedures [11].

In this paper, a dual user haptic surgery training system is proposed in which an expert surgeon (trainer) supervises a novice surgeon (trainee) to perform a surgical task. In this scheme, the trainee is in control of the operation as long as the trainer is satisfied with his (her) performance. Trainer may interfere into the procedure at any time to either guide the trainee or suppress his (her) authority. The idea of using the trainer’s expertise to develop more advanced control structures for dual user haptic systems have been proposed in [12] and the stability is analyzed for a simple linear case. This paper aims at developing a more advanced control structure to harness the trainer’s expertise in a real–time operation with nonlinear stability analysis.

In the proposed control scheme, the trainer is needed to receive the position of the trainee and to detect his/her probable incorrect motions. Meanwhile, the trainee should be provided with some guidance and haptic cues from the trainer. In fact, through such haptic cues, the unanticipated mistakes made by the trainee can be corrected. As a result, two control objectives are considered. The first one is that the position of trainer haptic console tracks the position of trainee haptic console. Due to the interaction between trainer and the respective haptic console, the motion and the force are dynamically dependent, and therefore, the dynamic relation between force and motion variables may be considered as the control objective [13]. As it is widely known, when a robotic manipulator is dynamically in contact with an exogenous force, impedance control is preferred rather than a pure motion control [14]. Therefore, an impedance control is developed for the control objective in the trainer side. Another important control objective is that the trainer should be provided with the ability to interfere into the procedure either to guide the trainee or suppress his (her) authority in a real–time operation. The swift nature of force reflection control structures is utilized for this objective to develop an efficient control structure. The stability of the
system is analyzed using Input-to-State Stability (ISS) as it is a commonly used method to prove the stability of a force reflecting haptic configuration [12], [15].

The paper starts by explaining the control structure in Section II. The stability analysis is presented in Section III, followed by investigating the effectiveness of the proposed method via simulation in Section IV. Finally, in Section V the conclusions are stated.

II. CONTROL STRATEGY

In the proposed training scheme, as it is depicted in Fig. 1, the trainee is assigned to perform a surgical task on a virtual environment. The framework allows the trainer to correct the trainee’s movements at anytime. On the trainee’s side, the haptic feedback from trainer acts as a performance correction index and helps trainee to learn the correct sequence of moves.

The dynamics of a general n-DOF haptic device can be modeled as [16]

\[ M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) = u_i + f_{hi} \]  

where the subscript \( i = 1, 2 \) refers to the haptic console 1 and haptic console 2. \( q_i \in \mathbb{R}^{n \times 1} \) is the position vector of each haptic in their joint space, \( M_i \in \mathbb{R}^{n \times n} \) is the inertia matrix of each device, \( C_i \in \mathbb{R}^{n \times n} \) denotes the centrifugal and Coriolis torques, \( G_i \in \mathbb{R}^{n \times 1} \) represents the gravitational torques. Also, \( u_i \in \mathbb{R}^{n \times 1} \) is the control effort vector, \( f_{hi} \in \mathbb{R}^{n \times 1} \) is the force implied by the operator on his (her) respective haptic device.

Note that, the Cartesian space positions of the haptic consoles \( x_i \in \mathbb{R}^{n \times 1} \) are obtained through the forward kinematics relation [16]

\[ x_i = k_i(q_i) \]  

where \( k_i(\cdot) \) is the nonlinear function that indicates the forward kinematics relation of the robot. In addition, the Jacobian matrix \( J_i(q_i) \in \mathbb{R}^{n \times n} \) expresses the relationship between the Cartesian space velocity \( \dot{x}_i \in \mathbb{R}^{n \times 1} \) and the joint space velocity \( \dot{q}_i \) according to

\[ \dot{x}_i = J_i(q_i)\dot{q}_i. \]  

If (3) is differentiated with respect to time, the task space acceleration denoted by \( \ddot{x}_i \in \mathbb{R}^{n \times 1} \) is obtained as

\[ \ddot{x}_i = J_i(q_i)\ddot{q}_i + J_i(q_i)\dot{q}_i. \]  

There are some useful properties involved with the dynamic equation (1) mentioned in [13].

Property 1. The inertia matrix \( M_i \) is positive definite for all \( q_i \in \mathbb{R} \) and \( \forall \lambda_m, \lambda_M \in \mathbb{R} \) such that:

\[ \lambda_m I_{n \times n} \leq M(q) \leq \lambda_M I_{n \times n}. \]  

Property 2. The matrix \( \dot{M}_i - 2C_i \) is skew-symmetric, meaning that for all \( x \in \mathbb{R} \):

\[ x^T(\dot{M}_i - 2C_i)x = 0. \]  

By this means, the control law is defined as

\[ u_1 = \dot{M}_i(q_1)\dot{v}_1 + \dot{C}_i(q_1, \dot{q}_1)v_1 + \dot{G}_1(q_1) - K_1 r_1 \]  

where \( \dot{\Lambda}_1 \in \mathbb{R}^{n \times n} \) and \( K_1 \in \mathbb{R}^{n \times n} \) are positive definite matrices and the notation \((\cdot)\) denotes the computed value of \((\cdot)\) that will be detailed later. It can be concluded from Property 3 that the control law (11) is equivalent to

\[ u_1 = Y_1 \dot{\theta}_1 - K_1 r_1 \]
Next, if the control law (12) is combined with the dynamic equation (1), the closed loop dynamic equation is obtained as:

$$M_1(q_1)\dot{r}_1 + C_1(q_1, \dot{q}_1) r_1 + K_1 r_1 = Y_1(\dot{\theta}_1 - \theta_1) + f_{h1}$$  \( (13) \)

Now, the proposed control law for the trainee haptic console is developed. In contrast to the trainer haptic console which needs to loosely track the surgical tool position, the control of trainee haptic console should be designed to swiftly transform task authority to the trainer and overrule trainee’s commands in the case of any mistake made by the trainee. Thus, owing the fast action of force feedback, a force reflection controllers is proposed for the trainee haptic console. In order to ensure the system’s stability, a stabilizing control law is also designed for the trainee haptic console. Thus, control law for the trainee haptic console is expressed as:

$$u_2 = \dot{M}_2(q_2)\dot{q}_2 - \dot{C}_2(q_2, \dot{q}_2) + K_2 \dot{q}_2 + f_{r2},$$  \( (14) \)

where the gain matrices $A_2 \in \mathbb{R}^{n \times n}$ and $K_2 \in \mathbb{R}^{n \times n}$ are symmetric and positive definite and the notation $\dot{\cdot}$ denotes the estimated value of $\cdot$ that will be detailed later. The force reflection term $f_{r2}$ is defined as:

$$f_{r2} = (f_{h1} - f_{h2}) \psi(\|f_{h1} - f_{h2}\|)$$  \( (15) \)

where $\psi(\cdot)$ is defined as:

$$\psi(\omega) = (1 - e^{-\alpha(\omega - \omega_0)}) u(\omega - \omega_0)$$  \( (16) \)

where $\omega = \|f_{h1} - f_{h2}\|$, $u(\cdot)$ is the unit step function, $\alpha$ is a positive value to adjust the sharpness of the transition phase, and $\omega_0$ is the upper bound of the acceptable force error. The presented function means that if the force error between the trainer and the trainee goes beyond the predefined value $\omega_0$, the trainee’s force is blocked and the task authority is transformed to the trainer. Besides, to ensure the smooth behavior of the system, the transforming of task authority is accomplished through a soft transition. Similar to the trainer’s haptic console, by using Property 3, the control (14) is expressed as:

$$u_2 = Y_2\dot{\theta}_2 - K_2(\dot{q}_2 + A_2 \dot{q}_2) + f_{r2} + f_e.$$  \( (17) \)

Now, define:

$$r_2 = \dot{q}_2 + A_2 \dot{q}_2$$  \( (18) \)

Then, after combining the control law (17) with the dynamic equation (1) and using (18) we have:

$$M_2(q_2)\ddot{r}_2 + C_2(q_2, \dot{q}_2) \dot{r}_2 + K_2 \ddot{r}_2 = Y_2(\dot{\theta}_2 - \theta_2) + f_{h2} + f_{r2} + f_e$$  \( (19) \)

Let us define the term $\dot{\theta}_1$ in (12) and (17) as:

$$\dot{\theta}_1 = \theta_1^* + \delta \theta_1$$  \( (20) \)

where $\theta_1^*$ is the nominal values of $\theta_1$ and it is supposed that:

$$\|\theta_1 - \theta_1^*\| \leq \xi_1$$  \( (21) \)

where $\xi_1$ is the uncertainty bound. In addition, the control term $\delta \theta_1$ is defined as:

$$\delta \theta_1 = -\xi_1 \text{sat}(Y_2^T r_2)$$  \( (22) \)

The stability of such control scheme will be analyzed in the following section.

### III. Stability Analysis

In this section, the stability of the dual user haptic system is investigated. The ISS stability of the haptic console #1 and haptic console #2 are studied in Proposition 1 and Proposition 2, respectively. Then, the stability of the overall system is presented in Theorem 1.

**Proposition 1:** The haptic console #1 subsystem is ISS with respect to the state $[\tilde{x}_1, \hat{x}_1, \dot{x}_1]^T$ and the input $[\xi_1, x_2, f_{h1}, f_e]^T$.

**Proof:** Consider the Lyapunov function candidate:

$$V_1 = \frac{1}{2} r_1^T M_1(q_1) r_1 + c_0 \tilde{x}_1^T P \tilde{x}_1 + \frac{1}{2} \hat{x}_1^T M_1 \hat{x}_1 + \frac{1}{2} \hat{x}_1^T K_1 \hat{x}_1 + \frac{1}{2} \hat{x}_1^T r_1$$  \( (23) \)

where $c_0$ is a positive number and $P$ is the solution of Lyapunov equation $A^T P + PA = -I$. Calculating $\dot{V}_1$ and using Property 2 yields:

$$\dot{V}_1 = r_1^T M_1(q_1) \dot{r}_1 + c_0 \tilde{x}_1^T P \ddot{\tilde{x}}_1 + \frac{1}{2} \hat{x}_1^T M_1 \dot{\hat{x}}_1 + \frac{1}{2} \hat{x}_1^T K_1 \dot{\hat{x}}_1 + \frac{1}{2} \hat{x}_1^T r_1$$

Next, from the Young’s quadratic inequality and after some manipulation we have:

$$\dot{V}_1 \leq -\frac{3}{4} \lambda_{min}(K_1) \|r_1\|^2 - c_0 \|\tilde{x}_1\|^2 - \frac{3}{4} \lambda_{min}(B_1) \|\hat{x}_1\|^2 + r_1^T Y(\dot{\theta} + \delta \theta) + 2c_0 \|P\| \|J_{M1}\| \|\tilde{x}_1\| \|r_1\| + \frac{1}{\lambda_{min}(B_1)} \|f_{h1}\|^2 + \frac{1}{\lambda_{min}(B_1)} \|f_e\|^2 + \frac{\lambda_{max}(K_1)}{\lambda_{min}(B_1)} \|x_2\|^2$$  \( (24) \)

Let us define the state $J_{M1} = \sup \|J(q_1)\|$. Then by choosing:

$$c_0 = \frac{\lambda_{min}(K_1)}{8\|P\|^2 J_{M1}^2}$$

where $J_{M1}$ is the uncertainty bound. In addition, the control term $\delta \theta_1$ is defined as:

$$\delta \theta_1 = -\xi_1 \text{sat}(Y_2^T r_2)$$  \( (22) \)

The stability of such control scheme will be analyzed in the following section.
the following inequality is obtained

\[
\dot{V}_1 \leq -\frac{1}{2}\lambda_{\min}(K_1)\|r_1\|^2 - \frac{\epsilon_0}{2}\|\ddot{x}_1\|^2 - \frac{3}{4}\lambda_{\min}(B_{r1})\|\ddot{x}_{r1}\|^2 \\
+ \rho T \gamma (\dot{\theta} + \delta \dot{\theta}) + \frac{1}{\lambda_{\min}(B_{r1})}\|f_{e1}\|^2 \\
+ \frac{1}{\lambda_{\min}(K_1)}\|f_{h1}\|^2 + \frac{1}{\lambda_{\min}(B_{r1})}\|f_e\|^2 \\
\]

(26)

Let us define \( \eta_1 = Y^T r_1 \). Then, (21) is used to conclude [5]

\[
\dot{V}_1 \leq -\frac{1}{2}\lambda_{\min}(K_1)\|r_1\|^2 - \frac{\epsilon_0}{2}\|\ddot{x}_1\|^2 - \frac{3}{4}\lambda_{\min}(B_{r1})\|\ddot{x}_{r1}\|^2 \\
+ \frac{\mu}{2}\|x_1\| + \frac{\lambda_{\max}(K_1)}{\lambda_{\min}(B_{r1})}\|x_{r1}\|^2 \\
+ \frac{1}{\lambda_{\min}(K_1)}\|f_{h1}\|^2 + \frac{1}{\lambda_{\min}(B_{r1})}\|f_e\|^2 \\
\]

(27)

From (23) and (27) we conclude that the closed loop of the haptic console #1 subsystem is ISS with respect to state \([\dot{x}_1^T, \dot{x}_{r1}^T, \ddot{x}_1^T]\) and the input \([\dot{x}_1, \dot{x}_{r1}, f_{h1}, f_e]^T\). □

**Proposition 2**: The haptic console #2 subsystem is Input-to-State Stable (ISS) with respect to the state \([q_{21}^T, q_{22}^T]\) and the input \([\dot{q}_2, f_{h2}, f_e]^T\).

**Proof**: The Lyapunov function candidate is considered as

\[
V_2 = \frac{1}{2}r_{2}^TM_2(q_2)r_2 + q_2^TM_2K_2q_2. \\
\]

(28)

Then, using a similar but less complicated reasoning as in the proof of Proposition 1, \( \dot{V}_2 \) is computed as

\[
\dot{V}_2 \leq -\frac{1}{4}\lambda_{\min}(K_2)\|q_2\|^2 - \frac{1}{4}\lambda_{\min}(K_2)^2\|\dot{q}_2\|^2 \\
+ \frac{\mu}{2}\|\dot{q}_2\| + \left(\frac{\lambda_{\max}(K_1)}{\lambda_{\min}(K_2)} + \frac{1}{\lambda_{\min}(K_1)}\right)\|\dot{x}_{r1}\|^2 \\
\times \left(\|f_{h1}\|^2 + \|f_e\|^2\right) \\
\]

(29)

It is clear from (28) and (29) that the haptic console #2 subsystem is ISS with respect to state \([q_{21}^T, q_{22}^T]\) and the input \([\dot{q}_2, f_{h2}, f_e]^T\). □

**Theorem 1**: The overall haptic system with the dynamic equation (1) and control laws (11) and (14) is ISS.

**Proof**: In order to study the stability of overall system, the ISS small gain approach of [17] is utilized. **Proposition 1** shows that the closed loop system of the haptic console #1 is Input-to-Output Stable (IOS) with state \([\dot{x}_1, \dot{x}_{r1}, f_{h1}, f_e]^T\) and output \([f_{h1}^T, \dot{x}_1^T, \dot{x}_{r1}^T, \ddot{x}_1^T]\) and the IOS gain \( \gamma_1 \). Besides, **Proposition 2** proves that the haptic console #2 system is IOS with state \([\dot{x}_2, \dot{x}_{r2}, f_{h2}, f_e]^T\) and output \([\dot{x}_2^T, \ddot{x}_2^T, \ddot{x}_{r2}^T]\) with the IOS gain \( \gamma_2 \). Utilizing the small gain theorem, the overall system is ISS, provided that \( \gamma_1 \gamma_2 < 1 \). Owing to the fact that the values of \( \gamma_1 \) and \( \gamma_2 \) can be obtained as a function of control parameters \( K_1 \) and \( \Lambda_1 \), the ISS stability is preserved upon the appropriate choice of those control parameters. □

### IV. Simulation Results

The proposed control structure is applied to a dual user haptic system composed of two identical Geomatic Touch™ haptic interfaces. The dynamic relations of such robot is expressed in [18]. Both human operators are supposed to have impedance parameters as \( M_{hi} = m_{hi}I, B_{hi} = b_{hi}I, \) and \( K_{hi} = k_{hi}I \) where \( m_{hi} = 9g, b_{hi} = 25N.s/m, \) and \( k_{hi} = 200N/m \) all for \( i = 1, 2, \) and the contact force of the environment in direction \#1 which is the penetration direction is defined as

\[
f_{ei} = \begin{cases} 
-k_ex_i, & x_i \geq 0 \\
0, & x_i < 0 
\end{cases}
\]

where \( k_e = 10N/m \). In order to ensure the robustness of the system, the dynamic parameters with 20% perturbation are considered in the control laws. In addition, the control gains of the haptic console #1 are selected as \( K_1 = 10I_{3 \times 3}, \Lambda_1 = 20I_{3 \times 3}, K_2 = 0.01I_{2 \times 2}, \Lambda_2 = 0.02I_{2 \times 2} \). An important consideration is that, the controller of the trainer side is an impedance controller with position tracking requirement, while the controller of the trainee side is only a stabilizing controller without any tracking objective. Therefore, the control gains of haptic console #1 are set larger and the respective bound of tracking error is more strict than the haptic console #2. Besides, the reference impedance parameters of trainer side are set as \( M_{r1} = m_{r1}I, B_{r1} = b_{r1}I, \) and \( K_{r1} = k_{r1}I \) where \( m_{r1} = 10g, b_{r1} = 20N.s/m, \) and \( k_{r1} = 150N/m \).

In the simulations, a trajectory following task in \( z \) direction is considered. The simulation results are shown in Fig. 2, Fig. 3, and Fig. 4. First, the force signals applied by the human operators are shown in Fig. 2 in which the trainer and the trainee signals are shown by blue (solid) and red (dashed) line, respectively. The trainer signal is a square wave with the amplitude 1N, period 10s, and duty cycle 50% passed from the low-pass filter \( 1/(0.1s + 1) \). In order to obtain the trainee force, the trainer force is perturbed by \(-60\% \) and \( 100\% \) in the first and second cycles, respectively. In the third cycle, i.e., from \( t = 20s \) to \( t = 30s \), the trainee force is the same as the trainer force. The trainer supposedly applies right force commands at all times. Besides, from the explanations about the force of the operators, the trainee does not apply correct force signals from \( t = 0s \) to \( t = 10s \) and from \( t = 10s \) to \( t = 20s \) and only the trainee force from \( t = 20s \) to \( t = 30s \) is correct. Notwithstanding the incorrect commands applied by the trainee, the proposed force reflection scheme is able to use the trainer’s skill to correct the trainee’s wrong movements. Therefore, a same position profile is expected in all the three cycles. This issue is apparent in the position signals as shown in Fig. 3. Moreover, the proposed impedance control impedance provides the position tracking objective. This issue is also apparent in the position error signals as shown in Fig. 4.

### V. Conclusions

In this paper, an impedance control structure together with a force reflection scheme is developed for the dual user haptic
training system. The stability of the system is studied using the ISS stability approach and small gain theorem. Through simulation results, it is shown that the proposed control scheme is completely effective for the dual user training system. Our next steps include studying the effect of kinematic and unstructured uncertainties as well as implementation of the proposed controller in our experimental setup.

REFERENCES


