

# Stabilization of Cable Driven Robots Using Interconnection Matrix: Ensuring Positive Tension

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**Abstract**—Cable driven parallel robots are closed-loop kinematic chains, where the end-effector is attached to the base by a number of cables. Because of the nature of cables, a major issue in control of these robots is ensuring positive tensions. When the robot is not redundant, this issue is of utmost importance for the controller synthesis. In the case of point to point motion control, the trajectory is not specified beforehand, thus negative tension may occur. In order to resolve this issue, interconnection and damping assignment passivity based control (IDA-PBC) is employed. IDA-PBC is a well-known method for control of underactuated systems where desired damping and interconnection between subsystems are imposed. Interconnection matrix is a skew-symmetric matrix which doesn't have an effect on stability, but it greatly influences the transient response of the system. In this paper, the interconnection matrix is exploited to ensure positive tension in fully actuated cable driven robots. The results are verified through simulation on a 3-DOF suspended cable driven robot.

**Index Terms**—cable driven robots, IDA-PBC, interconnection matrix, regulation.

## I. INTRODUCTION

Parallel robots are mechanisms where the end-effector of the robot is attached to the base by some kinematic chains of limbs [1]. These robots are extremely precise and agile, however, their workspace is limited [2]. In order to overcome this shortcoming, cable driven robots are introduced to the industry. Cable Driven Parallel Robots (CDPRs) deliver numerous desirable characteristics such as low inertia and high payload to weight ratio, transportability and ease of disassembly and reassembly, large workspace and economical construction [3]. Consequently, these robots can cover vast applications including cleanup of disaster sites, pick and place in factories, manipulation of heavy payloads in docks, spidercam etc [4]. Therefore, the intention of many researchers are focused on design, construction and control of these robots [5].

Since cables are able to pull but not push, one of the most important challenges in CDPRs is to ensure positive tension in cables. It has been shown that in order to control an  $n$  Degree Of Freedom (DOF) fully constrained CDPR, at least  $n+1$  cables are required [6], [7]. However, in under constraint cases or even in the case of underactuated robots [8], [9], it is possible to design a robot in which the number of cables are equal or less than the degrees of freedom of the robot [10].

In contrast to some researches which haven't considered positive tension such as [9], [11], many researchers ensure

this through various proposal [5], [7], [12]. The most common method is to design redundant CDPR in such a way that considering the null space of Jacobian matrix, at least in fully constrained robots, a minimum value for the actuators can be assigned [13], [5]. Additional cost of construction is the most important disadvantage of this method. Another method is to analyze the workspace of the robot. Several types of workspaces have been studied in the literature, such as static workspace [12], [7], [6] which presents a set of all possible configurations in which a CDPR can remain static with positive tensions in all cables. Force and wrench feasible workspaces are respectively studied in [14], [15]. This approach needs a precise initial analysis and is also highly dependent on the structure of the robot. Another approach which is based on designing a reference governor signal that restricts the reference signal to avoid laxity of the cables as proposed in [16]. This method is based on feedback linearization and prediction of the system behavior which leads to high computational cost and possibly inaccurate responses. Oh et al. [17] propose a regulator by Lyapunov control approach for a general suspended planar CDPR. The proposed control law contains a switching structure, while the stability analysis is not reported. In this paper, we propose a new method based on passivity based control (PBC) to guarantee convergence to the desired point while ensuring positive tensions.

Passivity based control is a well-known approach to control of physical systems with respect to a storage function. Classic PBC has been applied successfully to simple mechanical systems, while their stability may be guaranteed by only shaping potential energy [18]. In order to extend PBC to a broader class of systems and remedy some of weaknesses, a second class of PBC has been introduced in [19]. In this class, the closed loop storage function is considered in order to suitably shape, the desired structure of the closed-loop system. One of the most notable representative of this class of design is Interconnection and Damping Assignment (IDA), where the closed-loop system is represented in a Hamiltonian model [20]. In [21] IDA-PBC has been designed for simple mechanical systems. In this research, dynamic equation of the robot is represented in Port-Controlled Hamiltonian (PCH) model and a desired interconnection and damping matrix is assigned to the robot. Desired skew-symmetric interconnection and positive semidefinite damping matrices are the design parameters which may be freely chosen based on different

purposes.

In this paper, point to point control of fully actuated CDPR with IDA-PBC approach is elaborated in detail. First, a special case of IDA-PBC method which coincides with well-known PD with gravity compensator is considered. Then, using the desired interconnection matrix, which does not contribute to the stability proof of the system, the positive tension in the cables are ensured. It is shown that the new representation of interconnection matrix is not full rank, thus a least squares method is employed to achieve optimal result with a minimum norm solution.

The organization of this paper is as follows. In section II the IDA-PBC method for mechanical systems is described. A novel method based on determining desired interconnection matrix is introduced in section III to ensure stability with bounded inputs. Simulation results are given in section IV. Finally, conclusion and future works are proposed in section V.

## II. BACKGROUND

In this section IDA-PBC method for simple mechanical systems is proposed. We refer the reader to [22], [21] for more details. Dynamic equations of a robot in PCH model may be written in the form of [21]:

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0_{n \times n} & I_n \\ -I_n & 0_{n \times n} \end{bmatrix} \begin{bmatrix} \nabla_q H \\ \nabla_p H \end{bmatrix} + \begin{bmatrix} 0_{n \times m} \\ G(q) \end{bmatrix} \tau \quad (1)$$

where  $H(q, p) = \frac{1}{2}p^T M^{-1}(q)p + V(q)$  is total energy of the system, equivalent to the sum of kinetic and potential energy,  $q, p \in \mathbb{R}^n$  are generalized position and momenta respectively,  $M^T(q) = M(q) > 0$  is the inertia matrix and  $G(q) \in \mathbb{R}^{n \times m}$  is the input coupling matrix. Assume that  $H_d = \frac{1}{2}p^T M_d^{-1}(q)p + V_d(q)$ , and  $q_* = \arg \min V_d(q)$  denotes the desired point. Desired structure of the closed-loop system is considered as follows

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0_{n \times n} & M^{-1}M_d \\ -M_d M^{-1} & J_d - R_d \end{bmatrix} \begin{bmatrix} \nabla_q H_d \\ \nabla_p H_d \end{bmatrix} \quad (2)$$

in which  $J_d(q, p), R_d(q) \in \mathbb{R}^{n \times n}$  are desired interconnection and damping matrices, respectively. Note that  $R_d$  is positive semidefinite and  $J_d$  is a skew-symmetric matrix. IDA-PBC method is developed basically to control underactuated robots. However, in this paper it is applied to fully actuated CDPRs with the constraint of positive tension in cables.. Control law for the case of  $m = n$  is derived as follows

$$\tau = G^{-1} \left( \nabla_q H - M_d M^{-1} \nabla_q H_d + (J_d - R_d) \nabla_p H_d \right). \quad (3)$$

Note that total energy shaping in control of fully actuated robots is not required, thus merely potential energy is shaped, i.e.  $M_d$  is chosen equal to  $M$ .

If  $H_d$  is considered as Lyapunov candidate, its derivative is

$$\begin{aligned} \dot{H}_d &= (\nabla_q H_d)^T \dot{q} + (\nabla_p H_d)^T \dot{p} = (\nabla_q H_d)^T \nabla_p H_d + \\ & (\nabla_p H_d)^T (-\nabla_q H_d + (J_d - R_d) \nabla_p H_d) \\ &= -(\nabla_p H_d)^T R (\nabla_p H_d). \end{aligned} \quad (4)$$

Hence,  $q_*$  is stable equilibrium. Asymptotic stability is ensured using the Krasovski-Lasalle theorem. Therefore, it is clear that stability with control law (3) is ensured whereas there are no bounds on  $\tau$ .

Here, a brief description about interconnection matrix will be addressed before discussing the main results.

**Definition 1.** [23] Consider a finite-dimensional linear space  $\mathcal{F}$  with  $\mathcal{E} = \mathcal{F}^*$ . A subspace  $\mathcal{D} \subset \mathcal{F} \times \mathcal{E}$  is a Dirac structure if

- 1-  $\langle e | f \rangle = 0$ , for all  $(f, e) \in \mathcal{D}$
- 2-  $\dim \mathcal{D} = \dim \mathcal{F}$

Property (1) corresponds to power conservation, moreover, it expresses the fact that the total power entering or leaving Dirac structure is zero. Roughly speaking, Dirac structure links various port variables  $f \in \mathcal{F}$  as flows and  $e \in \mathcal{E}$  as efforts such that the total power  $e^T f$  is equal to zero.

Here, the Dirac structure is given as the graph of the skew-symmetric matrix

$$\begin{bmatrix} 0_{n \times n} & M^{-1}M_d \\ -M_d M^{-1} & J_d(q, p) \end{bmatrix}$$

which shows that this structure is modulated by the state variables  $q, p$ . Furthermore,  $J_d$  generally represent a pseudo-Dirac structure which means that they do not necessarily satisfy the integrability condition. In other words, It is a generalized Poisson structure with generalized Poisson bracket  $\{\cdot, \cdot\}$  which is a bilinear map from  $C^\infty(\mathcal{X}) \times C^\infty(\mathcal{X})$  into  $C^\infty(\mathcal{X})$  that satisfies skew-symmetric property:

$$\{F, G\} = -\{G, F\}$$

and Liebniz rule [24]:

$$\{F, G \cdot H\} = \{F, G\} \cdot H + G \cdot \{F, H\}.$$

If in addition, Poisson bracket satisfies the Jacobi identity:

$$\{F, \{G, H\}\} + \{G, \{H, F\}\} + \{H, \{F, G\}\} = 0$$

then it is possible to assign a canonical coordinate  $(q', p')$  such that structure matrix  $J(q', p')$  becomes:

$$\begin{bmatrix} 0_{n \times n} & I_n \\ -I_n & 0_{n \times n} \end{bmatrix}$$

In [25] It is shown that if  $J_d$  has a special structure, then it is possible to transform generalized Hamiltonian system (2) in the form of standard Hamiltonian. Furthermore, the relation between  $J_d$  and gyroscopic terms has been elaborated.

In this paper we consider pseudo-Dirac structure for  $J_d$ , i.e. Jacobi identity is not necessarily satisfied. In order to gain a better insight about this structure, consider an electrical circuit. In this case, Dirac structure represents the wire between the elements. In other words, Dirac structure is equivalent to the oriented graph of network.  $(i, j)$ -th element of  $J_d$  conceivably represents the current between nodes  $i$  and  $j$ . Skew-symmetric property of this matrix means that the current between nodes  $i$  and  $j$  is the negative of the current between nodes  $j$  and  $i$ .

### III. MAIN RESULTS

In this section, control law 3 is modified in order to ensure stability and boundedness of the inputs. Dynamic equation of a CDPR in the form of Euler-Lagrange may be written as in task coordinate

$$M(X)\ddot{X} + C(X, \dot{X})\dot{X} + G(X) = J_a^T(X)\tau,$$

where  $X$  is task space position and orientation,  $C(X, \dot{X})$  denotes the Coriolis and centrifugal terms,  $G(X) = \frac{\partial V}{\partial X}$  denotes the vector of gravity terms and  $J_a(X)$  is Jacobian matrix of the robot. Note that it is equivalent to the form of PCH (1) with  $G = J_a^T(X)$ .

As explained before, only the potential energy is modified, thus  $M_d = M$  and  $\nabla_p H_d = M_d^{-1}p = \dot{q}$ . To ensure that  $q_* = \arg \min V_d(q)$ ,  $V_d$  is chosen as follows

$$V_d = \sum_{i=1}^n \frac{k_i}{2} (q(i) - q_*(i))^2. \quad (5)$$

Moreover, damping matrix is assumed to be a diagonal matrix  $R_d = \text{diag}[r_1, \dots, r_n]$ . Interconnection matrix is considered as

$$J_d(q, p) = \begin{bmatrix} 0 & J_1 & \dots & J_{n-1} \\ -J_1 & 0 & \dots & J_{2n-2} \\ \vdots & \ddots & \ddots & \vdots \\ -J_{n-1} & -J_{2n-2} & \dots & 0 \end{bmatrix}. \quad (6)$$

Note that if  $J_d$  is set equal to zero, this controller is coincident with well-known PD alongside a gravity compensator controller. Assume that  $\tau$  is constrained as follows

$$T_m \leq \tau \leq T_M \quad (7)$$

where  $T_m$  and  $T_M$  are lower and upper bounds, respectively. Control law represented in (3) may be separated into three parts

$$\begin{aligned} \tau &= \tau_1 + \tau_2 + \tau_3 \\ \tau_1 &= J_a^{-1} \nabla_q V \\ \tau_2 &= -\alpha J_a^{-1} \left( \nabla_q V_d + R_d \nabla_p H_d \right) \\ \tau_3 &= J_a^{-1} J_d \nabla_p H_d. \end{aligned} \quad (8)$$

where  $\alpha$  is a designer positive scalar gain.

In this research, all parameters in  $V_d$  and  $R_d$  are assume to be constant. The aim of this paper is to design  $J_d$  and  $\alpha$  in such a way that input constraint proposed in (7) is satisfied. Hence,

$$T_m \leq \tau_1 + \tau_2 + \tau_3 \leq T_M. \quad (9)$$

shall be held.  $J_d \dot{q}$  is a vector whose undefined parameters are in the matrix  $J_d$ . Thus this expression may be rewritten as follows

$$J_d \dot{q} = P \mathfrak{J} \quad (10)$$

where  $P(q, p) \in \mathbb{R}^{n \times \frac{n(n-1)}{2}}$  is matrix-form representation of  $\dot{q}$  and  $\mathfrak{J} \in \mathbb{R}^{\frac{n(n-1)}{2}}$  is vector-form representation of  $J_d$  as follows

$$\mathfrak{J} = \begin{bmatrix} J_1 & J_2 & \dots & J_{\frac{n(n-1)}{2}} \end{bmatrix}^T.$$

Thus

$$\tau_2 = J_a^{-1} P \mathfrak{J} \triangleq A \mathfrak{J}. \quad (11)$$

Notice that lossless property of pseudo-Dirac structure should be complied in the new representation. Therefore, the following equality holds

$$\dot{q}^T J_d \dot{q} = 0 \quad \rightarrow \quad \dot{q}^T P \mathfrak{J} = 0.$$

Since this equality is preserved for every  $\mathfrak{J}$ , therefore,

$$P^T \dot{q} = 0 \quad (12)$$

which guarantees power conservation. Unfortunately, (12) shows that matrix  $P$  has a null space, thus  $P$  has a rank of  $n-1$ . This restricts  $\tau_2$  in a subspace of  $\mathbb{R}^{n-1}$ , hence assurance of (9) may not be possible. In order to remove this bottle neck, the following assumption like [26], [27] is required.

**Assumption 1.** Gravity term of the robot may be compensated by the actuators in all of its workspace. In other words,

$$T_m \leq J_a^{-1} \nabla_q V \leq T_M,$$

or with some more conservativeness:

$$\|J_a^{-1} \nabla_q V - \frac{T_m + T_M}{2}\| \leq \left\| \frac{T_m - T_M}{2} \right\|. \quad (13)$$

Notice that this assumption is reasonable, because without it, the robot is not stabilizable in every position inside its workspace.

Equation (9) may be rewritten as follows

$$\frac{T_m - T_M}{2} \leq \tau_1 + \tau_2 + \tau_3 - \frac{T_m + T_M}{2} \leq \frac{T_m - T_M}{2}. \quad (14)$$

This inequality may be represented in the following form

$$\left| \tau_{1_i} + \tau_{2_i} + \tau_{3_i} - \frac{T_{m_i} + T_{M_i}}{2} \right| \leq \frac{T_{M_i} - T_{m_i}}{2} \quad (15)$$

where index  $i$  represents  $i$ -th element of a vector. It is clear that

$$\left| \tau_{1_i} + \tau_{2_i} + \tau_{3_i} - \frac{T_{m_i} + T_{M_i}}{2} \right| \leq \max_{j=1, \dots, m} \left\{ \left| \tau_{1_j} + \tau_{2_j} + \tau_{3_j} - \frac{T_{m_j} + T_{M_j}}{2} \right| \right\} = \left\| \tau_1 + \tau_2 + \tau_3 - \frac{T_m + T_M}{2} \right\|_\infty \quad \text{for } i = 1, \dots, n.$$

Therefore, if the following inequality holds

$$\left\| \tau_1 + \tau_2 + \tau_3 - \frac{T_m + T_M}{2} \right\|_\infty \leq \min_{j=1, \dots, m} \left\{ \frac{T_{M_j} - T_{m_j}}{2} \right\} \quad (16)$$

then (15) is satisfied. In other words, inequality (16) is a conservative representation of (15). Since  $l_2$  norm is larger than  $l_\infty$  norm, a more conservative representation of (15) is as follows

$$\left\| \tau_1 + \tau_2 + \tau_3 - \frac{T_m + T_M}{2} \right\|_2 \leq \min_{j=1, \dots, m} \left\{ \frac{T_{M_j} - T_{m_j}}{2} \right\}. \quad (17)$$

Using triangular inequality, (17) may be replaced by

$$\left\| \tau_1 - \frac{T_m + T_M}{2} \right\|_2 + \|\tau_2 + \tau_3\|_2 \leq \min_{j=1, \dots, m} \left\{ \frac{T_{M_j} - T_{m_j}}{2} \right\}. \quad (18)$$

Now notice that this inequality may be satisfied easily using the least squares method. First, (17) may be rewritten in the following compact form

$$\|A\mathfrak{J} - b\|_2 \leq \beta \quad (19)$$

in which

$$b = -\tau_2, \quad \beta = \min_{j=1, \dots, m} \left\{ \frac{T_{M_j} - T_{m_j}}{2} \right\} - \left\| \tau_1 - \frac{T_m + T_M}{2} \right\|_2 \quad (20)$$

Note that the right hand side of (19) is always a positive scalar due to Assumption 1. Rank  $A$  is always  $n - 1$  and it has more columns than rows for  $n > 3$ . Therefore, we can just minimize the left hand side. Moreover, in order to ensure this inequality,  $\alpha$  shall be suitably designed. In order to convert (19) to a standard least squares problem,  $\mathfrak{J}$  is set as follows

$$\mathfrak{J} = A^T{}^n (AA^T)\mathfrak{J}' \quad (21)$$

in which  ${}^n(AA^T)$  means  $n$ -th column of  $AA^T$  is eliminated and  $\mathfrak{J}' \in \mathbb{R}^{n-1}$  is the new variable that should be determined. Now (19) is in the following form

$$\|AA^T{}^n (AA^T)\mathfrak{J}' - b\|_2 \triangleq \|\mathfrak{A}\mathfrak{J}' - b\|_2 \leq \beta \quad (22)$$

which is in the standard form of the least squares problem and the solution is

$$\mathfrak{J}' = \mathfrak{A}^T(\mathfrak{A}^T\mathfrak{A})^{-1}b. \quad (23)$$

This minimizes the left hand side of (22), however, the resulting value might not satisfy this inequality.  $\alpha$  has the important role to remedy this problem. First, notice that  $b$  and  $\mathfrak{J}'$  are both linear with respect to  $\alpha$ . Hence,

$$\alpha = \begin{cases} 1 & \text{if } \|\mathfrak{A}\mathfrak{J}' - b\|_2 < \beta \\ \frac{\beta}{\|\mathfrak{A}\mathfrak{J}' - b\|_2} & \text{else} \end{cases} \quad (24)$$

Note that  $\alpha$  in default, is equal to one and through its redesign in (24), it is possible to ensure input constraint. The results are presented in the following theorem.

**Theorem 1.** Consider a fully actuated CDPR with dynamic equation (1) and control law (3). Suppose that  $R_d$  is a diagonal matrix and  $V_d$  is as given in (5). Input constraint (9) is satisfied with  $\mathfrak{J}$  proposed in (21) and (23) and  $\alpha$  in (24).

*Proof.* Convergence to the desired position is shown in (4). Constraint (9) is replaced by conservative inequality (17). Considering Assumption 1, ensuring inequality (20) is required. Minimum solution is obtained by least squares method and is proposed in (21) and (23). Because this solution doesn't guarantee (20), gain  $\alpha$  is designed in (24) to ensure input constraint. Notice that there always exists suitably small  $\alpha$  to satisfies (20).  $\square$

**Remark 1.** In CDPRs the actuators are usually similar. Thus

$$\min_{j=1, \dots, m} \left\{ \frac{T_{M_j} - T_{m_j}}{2} \right\} = \frac{T_{M_i} - T_{m_i}}{2} \quad \text{for } i = 1, \dots, n$$

which reduces conservatism.

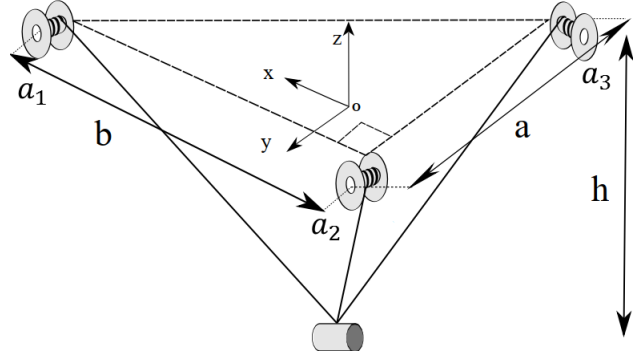


Fig. 1. Schematic of a 3-DOF suspended CDPR. All of the anchor points are in a horizontal plane.

**Remark 2.** At the start, velocity of the robot is usually zero. Hence, matrix  $A$  is a zero matrix and the proposed method fails. Therefore, as a solution, in  $t = 0$  we saturate the control law. Note that because  $t = 0$  is measure zero<sup>1</sup>, it doesn't contradict the stability proof.

**Remark 3.** It may seem that with just modifying  $k_i$ s and  $r_i$ s, input constraints will be satisfied. However, this will not be possible in the case of underactuated robots. In other words, the main reason we incorporate interconnection matrix is that it is possible to apply the proposed method for stabilization of underactuated robots with input constraint. IDA-PBC method alongside ensuring input constraint is our major future purpose.

#### IV. SIMULATION RESULTS

In this section we apply the preceding design methodology to a 3-DOF cable driven robot with three actuators through the simulation. Schematic of this robot is shown in Fig. 1.

Configuration variables of the robot are  $X = [x, y, z]^T$ . Dynamic parameters are considered as follows

$$M = mI_3 \quad V = mgz.$$

where  $m = 4Kg$  is the mass of end-effector. Jacobian matrix of this robot is [5]:

$$J_a = \begin{bmatrix} \frac{x-x_{a1}}{l_1} & \frac{y-y_{a1}}{l_1} & \frac{z-z_{a1}}{l_1} \\ \frac{x-x_{a2}}{l_2} & \frac{y-y_{a2}}{l_2} & \frac{z-z_{a2}}{l_2} \\ \frac{x-x_{a3}}{l_3} & \frac{y-y_{a3}}{l_3} & \frac{z-z_{a3}}{l_3} \end{bmatrix},$$

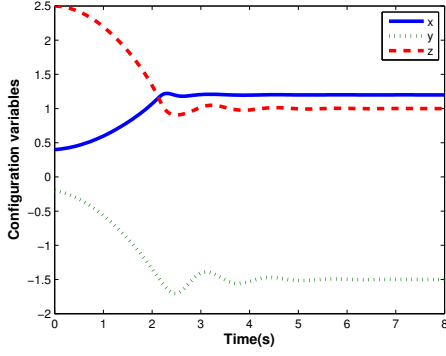
in which  $l_i$ s are the length of cables computed as follows

$$\begin{aligned} l_1 &= \sqrt{(x-x_{a1})^2 + (y-y_{a1})^2 + (z-z_{a1})^2} \\ l_2 &= \sqrt{(x-x_{a2})^2 + (y-y_{a2})^2 + (z-z_{a2})^2} \\ l_3 &= \sqrt{(x-x_{a3})^2 + (y-y_{a3})^2 + (z-z_{a3})^2}, \end{aligned}$$

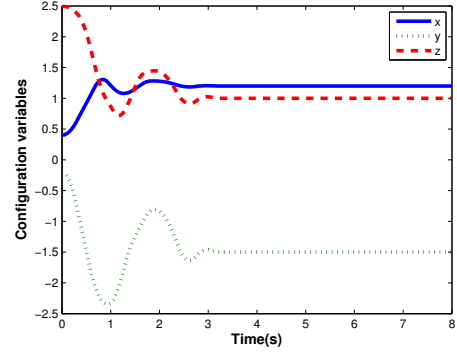
and kinematic parameters are

$$\begin{aligned} x_{a1} &= x_{a2} = -x_{a3} = b/2 = 3.56/2 \\ y_{a1} &= -y_{a2} = -y_{a3} = a/2 = 7.05/2 \\ z_{a1} &= z_{a2} = z_{a3} = h = 4.26 \end{aligned} \quad (25)$$

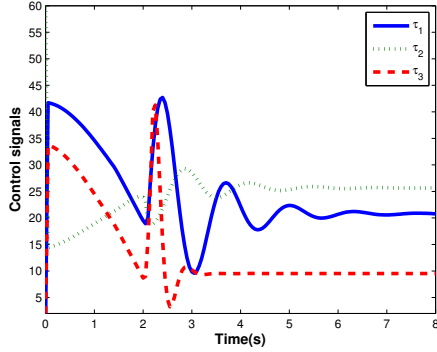
<sup>1</sup>For more information about Lebesgue measure, see [28]



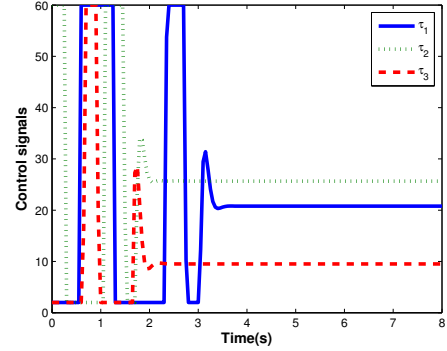
(a) Configuration variables of the robot with modification of interconnection matrix. All of the variables converge to their desired values.



(a) Configuration variables of the robot by saturating the control signals. Although all of the variables converge to their desired values, convergence is not guaranteed.



(b) Control efforts of the cable driven robot with modification of interconnection matrix. All of the inputs are in the range (2,60).



(b) Control efforts of the cable driven robot with a simple saturation function. Several sharp changes in control signals may lead to fluctuation in cables and therefore, this is not practical.

Fig. 2. Simulation results of the proposed method on a 3-DOF cable driven robot. It is clear that all of the variables converge to desired values while control signals remain in the determined bound.

Fig. 3. Simulation results of a simple PD plus gravity compensation with saturated inputs on a 3-DOF cable driven robot. Control signals change sharply and also proof of convergence is not established.

Here, initial and desired position of the robot as well as constant gains of the controller are considered in such a way that without modification of  $J_d$  positive tensions are not ensured. these parameters are set as

$$\begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} = \begin{bmatrix} 0.4 \\ -0.2 \\ 2.5 \end{bmatrix} \quad \begin{bmatrix} x_* \\ y_* \\ z_* \end{bmatrix} = \begin{bmatrix} 1.2 \\ -1.5 \\ 1 \end{bmatrix}$$

$$k_1 = k_2 = k_3 = 1200 \quad R_d = 100I_3.$$

The lower and upper bounds for all actuators are set equal to 2 and 60, respectively<sup>2</sup>. Matrix  $P$  in (10) is as follows

$$P = \begin{bmatrix} \dot{y} & \dot{z} & 0 \\ -\dot{x} & 0 & \dot{z} \\ 0 & -\dot{x} & -\dot{y} \end{bmatrix}.$$

Fig. 2 shows the results of proposed method and Fig. 3 illustrates the simulation results with a well-known PD plus gravity compensation and saturating inputs. Fig. 2(a) shows that motion of the robot is very smooth and the settling

time is about 5 seconds. As depicted in Fig. 2(b), using interconnection matrix and the proposed method, all of  $\tau_i$ s remain in the determined bound. Note that just at  $t = 0$  the control signals are saturated which is discussed in Remark 2. In Fig. 3 without modifying interconnection matrix and using a simple saturation function, control signals change rapidly and the transient response is not as smooth as in Fig. 2(a). Notice that because of high gains which are considered, without saturating inputs, all of control signals will be out of the bound.

## V. CONCLUSION AND FUTURE WORKS

This article presents a new method for stabilization of cable driven robots with input constraint. Interconnection and damping assignment passivity based control method is used and it is proved that interconnection matrix can be designed in such a way to avoid slackness in tensions. The proposed method is based on minimization of a control law based on the least squares method. Then, the proposed method was applied to a 3-DOF suspended cable robot through simulation and the results confirm the robot converges to the desired point while all the cables are in tension.

<sup>2</sup>Kinematic, dynamic and the bound of control law are considered based on ARAS cable driven robot proposed in [5].

Future works include minimization of infinity norm of control law by interconnection matrix, investigating the effect of this matrix on transient response of the system, applying this method on underactuated robots and experimental implementation of it on ARAS cable driven robot.

## REFERENCES

- [1] S. Khalilpour, A. Bourbour, R. Khorrambakht, S. Kariminasab, and H. Taghirad, "Forward kinematics resolution of a deployable cable robot," in *2017 5th RSI International Conference on Robotics and Mechatronics (ICRoM)*, pp. 27–32, IEEE, 2017.
- [2] M. R. J. Harandi and H. D. Taghirad, "Motion control of an underactuated parallel robot with first order nonholonomic constraint," in *2017 5th RSI International Conference on Robotics and Mechatronics (ICRoM)*, pp. 582–587, IEEE, 2017.
- [3] S. Khalilpour, H. Taghirad, and H. Habibi, "Wave-based control of suspended cable driven parallel manipulators," in *2017 5th International Conference on Control, Instrumentation, and Automation (ICCIA)*, pp. 173–178, IEEE, 2017.
- [4] S. Khalilpour, R. Khorrambakht, M. Harandi, H. Taghirad, and P. Cardou, "Robust dynamic sliding mode control of a deployable cable driven robot," in *Electrical Engineering (ICEE), Iranian Conference on*, pp. 863–868, IEEE, 2018.
- [5] S. Khalilpour, R. Khorrambakht, H. Taghirad, and P. Cardou, "Robust cascade control of a deployable cable-driven robot," *Mechanical Systems and Signal Processing*, vol. 127, pp. 513–530, 2019.
- [6] S. A. Khalilpour, A. Z. Loloie, H. D. Taghirad, and M. T. Masouleh, "Feasible kinematic sensitivity in cable robots based on interval analysis," in *Cable-Driven Parallel Robots*, pp. 233–249, Springer, 2013.
- [7] S. Khalilpour, A. Z. Loloie, M. T. Masouleh, and H. Taghirad, "Kinematic performance indices analyzed on four planar cable robots via interval analysis," in *2013 First RSI/ISM International Conference on Robotics and Mechatronics (ICRoM)*, pp. 313–318, IEEE, 2013.
- [8] N. Zoso and C. Gosselin, "Point-to-point motion planning of a parallel 3-dof underactuated cable-suspended robot," in *2012 IEEE International Conference on Robotics and Automation*, pp. 2325–2330, IEEE, 2012.
- [9] M. R. J. Harandi, H. Damirchi, H. D. Taghirad, *et al.*, "Point-to-point motion control of an underactuated planar cable driven robot," in *2019 27th Iranian Conference on Electrical Engineering (ICEE)*, pp. 979–984, IEEE, 2019.
- [10] A. B. Alp and S. K. Agrawal, "Cable suspended robots: design, planning and control," in *Proceedings 2002 IEEE International Conference on Robotics and Automation (Cat. No. 02CH37292)*, vol. 4, pp. 4275–4280, IEEE, 2002.
- [11] M. I. Hosseini, M. J. Harandi, S. A. K. Seyedi, *et al.*, "Adaptive fast terminal sliding mode control of a suspended cable-driven robot," in *2019 27th Iranian Conference on Electrical Engineering (ICEE)*, pp. 985–990, IEEE, 2019.
- [12] M. Gouttefarde, J.-F. Collard, N. Riehl, and C. Baradat, "Simplified static analysis of large-dimension parallel cable-driven robots," in *2012 IEEE International Conference on Robotics and Automation*, pp. 2299–2305, IEEE, 2012.
- [13] S.-R. Oh and S. K. Agrawal, "Cable suspended planar robots with redundant cables: Controllers with positive tensions," *IEEE Transactions on Robotics*, vol. 21, no. 3, pp. 457–465, 2005.
- [14] A. T. Riechel and I. Ebert-Uphoff, "Force-feasible workspace analysis for underconstrained, point-mass cable robots," in *IEEE International Conference on Robotics and Automation, 2004. Proceedings. ICRA'04. 2004*, vol. 5, pp. 4956–4962, IEEE, 2004.
- [15] P. Bosscher, A. T. Riechel, and I. Ebert-Uphoff, "Wrench-feasible workspace generation for cable-driven robots," *IEEE Transactions on Robotics*, vol. 22, no. 5, pp. 890–902, 2006.
- [16] S.-R. Oh and S. K. Agrawal, "A reference governor-based controller for a cable robot under input constraints," *IEEE transactions on control systems technology*, vol. 13, no. 4, pp. 639–645, 2005.
- [17] S.-R. Oh and S. K. Agrawal, "A control lyapunov approach for feedback control of cable-suspended robots," in *Proceedings 2007 IEEE International Conference on Robotics and Automation*, pp. 4544–4549, IEEE, 2007.
- [18] R. Ortega, A. J. Van Der Schaft, I. Mareels, and B. Maschke, "Putting energy back in control," *IEEE Control Systems Magazine*, vol. 21, no. 2, pp. 18–33, 2001.
- [19] R. Ortega, A. Van Der Schaft, B. Maschke, and G. Escobar, "Interconnection and damping assignment passivity-based control of port-controlled hamiltonian systems," *Automatica*, vol. 38, no. 4, pp. 585–596, 2002.
- [20] R. Ortega and E. Garcia-Canseco, "Interconnection and damping assignment passivity-based control: A survey," *European Journal of control*, vol. 10, no. 5, pp. 432–450, 2004.
- [21] R. Ortega, M. W. Spong, F. Gómez-Estern, and G. Blankenstein, "Stabilization of a class of underactuated mechanical systems via interconnection and damping assignment," *IEEE transactions on automatic control*, vol. 47, no. 8, pp. 1218–1233, 2002.
- [22] J. A. Acosta, R. Ortega, A. Astolfi, and A. D. Mahindrakar, "Interconnection and damping assignment passivity-based control of mechanical systems with underactuation degree one," *IEEE Transactions on Automatic Control*, vol. 50, no. 12, pp. 1936–1955, 2005.
- [23] A. van der Schaft, D. Jeltsema, *et al.*, "Port-hamiltonian systems theory: An introductory overview," *Foundations and Trends® in Systems and Control*, vol. 1, no. 2-3, pp. 173–378, 2014.
- [24] S. Stramigioli, B. Maschke, and A. Van Der Schaft, "Passive output feedback and port interconnection," *IFAC Proceedings Volumes*, vol. 31, no. 17, pp. 591–596, 1998.
- [25] G. Blankenstein, R. Ortega, and A. J. Van Der Schaft, "The matching conditions of controlled lagrangians and ida-passivity based control," *International Journal of Control*, vol. 75, no. 9, pp. 645–665, 2002.
- [26] A. Zavala-Rio and V. Santibanez, "A natural saturating extension of the pd-with-desired-gravity-compensation control law for robot manipulators with bounded inputs," *IEEE Transactions on Robotics*, vol. 23, no. 2, pp. 386–391, 2007.
- [27] H. Wang and Y. Su, "Global finite-time regulation of robot manipulators using only position measurements," *Transactions of the Institute of Measurement and Control*, vol. 40, no. 5, pp. 1681–1690, 2018.
- [28] F. Burk, *Lebesgue measure and integration: an introduction*, vol. 32. John Wiley & Sons, 2011.