Numerical Methods for Computing the Forward Kinematics of a Redundant Parallel Manipulator

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Abstract – **In this paper, different approaches are presented to solve the forward kinematics of a three DOF actuator redundant hydraulic parallel manipulator. It is known, that on the contrary to series manipulators, the forward kinematic map of parallel manipulators involves highly coupled nonlinear equations, which are almost impossible to solve analytically. The proposed methods are using mainly numerical computations, with different ideas to solve the problem. The accuracy of the results of each method are compared in detail and the advantages and the disadvantages of them in computing the forward kinematic map of the given mechanism is discussed in detail. It is concluded that the 4th order Taylor series approximation has the best acceptable prediction errors for robotic applications, compared to that of the different structures of neural networks and the quasi-closed solution method considered in this paper.**

Keywords- Parallel Manipulator; Forward Kinematics; Taylor Series; Neural Networks; Numerical solution; Closed-Form solution; Performance comparison

I. INTRODUCTION

Over the last two decades, parallel manipulators have been among the most considerable research topics in the field of robotics. These robots are now applied in real-life applications such as force sensing robots, fine positioning devices, and medical applications [1, 2].

As in the case of conventional serial robots, kinematics analysis of parallel manipulators is also performed in two phases. In forward or direct kinematics the position and orientation of the mobile platform is determined given the leg lengths. This is done with respect to a base reference frame. In inverse kinematics we use position and orientation of the mobile platform to determine actuator lengths. It is known that unlike serial manipulators, inverse position kinematics for parallel robots is usually simple and straightforward. In most cases joint variables (actuator displacements) may be computed independently using the given pose of the movable platform. The solution to this problem is in most cases uniquely determined. But forward kinematics of parallel manipulators is generally very complicated. Its solution usually involves systems of nonlinear equations which are highly coupled and in general have no closed form and unique solution. Different approaches are provided in literature to solve this problem either generally or in special cases. There are also numerous cases in which the solution to this problem is provided for a special or novel architecture. In general, different solutions to this problem can be found using numerical approaches, analytical approaches, and closed form solution for special architectures, [3, 4].

In this paper, representatives from the first class are being used to solve the kinematics problem in a 3DOF actuator redundant hydraulic parallel manipulator. The paper is organized as following. Section 2 contains the mechanism description. Kinematic modeling of the manipulator is discussed in section 3, where inverse and forward kinematics is studied and the need for appropriate method to solve the forward kinematics is justified. In section 4, three different methods to solve the forward kinematics problem are discussed; First, two different but mostly common neural networks are used to estimate the forward kinematic map of the given mechanism. In the second method a quasi-closed form is provided for the same purpose which combines the numerical and analytical schemes. Finally, with a new approach, conventional Taylor Series expansion is considered to approximate the nonlinear map with required precision. In section 5, these methods are simulated and compared regarding the problem in hand in order to identify the benefits and drawbacks of each scheme. It is concluded that the 4^{th} order Taylor series approximation has the best approximation accuracy compared to that of other methods for robotic applications.

II. MECHANISM DESCRIPTION

A three DOF actuator redundant hydraulic parallel manipulator is used as the basis of our study. The mechanism is designed by Dr. V. Hayward [5, 6, and 7], borrowing design ideas from biological manipulators and specially the biological shoulder. The interesting features of the mechanism and its similarity to human shoulder have made it a unique design, which can serve as a basis for a good experimental setup for parallel robot research. A schematic of the mechanism, which is currently under experimental studies in ARAS Robotics Lab, is shown in Fig. 1. The mobile platform is constrained to spherical motions. Four high performance hydraulic piston actuators are used to give three degrees of freedom in the mobile platform. Each actuator includes a position sensor of LVDT type and an embedded force sensor (Hall Effect). Simple elements like spherical and universal joints are used in the structure. A complete analysis of such a careful design will provide us with good results regarding the structure itself and its performance.

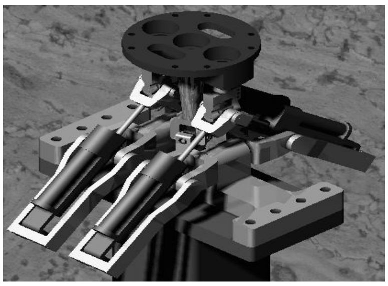

Figure1. A schematic of the hydraulic shoulder manipulator

 ISBN: 3-938153-30-X

From the structural point of view, the shoulder mechanism which, from now on, we call it "the Hydraulic Shoulder" falls into an important class of robotic mechanisms called parallel robots. In these robots, the end effector is connected to the base through several closed kinematic chains. The motivation behind using these types of robot manipulators was to compensate for the shortcomings of the conventional serial manipulators such as low precision, low stiffness, error accumulation and load carrying capability. Parallel structures are usually lighter and simpler than their serial counterparts. However, they have their own disadvantages, which are mainly smaller workspace and many singular configurations. The hydraulic shoulder, being a parallel structure, has the general features of these structures. It can be thought of as a shoulder for a light weighed seven DOF robotic arm, which can carry loads several times, its own weight. Simple elements, used in this design, add to its lightness and simplicity. The workspace of such a mechanism can be considered as part of a sphere surface. The orientation angles are limited to vary between $-\pi/6$ and $\pi/6$.

III. KINEMATICS

The hydraulic shoulder is kinematically over constrained. The inverse kinematics problem is easily solved, given the orientation of the mobile plate, similar to general parallel robots. The inverse kinematics problem has a unique solution, in our case meaning that, the hydraulic shoulder cannot be optimized by choosing between inverse kinematics solutions. But, in contrast to serial structures, the forward kinematics is very complicated and there is no closed form solution in general. Fig. 2, depicts a geometric model for the mechanism which will be used for its kinematics derivation. The parameters used in kinematics can be defined as:

$l_b : \left\|\overrightarrow{CA_i}\right\|$ $\qquad$ $l_p : \left\|\overrightarrow{CC_1}\right\|$

$l_d : \left\|\overrightarrow{C_1P_i}\right\|_{y1}$ $\qquad$ $l_k : \left\|\overrightarrow{C_1P_i}\right\|_{z1}$

α : The angle between CA_4 and y_0

C : Center of the reference frame

C_1 : Center of the moving plate

ρ_i : Actuator lengths i=1, 2, 3, 4

P_i : Moving endpoints of the actuators

A_i : Fixed endpoints of the actuators

Two coordinate frames are defined. The base frame $X_0Y_0Z_0$ is centered at C (rotation center) with its Z_0-axis perpendicular to the plane defined by $A_1A_2A_3A_4$ and an X_0 axis parallel to the bisector of angle $\angle A_1CA_4$. The second frame, namely $X_1Y_1Z_1$ is centered at C_1 (center of the moving plate) with its Z_1 axis perpendicular to the line defined by the actuators moving end points (P_1P_2) and horizontal Y axis along C_1P_2.

A. Inverse Kinematics

In modeling the inverse kinematics of the hydraulic shoulder we must determine actuator lengths (ρ_i) as the joint space variables given the task space variables, namely θ_x, θ_y and θ_z as the orientation angles of the moving platform. First we note that the fixed end points of the actuators (A_i) can be written in the base frame as:

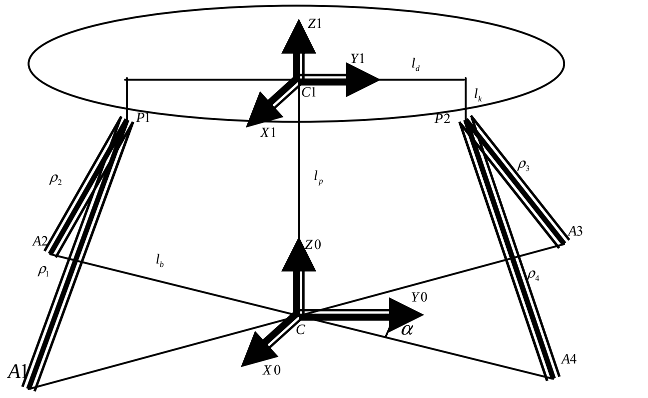

Figure2. A geometric model for the hydraulic shoulder manipulator

$$\begin{aligned} A_1^0 &= (l_b\sin\alpha \quad -l_b\cos\alpha \quad 0),\\ A_2^0 &= (-l_b\sin\alpha \quad -l_b\cos\alpha \quad 0),\\ A_3^0 &= (-l_b\sin\alpha \quad l_b\cos\alpha \quad 0),\\ A_4^0 &= (l_b\sin\alpha \quad l_b\cos\alpha \quad 0), \end{aligned} \tag{1}$$

Also:

$$P_1^1 = (0 \quad -l_d \quad -l_k),\ P_2^1 = (0 \quad l_d \quad -l_k), \tag{2}$$

These must be transferred to the base frame using the rotation matrix R_1^0;

$$P_i^0 = R_1^0 P_i^1, \tag{3}$$

Where:

$$S_{3\times3} = R_1^0 = R_z(\theta_z)R_y(\theta_y)R_x(\theta_x) \tag{4}$$

The rotation matrix components are computed as following:

$$\begin{aligned} S_{11} &= \cos(\theta_z)\cos(\theta_y)\\ S_{21} &= \sin(\theta_z)\cos(\theta_y)\\ S_{31} &= -\sin(\theta_y)\\ S_{12} &= \cos(\theta_z)\sin(\theta_y)\sin(\theta_x) - \sin(\theta_z)\cos(\theta_x)\\ S_{22} &= \sin(\theta_z)\sin(\theta_y)\sin(\theta_x) + \cos(\theta_z)\cos(\theta_x)\\ S_{32} &= \cos(\theta_y)\sin(\theta_x)\\ S_{13} &= \cos(\theta_z)\sin(\theta_y)\cos(\theta_x) + \sin(\theta_z)\sin(\theta_x)\\ S_{23} &= \sin(\theta_z)\sin(\theta_y)\cos(\theta_x) - \cos(\theta_z)\sin(\theta_x)\\ S_{33} &= \cos(\theta_y)\cos(\theta_x) \end{aligned} \tag{5}$$

So we have:

$$p_1^0 = \begin{pmatrix} -l_d s_{12} - l_k s_{13} \\ -l_d s_{22} - l_k s_{23} \\ -l_d s_{32} - l_k s_{33} \end{pmatrix},\ p_2^0 = \begin{pmatrix} l_d s_{12} - l_k s_{13} \\ l_d s_{22} - l_k s_{23} \\ l_d s_{32} - l_k s_{33} \end{pmatrix} \tag{6}$$

The final step is to translate the resulting vectors P_i^0 by l_p along the Z axis. Having P_i^0 and A_j^0 in hand, the actuator lengths $\left\|\overrightarrow{P_iA_j}\right\|$ can be easily computed as:

$$\rho_j = \sqrt{(p_x - a_x)^2 + (p_y - a_y)^2 + (p_z - a_z)^2}, \tag{7}$$

Where:

$$p_i^0 = [p_x\ p_y\ p_z] \quad , \quad A_j^0 = [a_x\ a_y\ a_z], \tag{8}$$

are defined in equations (6) and (1) respectively. From equations (7) and (8), the actuator lengths (ρ_i) are exactly computable by the orientation angles of the moving platform, θ_x, θ_y and θ_z, and

 ISBN: 3-938153-30-X

hence the inverse kinematic map is analytically computed. Therefore, it is clear that the manipulator doesn't have any kinematic redundancy, meaning that reaching a specific point in the task space can't be satisfied through different combinations of the actuator lengths.

B. Forward kinematics

Equations (7)–(8) can also be used for the forward kinematics of the hydraulic shoulder but with the actuator lengths as the input and orientation angles θ_x, θ_y, θ_z as the unknowns. In fact, we have four nonlinear equations to solve for three unknowns. Obviously, solving such a system of nonlinear equations for a unique closed-form analytic solution to the forward kinematic problem is very complicated, although three equations of the four could be used. Several inconclusive attempts have been made in this direction; therefore, we propose using a combination of the numerical and analytic schemes to solve the forward kinematic problem as a basic element in modeling and control of the manipulator. This is studied in detail in the next section.

IV. FORWARD KINEMATICS SOLUTION

A. Neural Network Estimation

1) Multilayer feed forward network

A simple feed forward network with back propagation learning was used in the first step. The input layer has as many nodes as the number of inputs to the map namely four actuator lengths. Similarly the output layer will have three nodes which represent the orientation of the moving plate ($\theta_x, \theta_y, \theta_z$). The number of neurons in the hidden layer was used as a design parameter. Sigmoid and linear transfer functions were selected for all hidden and output layer nodes respectively. Supervised learning scheme was used in which the manipulator is treated as a black box and the network is taught to learn the map by observing the inputs and outputs. Such a learning scheme will result in offline training. The target pattern for training, the three orientation angles, was randomly generated within the workspace of the robot and the input pattern, four actuator displacements, was found using the inverse kinematics model. The pair was then used to train the network and the weights were updated in a back propagation process. Random initialization was used for the weights. Different configurations of the feed forward network were tested by varying the number of neurons in the hidden layer between 5 and 35 and the performance of these networks was compared.

Different performance indices could be used in this case, the best of which could be the sum of square output errors, though other indices such as mean square or mean absolute error may also be used. Networks with best performance as indicated would be selected, from which the network with fewer hidden layer nodes will be the best choice since the number of weights and also the training time of the network will increase with more neurons in the hidden layer. As another configuration, the same multilayer feed forward network was used with two hidden layers. The activation function of the second hidden layer was also sigmoid. Different networks from each configuration were trained:

- About 30 multilayer feed forward networks with one hidden layer were trained by varying the number of neurons in the hidden layer from 5 to 35.
- About 20 multilayer feed forward networks with two hidden layers were trained by varying the number of neurons from 10 to 25 in the first hidden layer and from 5 to 15 in the second hidden layer.

All these networks were trained over 1000 training epochs with Bayesian regularization training. Each network was evaluated by comparing the predictions to the true outputs, resulting in a prediction error for each orientation angle. The autocorrelation coefficients were also computed for the prediction error in each angle. Using the whole stated criteria, five networks with best performance were selected from each configuration. Table (1) summarizes the performance of these networks. It can be seen that networks with two hidden layers have a better performance in general. It should be also noted that the mean square of error is approximately equal to the square of the maximum error, so a mean square error of 1e-5 will correspond to about 0.18 degree of accuracy for the forward kinematics solution. All the trainings and simulations of the neural networks were done on a Pentium4, 2 GHz.

TABLE (1): PERFORMANCE OF MULTILAYER FEED FORWARD NETWORKS

Network Structure	Multilayer Feed Forward One Hidden Layer				
Network Performance	No. of Hidden Layer Neurons	Training Time (sec)	MSE	SSE	MAE
	S=27	7.3e3	2.8e-5	0.644	0.0037
	S=29	8.2e3	2.9e-5	0.66	0.0035
	S=30	8.6e3	1.9e-5	0.428	0.0028
	S=34	1e4	1.1e-5	0.242	0.0022
	S=35	1.1e4	1.1e-5	0.26	0.0022
Network Structure	Multilayer Feed Forward Two Hidden Layers				
Network Performance	No. of Hidden Layer Neurons	Training Time (sec)	MSE	SSE	MAE
	S1=10 S2=15	9.5e3	6.8e-6	0.154	0.0018
	S1=12 S2=15	2.9e4	2.8e-6	0.062	0.0011
	S1=17 S2=15	6.1e4	8.1e-7	0.018	6e-4
	S1=17 S2=9	1e4	5.6e-6	0.12	0.0016
	S1=17 S2=12	2.3e4	1.9e-6	.044	9e-4

2) Radial Basis Function neural network

Radial basis function (RBF) neural network architecture was tested as another choice for computing the forward kinematics of the hydraulic shoulder. In general, RBF networks require more neurons but much less training time than feed forward back propagation networks. Input and output patterns were generated in a same procedure as in the multilayer feed forward network. Supervised learning method was used in a way to reduce the estimated error of the network. Other specifications such as weight initialization, network evaluation and performance indices were just the same as the multilayer feed forward network. About ten different configurations with different spread parameters were trained and compared, from

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which two networks with best performance were selected. The performance of these networks is shown in Table (2).

TABLE (2): PERFORMANCE OF RBF NETWORKS

Network Performance	Training Time (sec)	MSE	SSE	MAE
RBF1	750	1.3e-5	0.1	0.0019
RBF2	680	9.9e-6	0.074	0.0017

From the comparison of the selected structure in table (1) and (2) which are highlighted in gray, the multilayer feed forward with two hidden layers provides better approximation, with a mean square error of 2.8e-6, and mean absolute error of 0.0011. This corresponds to about 0.05 degree accuracy in forward kinematics, which is a generally suitable approximation in general applications. However, this accuracy may not meet the accurate robotic applications such as our redundant parallel manipulator, and for this means other methods are investigated and reported in next sections.

B. *Quasi-Closed Solution Method*

Next, we introduce a quasi-closed form solution method to estimate the forward kinematic map of the hydraulic shoulder. In this method analytic solution of the forward kinematics is pursued as far as possible and numerical approximation is only used to solve the remaining equations. To illustrate the structure of this method reconsider the kinematics equations of the mechanism as following:

$$\begin{aligned}\rho_1 &= \sqrt{A + Bs_{12} + Cs_{13} - Ds_{22} - Es_{23} - Fs_{32} - Gs_{33}}\\ \rho_2 &= \sqrt{A - Bs_{12} - Cs_{13} - Ds_{22} - Es_{23} - Fs_{32} - Gs_{33}}\\ \rho_3 &= \sqrt{A + Bs_{12} - Cs_{13} - Ds_{22} + Es_{23} + Fs_{32} - Gs_{33}}\\ \rho_4 &= \sqrt{A - Bs_{12} + Cs_{13} - Ds_{22} + Es_{23} + Fs_{32} - Gs_{33}}\end{aligned} \quad (9)$$

Where parameters A, B, C, D, E, F and G are used for simplicity of notations and depend only on the geometric features of the mechanism described in Fig. 2 as follows:

$$A = l_b^2 + l_d^2 + l_k^2 + l_p^2,\ B = 2l_b l_d \sin\alpha,\ C = 2l_b l_k \sin\alpha$$

$$D = -2l_b l_d \cos\alpha,\ E = -2l_b l_k \cos\alpha,\ F = -2l_d l_p,\ G = -2l_k l_p$$

The s_{ij} 's are the nine entries of the rotation matrix which represent the orientation of the moving platform, and $\rho_1, \rho_2, \rho_3, \rho_4$ are the actuator lengths. It is fairly easy to obtain the forward kinematic equations having the rotation matrix S in hand. So, the problem reduces to solving for the rotation matrix instead, with nine entries as the unknowns. Noting that these entries are not independent, there would be no need to compute all nine unknowns. Furthermore, the elements in the first column of S, namely: s_{11}, s_{21} and s_{31} are not present in the kinematic equations, which simplifies the problem as we can find the 2nd and 3rd columns of S and the 1st column will be simply computed as their cross product. Hence, the problem is solving (9) for the rotation matrix with the following constraints:

$$s_{12}^2 + s_{22}^2 + s_{32}^2 = 1, \quad (10)$$

$$s_{13}^2 + s_{23}^2 + s_{33}^2 = 1, \quad (11)$$

$$col(2).col(3) = 0, \quad (12)$$

Col (2) and Col (3) are defined as the second and third columns of the rotation matrix S, respectively. We must note that S is an orthonormal matrix so the 2nd and 3rd columns must be orthogonal with unit lengths. As the cross product of two orthonormal vectors would also be orthonormal, the other constraints on the rotation matrix entries would be trivial. From (9), we can solve for s_{12}, s_{13} as:

$$s_{12} = \frac{\rho_1^2 + \rho_3^2 - \rho_2^2 - \rho_4^2}{4B},\ s_{13} = \frac{\rho_1^2 + \rho_4^2 - \rho_2^2 - \rho_3^2}{4C} \quad (13)$$

These equations are in the form of an analytic closed form solution. Unfortunately the high coupling of the forward kinematic equations makes the closed form computation of other entries of S complicated. Several inconclusive attempts were made to find an analytic solution for these entries; therefore we tried a new approach combining the analytic and numerical methods to solve for the remaining entries of the rotation matrix in a quasi-closed form. We can relate the four remaining unknowns with the following equations:

$$\begin{aligned}s_{22} &= \frac{4A - \rho_1^2 - \rho_2^2 - \rho_3^2 - \rho_4^2 - 4G\,s_{33}}{4D},\\ s_{23} &= \frac{4F\,s_{32} + \rho_1^2 + \rho_2^2 - \rho_3^2 - \rho_4^2}{-4E},\end{aligned} \quad (14)$$

Or with much simpler notation:

$$s_{22} = \alpha + \beta\, s_{33},\ s_{23} = \gamma\, s_{32} + \eta \quad (15)$$

Where:

$$\begin{aligned}\alpha &= \frac{4A - \rho_1^2 - \rho_2^2 - \rho_3^2 - \rho_4^2}{4D}, \beta = -\frac{G}{D},\\ \eta &= \frac{\rho_1^2 + \rho_2^2 - \rho_3^2 - \rho_4^2}{-4E}, \gamma = -\frac{F}{E},\end{aligned} \quad (16)$$

Using (10)-(11), we have:

$$s_{12}^2 + (\alpha + \beta\, s_{33})^2 + s_{32}^2 = 1,\ s_{13}^2 + (\gamma\, s_{32} + \eta)^2 + s_{33}^2 = 1 \quad (17)$$

By this means from nine unknown parameters, using analytical manipulation, the problem is reduced to determine s_{32}, s_{33} using the above system of nonlinear equations, given s_{12}, s_{13}. Having s_{32}, s_{33} in hand, we can easily find the remaining entries s_{22}, s_{23} to obtain the 2nd and 3rd columns of the rotation matrix S, from which the first column would be determined by a cross product. The two entries s_{32}, s_{33} were obtained numerically from (17) solving a constrained optimization problem. This method is numerically implemented using optimization toolbox of Matlab, and in the next step the rotation matrix S was determined from which the orientation angles were easily obtained. Hence the forward kinematics problem was solved using the rotation matrix of the end effector as a means of simplifying the equations.

It should be noted that the degree of accuracy of the numerical optimization scheme to find an orthonormal rotation matrix is considered very important. The accuracy of the proposed method will definitely depend on the initial conditions used in the optimization process. This can be remedied in our application, by using the results of previous step approximation as the initial condition for the next step. We should also note that the proposed method uses a combination of analytic and numerical computation schemes, hence called a quasi-closed form solution method. The detail simulation results of this method are given in Section 5, where the effectiveness and accuracy of approximation is compared to other methods.

 ISBN: 3-938153-30-X

C. Taylor Series Expansion

As shown in section 3, the forward kinematic model of the hydraulic shoulder involves four nonlinear equations with actuator lengths ($\rho_1,...,\rho_4$) as the input and orientation angles of the end effector (θ_x, θ_y, θ_z) as the outputs, in other words:

$$\boldsymbol{\theta} = \mathbf{f}(\rho_1, \rho_2, \rho_3, \rho_4) \,, \qquad (18)$$

In which, $\mathbf{f}$ represents the forward kinematics map, that is subject of solution. A basic numerical approach to solve this problem is to approximate the nonlinear function $\mathbf{f}$ with a Taylor series expansion of arbitrary order:

$$\boldsymbol{\theta} = \mathbf{f}(\mathbf{0}) + \sum_{i=1}^{4} \frac{\partial \mathbf{f}}{\partial \rho_i} \rho_i + \sum_{i=1}^{4}\sum_{j=1}^{4} \frac{\partial^2 \mathbf{f}}{\partial \rho_i \partial \rho_j} \rho_i \rho_j + ... \qquad (19)$$

The number of the coefficients in the expansion is determined by the required degree of accuracy. Solving the forward kinematics problem, will hence be equal to computing these coefficients. In order to accomplish this task, different trajectories were considered for the end effector and the corresponding actuator displacements were determined using the inverse kinematics. The data pair was then used to compute the coefficients of the Taylor expansion using least square estimation. The trajectories in the task space must consider the whole workspace of the manipulator so that the estimated function for the forward kinematics could be used equally in different points of the workspace. Different orders of expansion up to 4^{th} ($O(n^5)$) were considered separately and the coefficients in each case were computed. The estimation error between the desired $\boldsymbol{\theta}$ and its estimate $\hat{\boldsymbol{\theta}}$, namely: $\mathbf{e} = \boldsymbol{\theta} - \hat{\boldsymbol{\theta}}$ was used as a performance index of each scheme. The results of estimations for different orders of expansion are compared in table (3), in which PE stands for Prediction Error, SSE stands for sum of square of error, MSE stands for mean square error, and MAE stands for mean absolute error. It can be seen that the number of the coefficients in the expansion will increase with the order of approximation resulting in a better degree of accuracy but usually with a slight increase in the computation time. This result enables the designer to choose the appropriate order, according to the performance requirement. The accuracy sought by the 4th order approximation seems to be a good compromise for robotic applications.

TABLE (3): MEASURES OF PREDICTION ERRORS

Approximation Order		2^{nd}	3^{rd}	4^{th}
No. of Coefficients		15	35	64
Max PE	θ_x	0.058	0.0116	0.0033
	θ_y	0.292	0.0475	0.014
	θ_z	0.025	0.0114	0.0011
SSE	θ_x	20.8	0.67	0.012
	θ_y	41.8	1.77	0.028
	θ_z	3.9	0.18	0.0025
MSE	θ_x	3.1e-4	1e-5	1.8e-7
	θ_y	6.3e-4	2.7e-5	4.2e-7
	θ_z	5.9e-5	2.8e-6	3.7e-8
MAE	θ_x	0.011	0.0024	2.9e-4
	θ_y	0.016	0.0038	4.5e-4
	θ_z	0.0056	0.0012	1.5e-4

V. SIMULATION RESULTS

A. Sample Trajectory Generation

We consider a smooth motion specified in terms of a desired pose of the moving platform of the hydraulic shoulder. The sample trajectory is easily defined given the initial and final points and the time to reach the final point. Fig. 3 shows the sample trajectory for each orientation angle in the task space of the hydraulic shoulder.

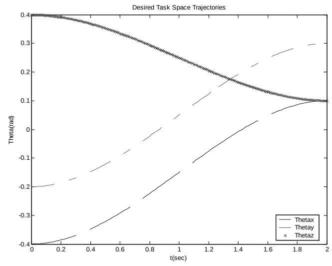

Figure (3): Sample Trajectory for Orientation Angles

B. Simulations

Fig.4 shows the simulation results using the trained neural networks of different structures. Best representatives from each structure, selected from tables (1) and (2) were tested with the sample trajectory along each orientation angle. Fig. 5 shows the simulation results for the quasi-closed method applied to follow the sample trajectory along each orientation angle. The simulation was performed using the following parameters, which are extracted from the geometrical configuration of the robot defined in equation (9).

A=0.0268m, B=0.0045m, C=0.0083m, D=0.0026m, E=0.0048m, F=0.0092m, G=0.0169m

Fig. 5 shows also the simulation results for the Taylor series method applied to follow the sample trajectory along each orientation angle. The tracking performance is seen to improve as the order of approximation increases, with the expense of larger numbers of parameters. Table (4) summarizes the statistics of approximation errors, and the accuracies obtained by each method for the considered trajectory. As it is observed through this simulation study for a typical robotic trajectory, the maximum approximation error reached by the suitable Neural network structures are limited to 0.03 radians (1.7 degrees) and 0.086 (5 degrees) for quasi-closed method, which are way beyond required in an accurate robotic application. The maximum error of approximation in 4^{th} order Taylor series is at least 10 times better that in other methods and typically about 0.005 radians (0.14 degrees).

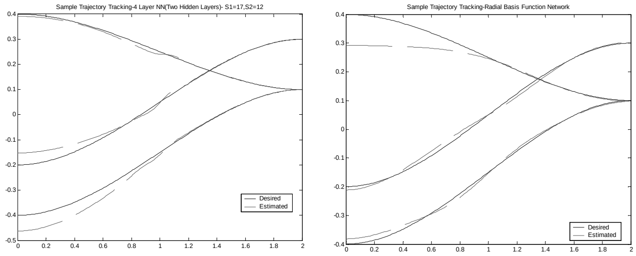

Figure 4. Tracking Performance for selected Structures of neural networks

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coefficients of the map. It is observed that the 4th order Taylor series approximation is the best compromise with acceptable prediction errors for robotic applications compared to the different structures of neural networks or the quasi-closed solution method proposed. Furthermore, despite the number of parameters obtained in the 4th order Taylor series approximation, its online digital implementation is with ease. Further attempts to increase the order of approximation worth considering only, when higher required accuracy is demanded.

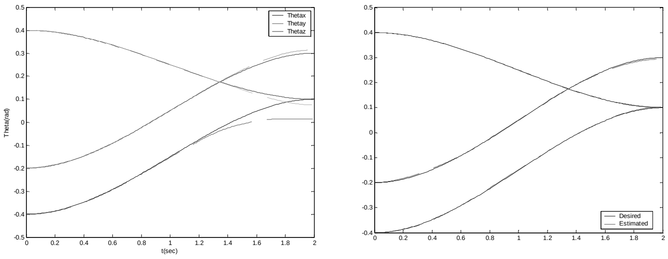

Figure 5. Sample Trajectory Tracking for Quasi-Closed Method, and Taylor Series Method with 4th Order Approximation

TABLE (4): MEASURES OF TRACKING ERRORS (SI UNITS)

Performance Index / Solution Method		Emax	SSE	MSE	MAE
4th order Taylor Expansion	θ_x	.0028	.0005	2.5e-6	.0013
	θ_y	.0056	.0018	9.1e-6	.0025
	θ_z	.0017	.00015	7.5e-7	.0006
3 Layer Feed-forward Neural Net (s=34)	θ_x	.0548	.124	.00061	.017
	θ_y	.0453	.056	.00028	.011
	θ_z	.0295	.025	.00013	.0074
4 layer FF Neural Net $s_1 = 12, s_2 = 15$	θ_x	.028	.032	.00016	.0091
	θ_y	.03	.069	.00034	.014
	θ_z	.032	.054	.00027	.012
RBF Neural Network	θ_x	.018	.019	9.9e-5	.008
	θ_y	.017	.016	8.3e-5	.0074
	θ_z	.1	.53	.0026	.033
Quasi-Closed Solution Method	θ_x	.086	.225	.0011	.0193
	θ_y	.014	.0072	3.6e-5	.0041
	θ_z	.025	.0171	8.5e-5	.0053

V. CONCLUSIONS

In this paper, three different approaches were presented to solve the forward kinematics problem in a three DOF actuator redundant hydraulic parallel manipulator. First, neural networks of different structures were introduced to solve the problem. Multilayer feed forward and Radial Basis networks were considered separately. Simulation results showed that multilayer feed forward networks with two hidden layers had a better performance compared to those with one hidden layer. The training time for RBF networks was shown to be much less than feed forward networks. Their tracking performance and estimation errors were also acceptable, but the weak point of such networks could be the big size leading to large number of neurons and weights. The main drawback of neural networks would be the long training times and the big size of the networks resulting in much more number of weights compared to the number of coefficients used in Taylor series expansion.

Alternatively, a quasi-closed solution method was developed which used the rotation matrix of the end effector to determine the corresponding orientation angles needed to solve the forward kinematics map. Finally, The Taylor series expansion was used in a least square estimation problem to solve for the unknown coefficients of the map.

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 ISBN: 3-938153-30-X