In the Transactions of the Canadian Society for Mechanical Engineering, 19(4):495--508, Dec 1995.

State-Feedback Control For Passenger Ride Dynamics

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Abstract

An analytical investigation of a half-car model with passenger dynamics, subjected to random road disturbance, is performed. Two different methods of defining the performance index for optimal controller design are proposed. Nondeterministic inputs are applied to simulate the road surface conditions more realistically. Results obtained illustrate that using an optimal state-feedback controller, with passenger acceleration included in the performance index, would exhibit not only an improved passenger ride comfort, but also, a better road handling and stability.

1 Introduction

Demands for better ride comfort and controllability of road vehicles are pursued by many automotive industries by seriously considering the use of active suspensions. These electronically controlled suspension systems can potentially improve the ride comfort as well as the road handling of the vehicle.

In any vehicle suspension system, there are a variety of performance parameters which need to be optimized. The trade off between ride comfort and handling characteristics is usually a trial and error procedure which represents an optimization problem. There are four important parameters which should be considered carefully in designing a suspension system; namely, *ride comfort, body motion, road handling*, and *suspension travel*.

No suspension system can simultaneously minimize all four of the above mentioned parameters. The advantage of controlled suspension is that a better set of design trade-offs are possible rather than with passive systems, [10]. State-feedback control for active suspension is a powerful tool for designing a controller, [6, 11]. In this approach a mathematical representation for ride comfort and road handling will be optimized considering the actuator limitations. Since body motion and suspension travel are functions of the system states, they will also be optimized during the design.

Linear optimal control theory provides a systematic approach to design the active suspension controllers, [2], and has been used by several investigators. Sinha et al., [12], and Caudill et al., [1], have used this method to design active suspension controllers for railroad vehicles. Esmailzadeh, [5, 6] investigated a pneumatic controlled active suspension for automobiles. Hrovat appends this method with the concept of dynamic absorber for improved performance for quarter-car and half-car models, [7, 9]. Elmadany considered using integral and state feedback controllers for active suspension for half-car model, [4]. Shannan et al. considered the lateral and longitudinal motion for a full car model, and implemented active controller with linear optimal control, [11].

This paper will emphasize the methodology of controller design based on optimal control theory. A half-car model is developed considering passenger dynamics which to our knowledge is given little attention in the existing literature. Passenger dynamics is more important when ride comfort has to be studied, and provides the necessary tool to design the controller which will effectively satisfy both the ride comfort and road handling performance.

2 Mathematical Modelling

This section is devoted to the mathematical modelling of vehicle, considering the passengers dynamics and road disturbance. A linear model is considered to represent the vehicle-passenger dynamics, while a normal random profile is used to model the road roughness.

2.1 Vehicle-Passenger Model

Figure 1 illustrates the half-car model of a passenger car, which has six degrees of freedom (6 dof). The model consisted of a body, two axles and two passengers. Body motions are considered to be bounce and pitch, with every axle having its own bounce. The passengers are considered to

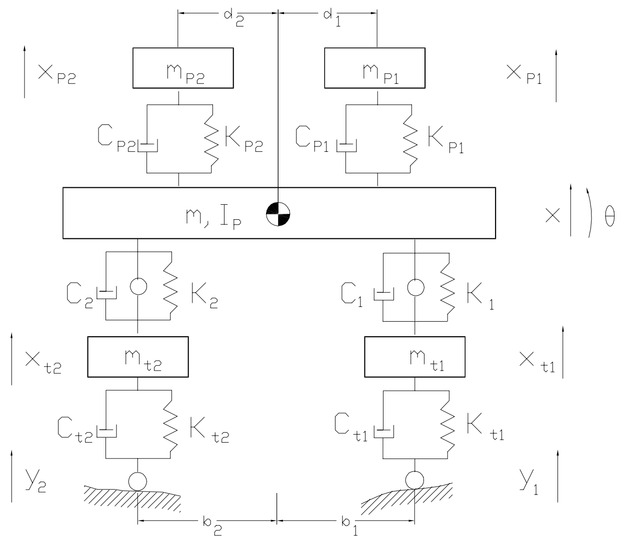

Figure 1: **Half-car dynamic model of road vehicle with six degrees of freedom**

have only vertical oscillations. The suspension, tire, and passengers seats are modelled by linear springs in parallel with viscous dampers. The actuators are considered to be a source of controllable force, and located parallel with the suspension spring and shock absorber.

The system variable notations with their corresponding values are presented in Nomenclatures. The parameters related to the tires are denoted with subscript t, while, the passenger parameters have subscript p.

The system with six degrees of freedom are represented by the following states:
body bounce, x; body pitch, θ; tire deflection, x_{t1}, x_{t2}; and passenger vertical motions, x_{p1}, x_{p2}.

The following equations of motion could be derived using Newton-Euler method:

$$
\begin{aligned}
\ddot{x} = \; & -\tfrac{1}{m}(k_1 + k_2 + k_{p1} + k_{p2})x - \tfrac{1}{m}(k_1 b_1 - k_2 b_2 + k_{p1} d_1 - k_{p2} d_2)\theta \\
& + \tfrac{k_{p1}}{m} x_{p1} + \tfrac{k_{p2}}{m} x_{p2} + \tfrac{k_1}{m} x_{t1} + \tfrac{k_2}{m} x_{t2} + \tfrac{c_{p1}}{m}\dot{x}_{p1} + \tfrac{c_{p2}}{m}\dot{x}_{p2} + \tfrac{c_1}{m}\dot{x}_{t1} + \tfrac{c_2}{m}\dot{x}_{t2} \\
& - \tfrac{1}{m}(c_1 + c_2 + c_{p1} + c_{p2})\dot{x} - \tfrac{1}{m}(c_1 b_1 - c_2 b_2 + c_{p1} d_1 - c_{p2} d_2)\dot{\theta} + \tfrac{f_1 + f_2}{m} \\
\ddot{\theta} = \; & -\tfrac{1}{I_p}(k_1 b_1 - k_2 b_2 + k_{p1} d_1 - k_{p2} d_2)x - \tfrac{1}{I_p}(k_1 b_1^2 + k_2 b_2^2 + k_{p1} d_1^2 + k_{p2} d_2^2)\theta \\
& - \tfrac{k_{p1} d_1}{I_p} x_{p1} + \tfrac{k_{p2} d_2}{I_p} x_{p2} - \tfrac{k_1 b_1}{I_p} x_{t1} + \tfrac{k_2 b_2}{I_p} x_{t2} \\
& - \tfrac{1}{I_p}(c_1 b_1 - c_2 b_2 + c_{p1} d_1 - c_{p2} d_2)\dot{x} - \tfrac{1}{I_p}(c_1 b_1^2 + c_2 b_2^2 + c_{p1} d_1^2 + c_{p2} d_2^2)\dot{\theta} \\
& - \tfrac{c_{p1} d_1}{I_p}\dot{x}_{p1} + \tfrac{c_{p2} d_2}{I_p}\dot{x}_{p2} - \tfrac{c_1 b_1}{I_p}\dot{x}_{t1} + \tfrac{c_2 b_2}{I_p}\dot{x}_{t2} + \tfrac{1}{I_p}(f_1 b_1 - f_2 b_2) \\
\ddot{x}_{p1} = \; & \tfrac{1}{m_{p1}}\left(k_{p1} x + k_{p1} d_1 \theta - k_{p1} x_{p1}\right) + \tfrac{1}{m_{p1}}\left(c_{p1}\dot{x} + c_{p1} d_1 \dot{\theta} - c_{p1}\dot{x}_{p1}\right) \\
\ddot{x}_{p2} = \; & \tfrac{1}{m_{p2}}\left(k_{p2} x - k_{p2} d_2 \theta - k_{p2} x_{p2}\right) + \tfrac{1}{m_{p2}}\left(c_{p2}\dot{x} - c_{p2} d_2 \dot{\theta} - c_{p2}\dot{x}_{p2}\right) \\
\ddot{x}_{t1} = \; & \tfrac{1}{m_{t1}}\left(k_1 x + k_1 b_1 \theta - (k_1 + k_{t1}) x_{t1} + k_{t1} y_1 - f_1\right) \\
& + \tfrac{1}{m_{t1}}\left(c_1 \dot{x} + c_1 b_1 \dot{\theta} - (c_1 + c_{t1})\dot{x}_{t1} + c_{t1}\dot{y}_1\right) \\
\ddot{x}_{t1} = \; & \tfrac{1}{m_{t2}}\left(k_2 x - k_2 b_2 \theta - (k_2 + k_{t2}) x_{t2} + k_{t2} y_2 - f_2\right) \\
& + \tfrac{1}{m_{t2}}\left(c_2 \dot{x} - c_2 b_2 \dot{\theta} - (c_2 + c_{t2})\dot{x}_{t2} + c_{t2}\dot{y}_2\right)
\end{aligned}
\qquad (1)
$$

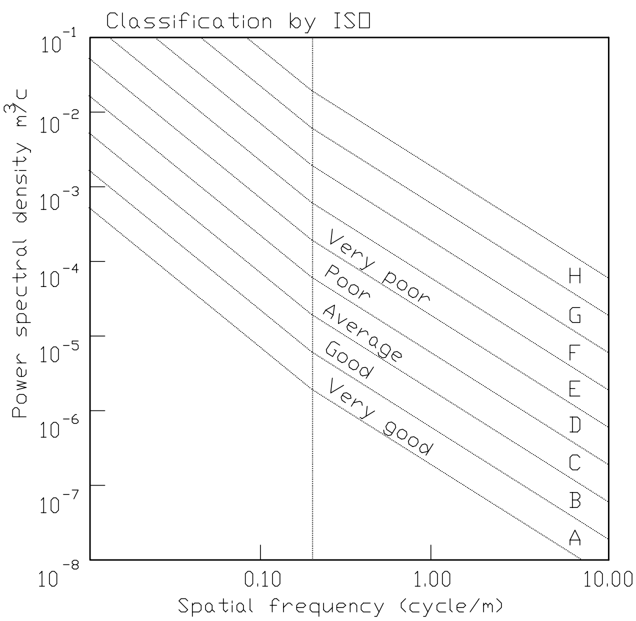

Figure 2: **Road roughness classification by ISO**

These equations could be simply written as a matrix equation:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{G}\mathbf{w} \tag{2}$$

where the state vector $\mathbf{x}$ is composed of:

$$\mathbf{x} = \begin{bmatrix} \mathbf{x_1} \\ \dot{\mathbf{x}}_1 \end{bmatrix} \quad ; \quad \mathbf{x_1} = [x \ , \ \theta \ , \ x_{p1} \ , \ x_{p2} \ , \ x_{t1} \ , \ x_{t2}]^T \tag{3}$$

The input vector $\mathbf{u}$ representing the two actuator forces, while the disturbance vector $\mathbf{w}$ consists of road disturbance.

$$\mathbf{u} = [f_1 \ , \ f_2]^T \quad ; \quad \mathbf{w} = [y_1 \ , \ y_2 \ , \ \dot{y}_1 \ , \ \dot{y}_2]^T \tag{4}$$

The system matrices could be simply derived using Equation 1. The matrix representation of Equation 2 would be the basis for linear optimal controller design.

2.2 Road Roughness Model

In the early days of studying the performance of vehicles on rough roads, simple functions such as sine waves, step functions, or triangular waves were generally applied as disturbances from the ground. While these inputs provide a basic idea for comparative evaluation of designs, it is recognized that the road surface is usually not represented by these simple functions, and therefore, the deterministic irregular shapes cannot serve as a valid basis for studying the actual behaviour of the vehicle.

Degree of Roughness $S(\Omega) \times 10^{-6}$		
Road Class	Range	Geometric mean
A (Very good)	< 8	4
B (Good)	$8 - 32$	16
C (Average)	$32 - 128$	64
D (Poor)	$128 - 512$	256
E (Very poor)	$512 - 2048$	1024
F	$2048 - 8192$	4096
G	$8192 - 32768$	16384
H	$32768 <$	

Table 1: **Road roughness values classified by ISO**

In this study a real road surface, taken as a random exciting function, is used as the input to the vehicle-road model. Power spectral density (PSD) analysis is used to describe the basic properties of random data.

Several attempts have been made to classify the roughness of a road surface. In this study, classifications are based on the International Organization for Standardization (ISO). The ISO has proposed road roughness classification using the PSD values, [8], as shown in Figure 2. The corresponding values are illustrated in Table 1.

To make use of the above mentioned classification, a normal random input is generated with a variable amplitude. Using fast Fourier transform(FFT), a trial and error attempt is proposed in order to obtain the desired PSD characteristics of the random input. Table 1 illustrates the stochastical characteristics of the final random input design, which corresponds to the poor road condition as being classified by ISO.

Random Input Characteristics		
	Spatial (mm)	PSD $(m^3/cycle)$
Min	-49	0
Max	40	2.06×10^{-3}
Mean	0	3.08×10^{-4}
STD	12	3.26×10^{-4}

Table 2: **Stochastical characteristic of road random disturbance**

3 Optimal Controller Design

The performance characteristics which are of most interest when designing the vehicle suspension are passengers ride comfort, body motion, road handling, and suspension travel. The passenger acceleration has been used here as an indicator of ride comfort. Suspension travel and body motion are the states of the system, but road handling is related to the tire deflection. The controller should minimize all these quantities.

The linear time-invariant system, (LTI), is described by Equation 2. For controller design it is assumed that all the states are available and also could be measured exactly. First of all let us consider a state variable feedback regulator:

$$\mathbf{u} = -\mathbf{K} \cdot \mathbf{x} \tag{5}$$

where $\mathbf{K}$ is the state feedback gain matrix. The optimization procedure consists of determining the control input $\mathbf{u}$, which minimizes the performance index. The performance index $\mathbf{J}$ represents the performance characteristic requirement as well as the controller input limitations.

In this paper two different approaches are taken in order to evaluate the performance index, and hence designing the optimal controller. The first approach is the conventional method, in which only the system states and inputs are penalized in the performance index. However, in the second approach special attention is paid to the ride comfort and hence, the passenger acceleration terms are also included in the performance index.

3.1 Conventional Method (CM)

In this method, the performance index $\mathbf{J}$ penalizes the state variables and the inputs; thus, it has the standard form of:

$$\mathbf{J} = \int_0^\infty \left(\mathbf{x}^T \mathbf{Q}\ \mathbf{x} + \mathbf{u}^T \mathbf{R}\ \mathbf{u}\right) dt \tag{6}$$

where $\mathbf{Q}$ and $\mathbf{R}$ are positive definite, being called weighting matrices. Here the passenger acceleration which is an indicator of ride comfort is not being penalized.

To obtain a solution for the optimal controller introduced in Equation 5 the LTI system must be stabilizable, [3]. This condition unlike controllability is rather more accessible. A system is defined stabilizable when only the unstable modes are controllable. Therefore, for a system with no unstable mode, being the case considered in this paper, the optimal solution is guaranteed.

Linear optimal control theory provides the solution of Equation 6 in the form of Equation 5. The gain matrix $\mathbf{K}$ is computed from:

$$\mathbf{K} = \mathbf{R}^{-1}\mathbf{B}^T\mathbf{P} \tag{7}$$

where the matrix $\mathbf{P}$ is evaluated being the solution of the Algebraic Riccati Equation, (ARE).

$$\mathbf{AP} + \mathbf{A}^T\mathbf{P} - \mathbf{PBR}^{-1}\mathbf{B}^T\mathbf{P} + \mathbf{Q} = \mathbf{0} \tag{8}$$

Equation 2 for the optimal closed-loop system, being used for computer simulation, may be written in the form of:

$$\dot{\mathbf{x}} = (\mathbf{A} - \mathbf{BK})\mathbf{x} + \mathbf{Gw} \tag{9}$$

3.2 Acceleration Dependent Method (ADM)

In this method the two passenger accelerations are included in the performance index. Suppose that the vector $\mathbf{z}$ represents the passengers acceleration, in the form of:

$$\mathbf{z} = \begin{bmatrix} \ddot{x}_{p1} \\ \ddot{x}_{p2} \end{bmatrix} \tag{10}$$

The performance index could be written in the following form

$$\mathbf{J} = \int_0^\infty \left(\mathbf{x}^T \mathbf{Q}\ \mathbf{x} + \mathbf{u}^T \mathbf{R}\ \mathbf{u} + \mathbf{z}^T \mathbf{S}\ \mathbf{z} \right) dt \tag{11}$$

The weighting matrix for acceleration terms in the simple case may be assumed diagonal:

$$\mathbf{S} = \begin{bmatrix} S_1 & 0 \\ 0 & S_2 \end{bmatrix} \tag{12}$$

Therefore, Equation 11 becomes

$$\mathbf{J} = \int_0^\infty \left(\mathbf{x}^T \mathbf{Q}\ \mathbf{x} + \mathbf{u}^T \mathbf{R}\ \mathbf{u} + \ddot{x}_{p1}^T S_1\ \ddot{x}_{p1} + \ddot{x}_{p2}^T S_2\ \ddot{x}_{p2} \right) dt \tag{13}$$

Equation 13 could be further modified, since both passenger accelerations are linearly dependent on the state variables. Note that from Equation 1 we can rewrite:

$$\ddot{x}_{p1} = \mathbf{v}_1\ \mathbf{x} \quad ; \quad \ddot{x}_{p2} = \mathbf{v}_2\ \mathbf{x} \tag{14}$$

where row vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ may be evaluated using Equation 1, namely:

$$\mathbf{v}_1 = \frac{1}{m_{p1}} \begin{bmatrix} k_{p1} & k_{p1}d_1 & -k_{p1} & 0 & 0 & 0 & c_{p1} & c_{p1}d_1 & -c_{p1} & 0 & 0 & 0 \end{bmatrix} \tag{15}$$

$$\mathbf{v}_2 = \frac{1}{m_{p2}} \begin{bmatrix} k_{p2} & k_{p2}d_2 & -k_{p2} & 0 & 0 & 0 & c_{p2} & c_{p2}d_2 & -c_{p2} & 0 & 0 & 0 \end{bmatrix} \tag{16}$$

Thus, Equation 13 could be written as:

$$\mathbf{J} = \int_0^\infty \left(\mathbf{x}^T \left(\mathbf{Q} + \mathbf{v}_1^T S_1 \mathbf{v}_1 + \mathbf{v}_2^T S_2 \mathbf{v}_2 \right) \mathbf{x} + \mathbf{u}^T \mathbf{R}\ \mathbf{u} \right) dt \tag{17}$$

or in the simple form of

$$\mathbf{J} = \int_0^\infty \left(x^T \mathbf{Q}_n\ x + u^T \mathbf{R}\ u \right) dt \tag{18}$$

where,

$$\mathbf{Q}_n = \mathbf{Q} + \mathbf{v}_1^T S_1 \mathbf{v}_1 + \mathbf{v}_2^T S_2 \mathbf{v}_2 \tag{19}$$

The optimal solution for Equation 18 could be found in a similar manner to that of Equation 6.

Equation 8 shows that the optimal solution and its final performance of the closed-loop system are directly related to the initial values of weighting matrices, **Q** and **R**.

4 Simulation Results

A program has been written using Matlab Software, to handle the controller design and simulation. To have a quantitative comparison between the different simulation results, a stochastical approach is followed. Since the input to the system is in the form of normal random distribution, it is expected to have normal distributed outputs. Therefore, we can calculate useful probability values for the signals.

For a Gaussian normal distribution, the probability function of the random signal $x(t)$ can be written, [13]:

$$\text{Prob}\left[-\lambda\sigma \leq x(t) \leq \lambda\sigma\right] = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\lambda\sigma}^{\lambda\sigma} e^{-\frac{x^2}{2\sigma^2}} dx = (\text{Erf}) \left(\frac{\lambda}{\sqrt{2}}\right) \tag{20}$$

and,

$$\text{Prob}\left[|x(t)| > \lambda\sigma\right] = 1 - \text{Prob}\left[-\lambda\sigma \leq x(t) \leq \lambda\sigma\right] = (\text{Erfc}) \left(\frac{\lambda}{\sqrt{2}}\right) \tag{21}$$

where σ is the standard deviation (STD), λ is a real number, (Erf) denotes error function and (Erfc) denotes complementary error function.

Variables	x	θ	x_{p1}	x_{p2}	x_{t1}	x_{t2}	$\ddot{x}_{p1}$	$\ddot{x}_{p2}$	f_1	f_2
Limits	0.01	0.05	0.01	0.01	0.05	0.05	0.2% g	0.2% g	500	500

Table 3: **Variable limits assigned for the controller design**

In this study, there are some limits assigned for all the states, both passenger acceleration, and actuator limits in order to satisfy the required ride comfort, road handling, and the design restriction. These limits are illustrated in Table 3. The first condition in designing the controller is to satisfy these limits. This can be examined by checking the probability values of the outputs.

For a quantitative comparison between the two controllers, for each variable the amount of bounding limit with 90% probability is calculated. This quantity can be easily obtained using standard deviation of the signal together with Equations 20 and 21. Let $(\text{Erf})^{-1}$ represents the inverse error function. Then Equation 20 will transform to:

$$\lambda_{90} = \sqrt{2} \;\; \{\text{Erf}\}^{-1} (0.9) \tag{22}$$

and

$$x_{90} = \lambda_{90} \times \sigma + \text{mean}[x(t)] \tag{23}$$

where x_{90} represents the bounding limit of the random signal x with 90% probability. This quantity can be used to compare different designs quantitatively.

The problem of controller design is then a challenge for finding suitable weightings that satisfies the design performances. This can be done by trying an arbitrary weighting matrix $\mathbf{W}$ and comparing the resultant x_{90} of the closed loop system to the prescribed limits, and adjusting the weighting elements due to this comparison. This methodology has been forwarded for a typical mid size car and the results are illustrated in Figures 3 to 5.

Let us first examine the effect of active system to remedy the drawbacks of the passive system. The numerical values used in the simulation for system parameters are given in Nomenclatures. Figure 3 illustrates clearly how the active suspension can effectively absorb the vehicle vibration in comparison to the passive system. There are the body motions, passengers acceleration, and

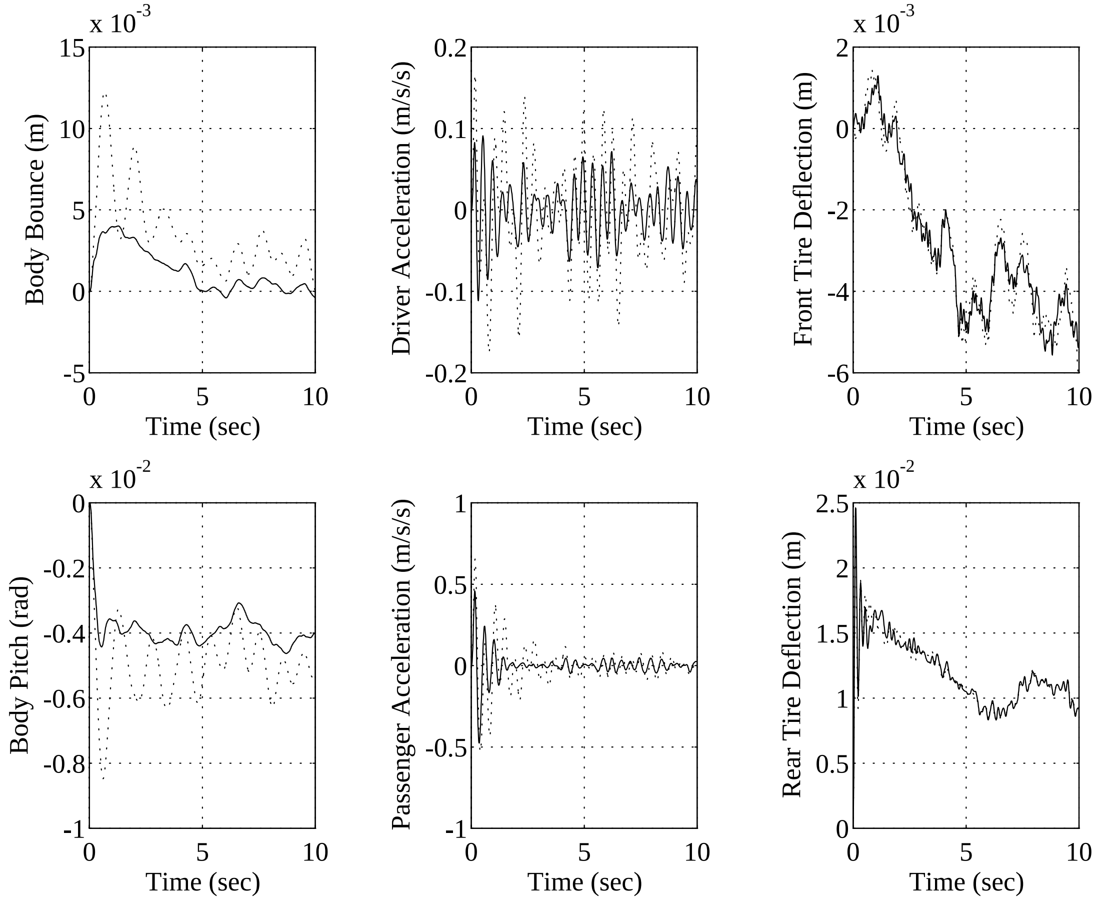

Figure 3: **Comparison between passive and active systems. Dotted: Passive, Solid: Active**

tires deflection compared in this figure. Conventional method is used for the controller design in the active system, where the variable weightings are illustrates in Table 5. The body motion and passenger accelerations in the active system are reduced significantly, which guarantee better ride comfort. Moreover, the tire deflection is also smaller in the active suspension system; therefore, it is concluded that the active system retain both better ride comfort and road handling characteristics compared to the passive system.

Tables 4 give quantitative comparison of these systems, which illustrates the system variables bounds with 90% probability for passive and both active systems. In this statistical comparison it is shown that the the body bounce and passenger acceleration in active case are reduced to about half of their values in passive system, and the tire deflection is also reduced 25%. This confirms the efficiency of the active suspension in both ride comfort and road handling performance.

Now let us compare the results of two different methods of controller design in detail. Figures 4 and 5 show the simulation results for the final controllers design. In Figure 4 body bounce and

90% Probability Bound			
States	Passive	CM	ADM
x	7.7418×10^{-3}	3.4327×10^{-3}	4.0220×10^{-3}
θ	6.8487×10^{-3}	4.8678×10^{-3}	5.8281×10^{-3}
x_{p1}	5.1935×10^{-3}	2.9151×10^{-3}	2.9559×10^{-3}
x_{p2}	1.6510×10^{-2}	9.0565×10^{-3}	9.7812×10^{-3}
x_{t1}	6.0335×10^{-3}	5.9441×10^{-3}	5.9433×10^{-3}
x_{t2}	1.6187×10^{-2}	1.6215×10^{-2}	1.6395×10^{-2}
$\ddot{x}_{p1}$	1.0745×10^{-1}	5.6089×10^{-2}	1.6890×10^{-2}
$\ddot{x}_{p2}$	2.1751×10^{-1}	1.3213×10^{-1}	5.9807×10^{-2}
f_1	0	111.93	150.86
f_2	0	125.66	111.56

Table 4: **Comparison of 90% probability bounds for passive and active systems**

variables	x	θ	x_{p1}	x_{p2}	x_{t1}	x_{t2}	$\ddot{x}_{p1}$	$\ddot{x}_{p2}$	f_1	f_2
Weightings	1	1	1	6	1	3	1	2	1	8

Table 5: **Variable weightings assigned for the controller design**

pitch, passengers accelerations and the tires deflection are compared. The weightings used in this case is given in Table 5. The body motions are lower in CM, the tires deflection are approximately the same in both methods; however, the passenger accelerations are significantly lower in the ADM approach. This implies that ADM could improve the ride comfort, while retaining the road handling performance.

Figure 5 compares the actuator forces of the two different methods. The actuator forces are well below the limits and practically implementable. In ADM approach gaining better passenger acceleration is possible by the cost of larger actuator forces. However, optimal controller design could limit the actuator forces in some realistic bounds.

Table 4 give a quantitative comparison of the above mentioned methods. This table gives the variable limits with 90% probability. Comparing the variable limits with the prescribed limits in both methods, the controllers are able to satisfy all the limits. However, ADM approach is more successful in reducing the passengers acceleration. The body motion and tire deflection limits do not have significant difference in both methods. These quantitative values could be as an effective tool for the designer to satisfy the required performance or to compare different designs.

5 Conclusion

The objective of this paper has been to examine the use of optimal state-feedback controllers for improving the ride comfort and stability performance of the road vehicles. The potential for improved vehicle ride comfort, and road handling resulting from controlled actuator forces, are examined. The performance characteristics of such suspension systems are evaluated by two

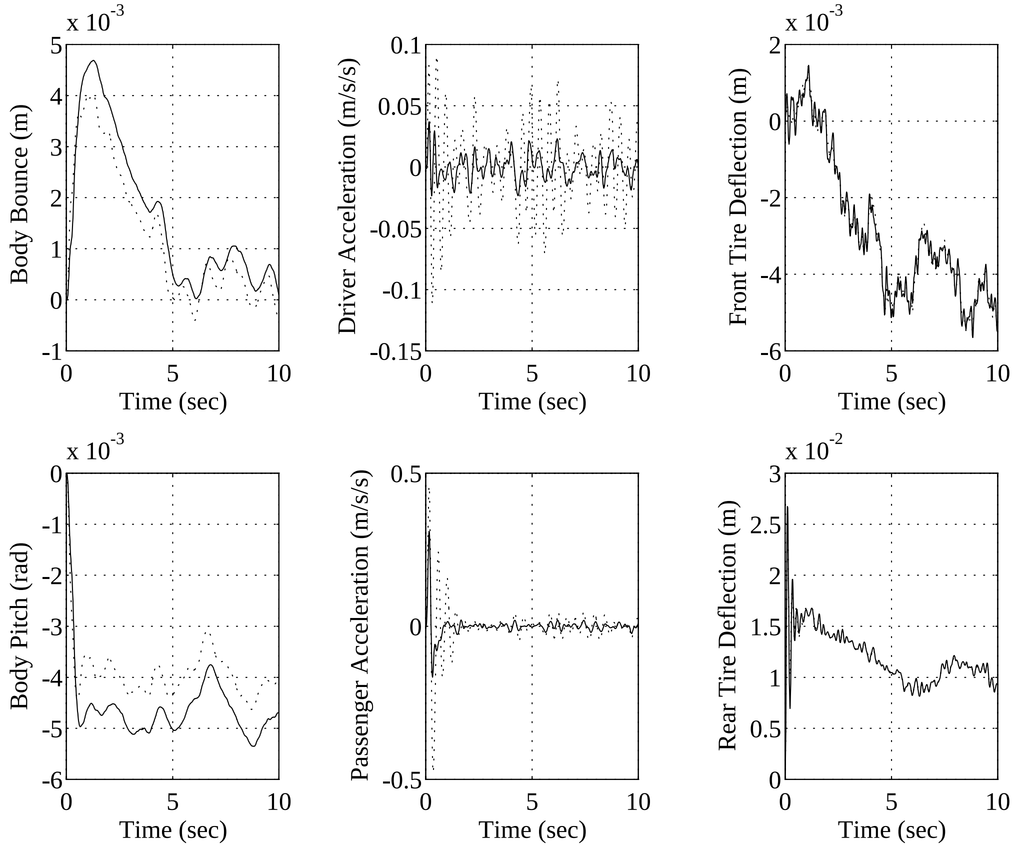

Figure 4: **Comparison of two controller designs. Dotted: Conventional, Solid: Acceleration dependent**

methods, and compared with a passive suspension system.

The result of comparison, presented in this paper, lead to the conclusion that the optimal control theory provides a useful mathematical tool for the design of active suspension systems. Using random inputs for road surface disturbances applied to the vehicle, make it possible to have a more realistic idea about the vehicle dynamic response to the road roughness.

The suspension designs which may have emerged from the use of optimal state-feedback control theory proved to be effective in controlling vehicle vibrations and achieve better performance than the conventional passive suspension. The stochastical comparison of the final designs could well be used in further improvement of the controller performance. Moreover, it provides either a detailed quantitative comparison between different designs, or a better degree of satisfaction for the required performances.

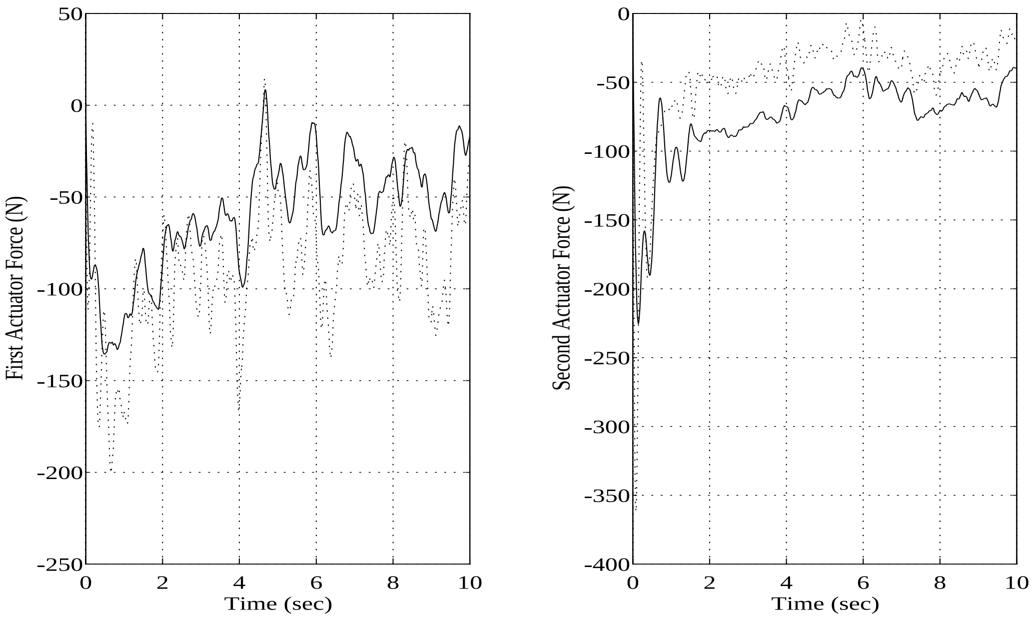

Figure 5: **Comparison of the actuator forces for two controller designs. Dotted: Conventional, Solid: Acceleration dependent**

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Nomenclature

Not.	Description	Units	Values	Not.	Description	Units	Values
I_p	Body inertia	$Kg \cdot m^2$	3443.05	m	Body mass	Kg	1794.4
m_{p1}	Driver mass	Kg	75	m_{p2}	Passenger mass	Kg	75
m_{t1}	Front axle mass	Kg	87.15	m_{t2}	Rear axle mass	Kg	140.04
k_1	Front main stiffness	N/m	66824.2	c_1	F.m. damping	$\frac{N \cdot s}{m}$	1190
k_2	Rear main stiffness	"	18615.0	c_2	R.m. damping	"	1000
k_{p1}	Front seat stiffness	"	14000	c_{p1}	F.s. damping	"	50.2
k_{p2}	Rear seat stiffness	"	14000	c_{p2}	R.s. damping	"	62.1
k_{t1}	Front tire stiffness	"	101115	c_{t1}	F.t. damping	"	14.6
k_{t2}	Rear tire stiffness	"	101115	c_{t2}	R.t. damping	"	14.6
b_1	Dimension	m	1.271	b_2	Dimension	m	1.713
d_1	Dimension	m	0.481	d_2	Dimension	m	1.313