Exercise 1: Analyze the stability of the origin as the equilibrium point of the following system.

\[
\dot{x}_1 = -x_1 + x_2^6 \\
\dot{x}_2 = x_1^6 + x_2^3
\]

Exercise 2: Using the Variable Gradient Method, find a Lyapunov function to show the origin is globally asymptotically stable equilibrium point for the following system.

\[
\dot{x}_1 = x_2 \\
\dot{x}_2 = -(x_1 + x_2) - h(x_1 + x_2)
\]

Where \(h(0) = 0\) and \(h(z) > 0\), \(\forall z \neq 0\).

Exercise 3: Consider the following nonlinear system

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\frac{(y + \beta x_2^2) \sin(x_1)}{\alpha + \beta \cos(x_1)} - u_d \\
\dot{x}_3 &= \frac{2(y + \beta x_2^2)\sin(x_1)}{\alpha + \beta \cos(x_1)} + u_d
\end{align*}
\]

where \(\alpha, \beta,\) and \(y\) are constant and positive and \(|x_1| < \frac{\pi}{2}\).

The control input is designed as follows:

\[
u_d = \frac{2x_2}{(\alpha + \beta \cos(x_1))^2} + \rho(2x_2 + x_3)
\]

where \(\rho\) is a positive constant. Use the following Lyapunov function to analyze the stability of the equilibrium point.

\[
V = \frac{y + \beta x_2^2}{\beta(\alpha + \beta \cos(x_1))^2} - \frac{y}{\beta(\alpha + \beta)^2} + 0.5\rho(2x_2 + x_3)^2
\]
Exercise 4: Consider the following nonlinear system

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -x_2 - \sin(x_1) - 2\text{sat}(x_1 + x_2)
\end{align*}
\]

1. Show that the origin is the unique equilibrium point of the system.
2. Use linearization method to analyze the stability of the origin.
3. Take \(\sigma = x_1 + x_2\), and show for \(|\sigma| \geq 1\) we have: \(\sigma \dot{\sigma} \leq -|\sigma|\)
4. Consider the function \(V = x_1^2 + 0.5x_2^2 + 1 - \cos(x_1)\) and the set \(M_c\) defined as

\[
M_c = \{x \in \mathbb{R}^2 | V(x) \leq c\} \cap \{x \in \mathbb{R}^2 | |\sigma| \leq 1\}.
\]

Show that \(M_c\) is a positive invariant set and every path inside \(M_c\) converges to the origin as \(t \to \infty\).
5. Show that origin is globally asymptotically stable.

Exercise 5:

1. Consider the system in below figure, where the nonlinear function is given by \(g(e) = e(t)^3\).

   ![System Diagram]

   a. Find the state space model.

   b. Show that the origin is asymptotically stable using the Lyapunov function:

\[
V(x) = x^T p x
\]