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Robust Control for Flexible Joint Robots with A Supervisory Control to Remedy Actuator Saturation

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Abstract: *In this paper a controller design method for flexible joint robots (FJR), considering actuator saturation is proposed and its robust stability is thoroughly analyzed. This method consists of a composite control structure, with a PD controller on the fast dynamics and a PID controller on slow dynamics. Moreover, the need of powerful actuator is remedied by decreasing the bandwidth of the fast controller during critical occasions, with the use of a supervisory loop. Fuzzy logic is used in the supervisory law, in order to adjust the proper gain in the forward path. It is then shown that UUB stability of the overall system is guaranteed in presence of uncertainties, provided that the PD and the PID gains are tuned to satisfy certain conditions.*

Keywords: Flexible Joint Robot, Actuator Saturation, Supervisory Control, Fuzzy Logic.

1. Introduction

The desire for higher performance from the structure and mechanical specifications of robot manipulators has been spurred designers to come up with *flexible joint robots* (FJR). Several new applications such as space manipulators and articulated hands necessitate using FJRs. This necessity has emerged new control strategies required, since the traditional controllers implemented on FJRs have failed in performance [1,2]. Since 1980's many attempts have been made to encounter this problem and now, several methods has been proposed including various linear, nonlinear, robust, adaptive and intelligent controllers [3, 4]. Among these, only a few researchers have considered practical limitations such as actuator saturation in the controller synthesis, as a real practical drawback to achieve a good performance [5].

On the other hand actuator saturation has been considered by the control community from early achievements of control engineering. During 50's and 60's at the beginning era of optimal control, researchers have been working on saturation, introducing bang-bang control methods. Over the last decade the control research community has shown a new interest in the study of the effects of saturation on the performance of systems. In fact it can be said that in the past, researchers were encountered a *drawback* identified as actuator saturation and developed methods to avoid it, while now, researchers develop methods to achieve a desirable performance in the presence of actuator saturation encountered as a *limitation*.

It can be said that saturation may cause two types of performance degradations:

1) Inevitable limitations such as slow responses, undesirable transitions, etc.
2) Removable problems such as instability, undesired steady state performance, etc.

The goal in considering saturation in controller synthesis is to decrease or remove the latter. A common classical remedy for systems with bounded control is to reduce the bandwidth of the control system such that saturation seldom occurs. This is a trivial weak solution, since even for small reference commands and disturbances the possible performance of the system is significantly degraded. This idea (reduction in bandwidth by reduction in the closed loop gain) is practical and "easy", so this motivates some researchers to propose an "adaptive" reduction in bandwidth consistent with the actuation levels [6]. The "adaptation" process is done under supervision of a *supervisory loop*, and as proposed in [6] can be accomplished through complex computations, which seem not to be practically implementable. In order to come up with an online implementable controller for FJRs, a fuzzy logic supervisory control has been proposed by authors in [7]. In this topology, the fuzzy logic is set to be "out of the main loop", at a supervisory level, at the aim of preserving the essential properties of the main controller. This idea is first published by the authors in [8 and 9] and is modified to use with composite controller for FJRs in [7]. It is observed in various simulations that by including this supervisory loop to the controller structure, the steady state performance of the system is preserved, and moreover, the stability of the overall system which may be defected by addition of a saturation block, will come back again. In other words the supervisor can remove instability due to saturation. The stability analysis of the overall system, however, is essential for the closed loop structure for susceptible applications of the FJRs such as space robots, which is analyzed thoroughly in this present paper.

This paper is organized as follows: Section 2 presents the modeling procedure taken for an FJR; Section 3 is devoted to description of the proposed controller structure; Section 4 is allocated for robust stability analysis; In section 5 a design procedure based on the analyses is presented, and finally, the conclusions are presented in Section 6.

2. FJR Modeling

In order to model an FJR, the state vector includes the link positions, as is the case with solid robots. However, actuator positions must also be added to the state vectors. Since, in contradiction to solid robots these are not statically related to the i'th position via just a gearbox gain, rather they are related through the dynamics of the flexile element. It is usual in the FJR literature to arrange these angles in a vector as follows:

$$\vec{Q} = [\theta_1, \theta_2, ..., \theta_n, \theta_{n+1} \mid \theta_{n+2}, ..., \theta_{2n}]^T = [\vec{q}_1^{\,T}, \vec{q}_2^{\,T}]^T \qquad (1)$$

Where $\theta_i : i = 1,2,...,n$ represents the position of the i'th link and the position of the i'th actuator is represented by $\theta_{i+n} : i=1,2,...,n$. Using this notation and taking into account some simplifying assumptions, Spong has proposed a model for FJRs [10] as follows:

$$\begin{cases} \mathbf{M}(\vec{q}_1)\ddot{\vec{q}}_1 + \vec{N}(\vec{q}_1, \dot{\vec{q}}_1) = -\mathbf{K}(\vec{q}_1 - \vec{q}_2) \\ \mathbf{J}\,\ddot{\vec{q}}_2 - \mathbf{K}(\vec{q}_1 - \vec{q}_2) = \vec{u} \end{cases} \qquad (2)$$

Where **M** is the matrix of the link inertias and **J** is that of the motors, $\vec{u}$ is the vector of input torques and $\vec{N}$ is the vector of all gravitational, centrifugal and Coriolis torques as follows:

$$\vec{N}(\vec{q}_1, \dot{\vec{q}}_1) = \mathbf{V_m}(\vec{q}_1, \dot{\vec{q}}_1)\,\dot{\vec{q}}_1 + \vec{G}(\vec{q}_1) + \mathbf{F_d}\dot{\vec{q}}_1 + \vec{F}_S(\dot{\vec{q}}_1) + \vec{T}_d \qquad (3)$$

In which, $V_m(q,\dot{q})$ consists of the Coriolis and centrifugal terms, G(q) is the gravity terms, F_d is the diagonal viscous friction matrix, F_s includes the static friction terms, and finally, T_d is the vector of disturbance and unmodeled but bounded dynamics. Including this last term in the model, enables us to encapsulate the modeling uncertainties into the picture. As it is demonstrated in [11], in spite of uncertainty in all parameters, the following quantities are bounded:

$$\underline{m}\,\mathbf{I} \le \mathbf{M}(\vec{q}_1) \le \overline{m}\,\mathbf{I} \qquad (4)$$

$$\left\|\vec{N}(\vec{q}_1, \dot{\vec{q}}_1)\right\| \le \beta_0 + \beta_1\left\|\dot{\vec{q}}_1\right\| + \beta_2\left\|\dot{\vec{q}}_1\right\|^2 \qquad (5)$$

$$\left\|\mathbf{V_m}(\vec{q}_1, \dot{\vec{q}}_1)\right\| \le \beta_3 + \beta_4\left\|\dot{\vec{q}}_1\right\| \qquad (6)$$

Where $\underline{m}, \overline{m}$ and β_i's are real positive constants and these uncertainty bounds will be used in robust stability analysis. It is assumed that all flexible elements are modeled by linear springs and without loss of generality [10], all springs assumed to have the same spring constant *k*. the matrix **K** is defined as **K** = *k***I** .The inertia matrices are non-singular so the model can be changed to the following singular perturbation standard form:

$$\begin{cases} \ddot{\vec{q}} = -\mathbf{M}^{-1}(\vec{q})\vec{z} - \mathbf{M}^{-1}(\vec{q})\vec{N}(\vec{q}, \dot{\vec{q}}) \\ \varepsilon\,\ddot{\vec{z}} = -(\mathbf{M}^{-1}(\vec{q}) + \mathbf{J}^{-1})\vec{z} - \mathbf{M}^{-1}(\vec{q})\vec{N}(\vec{q}, \dot{\vec{q}}) - \mathbf{J}^{-1}\vec{u} \end{cases} \qquad (7)$$

in which $\vec{q} = \vec{q}_1$, $\vec{z} = \mathbf{K}(\vec{q}_1 - \vec{q}_2)$ and $\varepsilon = 1/k$. Now if we choose $\varepsilon = 0$ then the slow behavior of $\vec{z}$ could be derived as:

$$\begin{aligned} \vec{z}_S &= -(\mathbf{M}^{-1}(\vec{q}) + \mathbf{J}^{-1})^{-1}[\mathbf{M}^{-1}(\vec{q})\vec{N}(\vec{q}, \dot{\vec{q}}) - \mathbf{J}^{-1}\vec{u}_S] \\ &= -(\mathbf{I} + \mathbf{M}(\vec{q})\mathbf{J}^{-1})^{-1}\vec{N}(\vec{q}, \dot{\vec{q}}) - (\mathbf{J}\,\mathbf{M}(\vec{q})^{-1} + \mathbf{I})^{-1}\vec{u}_S \end{aligned} \qquad (8)$$

By substitution of this into equation (7) after some matrix manipulations we reach to:

$$\ddot{\vec{q}}_S = -\mathbf{M}^{-1}(\vec{q})\vec{z}_f - [\mathbf{J} + \mathbf{M}(\vec{q})]^{-1}\vec{N}(\vec{q}, \dot{\vec{q}}) + [\mathbf{J} + \mathbf{M}(\vec{q})]^{-1}\vec{u}_S \qquad (9)$$

in which $\vec{z}_f$ represents the fast behavior of $\vec{z}$ which is defined as $\vec{z}_f = \vec{z} - \vec{z}_S$. Its dynamics could be found to be

$$\varepsilon\ddot{\vec{z}}_f = -(\mathbf{M}^{-1}(\vec{q}) + \mathbf{J}^{-1})\vec{z}_f - \mathbf{J}^{-1}\vec{u}_f \qquad (10)$$

having $\vec{u}_S$ and $\vec{u}_f$ we can solve the last three equations to find $\vec{q}_S, \vec{z}_S$ and $\vec{z}_f$ .Tikhonov theorem provides some stability conditions, under which, the overall behavior of the system can be determined from these variables as follows

$$\vec{q}(t) = \vec{q}_S(t) + O(\varepsilon) \quad t \in [0,T]$$

$$\vec{z}(t) = \vec{z}_S(t) + \vec{z}_f(t) \quad t \in [0,T]$$

$$\exists t_1 \ni \vec{z}(t) = \vec{z}_S(t) + O(\varepsilon) \quad t \in [t_1, T]$$

In which, $O(\varepsilon)$ determines the terms whose order is as the order of ε. The stability conditions can be satisfied by proper selection of $\vec{u}_f$. Taking into account the dynamics of $\bar{z}_f$ (equation (7)), a proper second order dynamics can be imposed to it by the use of a simple PD controller:

$$\bar{u}_f = [\mathbf{K}_{\mathbf{Pf}}\bar{z}_f + \mathbf{K}_{\mathbf{Df}}\dot{\bar{z}}_f] \tag{11}$$

Different control strategies can be used for the slow subsystem. Using a robust PID controller for the $\bar{u}_S$ has the following benefits:

1) No need for rate measurements,
2) No need for offline computations (specially derivations of the reference input), and
3) Guaranteed robust stability by the conditions detailed in [12].

for implementation purposes these are the three main requests so we propose using such structure:

$$\bar{u}_S = \mathbf{K}_{\mathbf{P}}\bar{e} + \mathbf{K}_{\mathbf{D}}\dot{\bar{e}} + \mathbf{K}_{\mathbf{I}}\int_0^t \bar{e}(\tau)d\tau \tag{12}$$

in which the error vector is defined as

$$\bar{e} = \bar{q}_d - \bar{q} \tag{13}$$

The overall control system is shown in figure (1) by which desirable performance can be achieved at the expense of high control effort. This may result in actuator saturation. In the next section a method will be introduced to remedy actuator saturation, by adding a term to this robust controller structure.

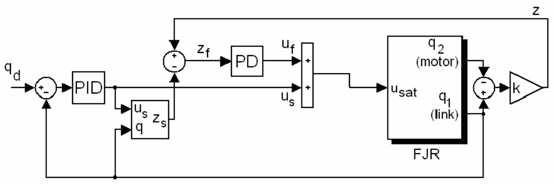

Figure 1: The FJR control system

3. The Supervisory loop

In this part we will first describe the idea of error governor as it is first proposed by the authors in [8 and 9]. Then the modifications needed to use this idea with the FJR model are elaborated. Without loss of generality one can assume that each element $u_i(t)$ of the control vector has a saturation limit of 1. In other words the saturation function can be defined as follows:

$$sat(u_i(t)) = \begin{cases} 1 & 1 \le u_i(t) \\ u_i(t) & -1 \le u_i(t) \le 1 \\ -1 & u_i(t) \le -1 \end{cases} \tag{14}$$

The proposed method is twofold, first the compensator is designed without considering any saturation limit, then a time varying scalar gain $0<\lambda(t)\le 1$ is added which modifies error and is adjusted via a supervisory loop in order to cope with saturation (figure 2).

Intuitively one can state the logic of adjustment as follows:

- If the system is to experience saturation make λ smaller, Otherwise increase λ up to one.

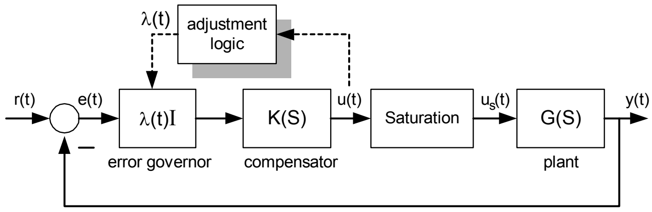

Figure 2: The closed loop system with error governor

This logic decreases the bandwidth when the system is to experience saturation and in normal conditions the effect of error governor is diminished by making λ=1. This configuration reduces the amplitude of the control effort as is done by saturation itself but there are some important differences:

- This is a dynamic compensator and not a hard nonlinearity as is the case with saturation.
- This approach limits the control effort by affecting the controller states while saturation will limit the control effort independent of the controller states. In other words, it acts in a closed loop fashion rather than an open loop structure of a saturation block. Hence, the dynamic behavior of it can be used to preserve stability in a systematic manner.

It is difficult to implement this logic with a rigorous mathematical model and if even this is accomplished, it will be difficult to implement. However fuzzy logic can be easily employed here as first has been proposed by authors in [8]. Details are as follows.

In order to sense the value of nearness to saturation, the absolute value of the amplitude of the control effort $|u(t)|$ is a good measure. To give a kind of prediction to the logic $\dot{u}(t)$ is also taken into account. The above logic thus can be interpreted with fuzzy notation as follows:

- If $|u(t)|$ is *NEAR* to one and $\dot{u}(t)$ is *POSITIVE* make λ *LESS* than one,
- When $|u(t)|$ is *OVER* one, make λ *SMALL* if $\dot{u}(t)$ is negative and *VERY SMALL* if $\dot{u}(t)$ is not negative,
- Otherwise make it *ONE* (see table 1).

TABLE 1 FUZZY RULES

$\dot{u}$ \ $\lvert u \rvert$	**Small**	**Near**	**Over**
Neg	One	One	S
Zero	One	One	VS
Pos	One	L	VS

To implement this logic, fuzzy sets are defined as in

[7 and 9] and the rest of decision making is done as usual in a fuzzy algorithm.

The proposed method can be implemented not only on FJRs but also on a variety of systems experiencing limitations in the actuators, since the logic is based on a model free routine. The effectiveness of this structure is verified in different applications [7 to 9]. It is observed that it can return the stability which may be lost due to saturation, and also the steady state behavior of the closed loop system remains unchanged. Theoretical reason will be elaborated in the next section.

In order to use this strategy for the FJR, some modifications should be made which is briefed as follows:

1) The supervisor is only applied for the fast subsystem, which mainly causes the instability when limited by saturation.
2) The saturation limit is not 1 in the FJR configuration, so the control effort u(t) must be attenuated by this factor before feeding to the supervisor.

The modified supervisory loop for the FJR is shown in figure (3). A filter is used to estimate $\dot{u}(t)$ from $u(t)$ so that the only measurement required remains $u(t)$.

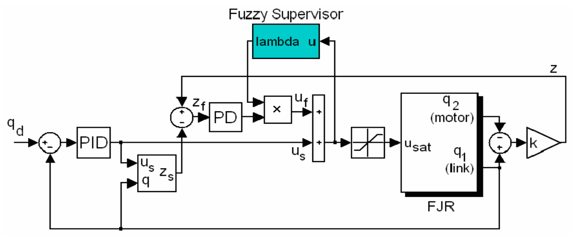

Figure 3: Fuzzy supervisor for the FJR

4. Robust Stability Analysis

Recall the system equations

$$\ddot{\bar{q}}_S = -\mathbf{M}^{-1}(\bar{q})\bar{z}_f - [\mathbf{J}+\mathbf{M}(\bar{q})]^{-1}\bar{N}(\bar{q},\dot{\bar{q}}) + [\mathbf{J}+\mathbf{M}(\bar{q})]^{-1}\bar{u}_S \quad (15)$$

$$\varepsilon\ddot{\bar{z}}_f = -(\mathbf{M}^{-1}(\bar{q})+\mathbf{J}^{-1})\bar{z}_f - \mathbf{J}^{-1}\bar{u}_f \quad (16)$$

and the control terms

$$\bar{u}_S = \mathbf{K_P}\bar{e} + \mathbf{K_D}\dot{\bar{e}} + \mathbf{K_I}\int_0^t \bar{e}(\tau)d\tau \quad (17)$$

$$\bar{u}_f = \lambda(t).[\mathbf{K_{Pf}}\bar{z}_f + \mathbf{K_{Df}}\dot{\bar{z}}_f] \quad (18)$$

the error dynamics could be found to be

$$\ddot{\bar{e}} = \ddot{\bar{q}}_d + \mathbf{M}^{-1}(\bar{q})\bar{z}_f + \mathbf{M_t}^{-1}(\bar{q})\mathbf{N}(\bar{q},\dot{\bar{q}}) + \mathbf{M_t}^{-1}(\bar{q})(\mathbf{K_P}\bar{e} + \mathbf{K_D}\dot{\bar{e}} + \mathbf{K_I}\int_0^t \bar{e}(\tau)d\tau) \quad (19)$$

and the fast dynamics is

$$\varepsilon\ddot{\bar{z}}_f = -(\mathbf{M}^{-1}(\bar{q})+\mathbf{J}^{-1})\bar{z}_f + \mathbf{J}^{-1}\lambda(t).[\mathbf{K_{Pf}}\bar{z}_f + \mathbf{K_{Df}}\dot{\bar{z}}_f] \quad (20)$$

We can rewrite this dynamics as

$$\ddot{\bar{z}}_f = -\mathbf{K_1}\dot{\bar{z}}_f - \mathbf{K_2}\bar{z}_f \quad (21)$$

in which

$$\mathbf{K_1} = \frac{\lambda(t)}{\varepsilon}\mathbf{J}^{-1}\mathbf{K_{Df}} \quad (22)$$

$$\mathbf{K_2} = (1/\varepsilon)[\mathbf{M}^{-1}(\bar{q}) + \mathbf{J}^{-1}(\mathbf{I}+\lambda(t)\mathbf{K_{Pf}})] \quad (23)$$

These equations can be rearranged into state space format

$$\dot{\bar{x}} = \mathbf{A}\bar{x} + \mathbf{B}\Delta\mathbf{A} + \mathbf{C}[\mathbf{I} \quad \mathbf{0}]\bar{y} \quad (24)$$

$$\dot{\bar{y}} = \mathbf{A_f}\bar{y} \quad (25)$$

in which

$$\bar{x} = \begin{bmatrix} \int_0^t \bar{e}(\tau)d\tau \\ \bar{e} \\ \dot{\bar{e}} \end{bmatrix}, \quad \bar{y} = \begin{bmatrix} \bar{z}_f \\ \dot{\bar{z}}_f \end{bmatrix} \quad (26)$$

and

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \\ -\mathbf{M_t}^{-1}(\bar{q})\mathbf{K_I} & -\mathbf{M_t}^{-1}(\bar{q})\mathbf{K_P} & -\mathbf{M_t}^{-1}(\bar{q})\mathbf{K_D} \end{bmatrix}, \quad (27)$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{M_t}^{-1}(\bar{q}) \end{bmatrix},$$

$$\Delta\mathbf{A} = (\mathbf{N}(\bar{q},\dot{\bar{q}}) + \mathbf{M_t}(\bar{q})\ddot{\bar{q}}_d), \quad \mathbf{C} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{M}^{-1}(\bar{q}) \end{bmatrix}$$

$$\mathbf{A_f} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{K_2} & -\mathbf{K_1} \end{bmatrix} \quad (28)$$

In the next subsection we study the effect of $\lambda(t)$ on the stability of the fast subsystem.

4.1. Stability of the fast subsystem

To study the stability of the fast subsystem we consider the following Lyaponov function candidate

$$V_f(\bar{y}) = \bar{y}^T\mathbf{S}\,\bar{y} \quad (29)$$

in which **S** is defined as

$$\mathbf{S} = 1/2\begin{bmatrix} 2\mathbf{I} & \mathbf{K_1}^{-1} \\ \mathbf{K_1}^{-1} & \mathbf{K_2}^{-1} \end{bmatrix} \quad (30)$$

Lemma 1: The matrix S is positive definite.

The proof is based on Shur complement and is the same as what can be found in [13] for the case $\lambda(t) = 1$. So the function V_f is positive.

■

Theorem 1: The fast subsystem of (25) with thw matrix $\mathbf{A_f}$ introduced in (28) is stable provided that condition (33) is met.

In other words, there are some bounds on the parameters $\mathbf{K_{PF}}$, $\mathbf{K_{DF}}$, $\lambda(t)$ used in control term (18) which if be met can stabilize the dynamics (16) in a closed loop configuration.

Proof: To prove stability using Lyaponov direct method consider the time derivative of V_f along trajectory (25)

$$\dot{V}_f(\bar{y}) = \dot{\bar{y}}^T\mathbf{S}\,\bar{y} + \bar{y}^T\mathbf{S}\,\dot{\bar{y}} + \bar{y}^T\dot{\mathbf{S}}\,\bar{y} = \bar{y}^T[\mathbf{A_f}^T\mathbf{S} + \mathbf{S}\mathbf{A_f}]\bar{y} + \bar{y}^T\dot{\mathbf{S}}\,\bar{y} \quad (31)$$

considering the first two terms, note that

$$\mathbf{A_f}^T\mathbf{S}+\mathbf{S}\mathbf{A_f} = -\begin{bmatrix}\mathbf{K_1}^{-1}\mathbf{K_2} & \mathbf{0}\\ \mathbf{0} & \mathbf{K_1}\mathbf{K_2}^{-1}-\mathbf{K_1}^{-1}\end{bmatrix} = -\mathbf{W} \qquad (32)$$

since matrices $\mathbf{K_1}$ and $\mathbf{K_2}$ are p.d., to make matrix $\mathbf{W}$ p.d. the following matrix should be p.d.

$$\mathbf{K_1}\mathbf{K_2}^{-1}-\mathbf{K_1}^{-1} > \mathbf{0} \qquad (33)$$

after some matrix manipulations this can be transformed to the following condition on PD gain matrices

$$\frac{1}{\varepsilon}\left[\underline{\lambda}(\lambda(t)\mathbf{J}^{-1}\mathbf{K_{Df}})\right]^2 > \bar{\lambda}(\mathbf{M}^{-1}(\bar{q})+\mathbf{J}^{-1}[\mathbf{I}+\lambda(t)\mathbf{K_{Pf}}]) \qquad (34)$$

if we let $\lambda(t) = 1$ this can be met by increasing $\mathbf{K_{DF}}$ and decreasing $\mathbf{K_{PF}}$. In other words a lower bound on $\mathbf{K_{DF}}$ and simultaneously an upper bound on $\mathbf{K_{PF}}$ should be satisfied (as is the case when there is not a supervisory loop). For example if we assume the gain matrices to be diagonal

$$\mathbf{K_{Df}} = k_{Df}\mathbf{I},\ \mathbf{K_{Pf}} = k_{Pf}\mathbf{I} \qquad (35)$$

then the following condition must be met

$$k_{Df} \ge \underline{k},\ k_{Pf} \le \bar{k} \qquad (36)$$

In presence of supervisory logic, as $\lambda(t)$ is always less than 1, the upper bound can be left unchanged. The lower bound can be satisfied assuming a lower bound on $\lambda(t)$

$$\lambda(t) > \Lambda_{\min} \qquad (37)$$

and then changing the lower bound as

$$k_{Df} \ge \frac{\underline{k}}{\Lambda_{\min}} \qquad (38)$$

Now consider the third term in equation (31)

$$\dot{\mathbf{S}} = -\tfrac{1}{2}\begin{bmatrix}\mathbf{0} & \dfrac{\dot{\lambda}(t)}{\lambda^2(t)k}\mathbf{K_{Df}}^{-1}\mathbf{J}\\ \dfrac{\dot{\lambda}(t)}{\lambda^2(t)k}\mathbf{K_{Df}}^{-1}\mathbf{J} & -d(\mathbf{K_2}^{-1})/dt\end{bmatrix} \qquad (39)$$

which is clearly negative definite, so one can write

$$\bar{y}^T[\mathbf{A_f}^T\mathbf{S}+\mathbf{S}\mathbf{A_f}]\bar{y} + \bar{y}^T\dot{\mathbf{S}}\bar{y} \le -\underline{\lambda}(\mathbf{W})\|\bar{y}\|^2 \qquad (40)$$

now if the condition (34) is satisfied the matrix $\mathbf{W}$ will be p.d. so from the above equation V_f is negative, thus it is a Lyaponov function and stability has been proved.

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In this way we can deduce that the supervisory loop will not essentially affect the stability results previously presented at [14]. With this fact in mind, in the next subsection we will study the stability of the complete system.

4.2. Necessary lemmas for stability analysis

To prove the robust stability of the closed loop system in presence of modeling uncertainty, the Lyapunov direct method is used. Let V be the Lyapunov function candidate as follows

$$V(\bar{x},\bar{y}) = \bar{x}^T\mathbf{P}\bar{x} + \bar{y}^T\mathbf{S}\bar{y} \qquad (41)$$

in which $\mathbf{S}$ is defined as before (equation (30)) and $\mathbf{P}$ is chosen to be

$$\mathbf{P} = \tfrac{1}{2}\begin{bmatrix}\alpha_2\mathbf{K_P}+\alpha_1\mathbf{K_I}+\alpha_2^2\mathbf{M} & \alpha_2\mathbf{K_D}+\mathbf{K_I}+\alpha_1\alpha_2\mathbf{M} & \alpha_2\mathbf{M}\\ \alpha_2\mathbf{K_D}+\mathbf{K_I}+\alpha_1\alpha_2\mathbf{M} & \alpha_1\mathbf{K_D}+\mathbf{K_P}+\alpha_1^2\mathbf{M} & \alpha_1\mathbf{M}\\ \alpha_2\mathbf{M} & \alpha_1\mathbf{M} & \mathbf{M}\end{bmatrix} \qquad (42)$$

in which α_i s are real positive constants. The above function has a quadratic form and it is positive due to positive definiteness of P and S. Positive definiteness of S has been shown in lemma 1 and the following lemma guarantees that $\mathbf{P}$ is also positive definite.

Lemma 2: The matrix $\mathbf{P}$ is positive definite if

$$\alpha_1 > 0,\ \alpha_2 > 0,\ \alpha_1 + \alpha_2 < 1 \qquad (43)$$

$$\alpha_2(k_P - k_D) - (1-\alpha_1)k_I - \alpha_2(1+\alpha_1-\alpha_2)\bar{m} > 0 \qquad (44)$$

$$k_P + (\alpha_1-\alpha_2)k_D - k_I - \alpha_1(1+\alpha_2-\alpha_1)\bar{m} > 0 \qquad (45)$$

in which

$$\mathbf{K_P} = k_P\mathbf{I},\quad \mathbf{K_I} = k_I\mathbf{I},\quad \mathbf{K_D} = k_D\mathbf{I} \qquad (46)$$

Proof is given in [15].

■

Now for the stability analysis Differentiate V along trajectories (24) and (25), which yields to

$$\begin{aligned}\dot{V}(\bar{x},\bar{y}) &= 2\bar{x}^T\mathbf{P}\dot{\bar{x}} + \bar{x}^T\dot{\mathbf{P}}\bar{x} + \dot{\bar{y}}^T\mathbf{S}\bar{y} + \bar{y}^T\mathbf{S}\dot{\bar{y}} + \bar{y}^T\dot{\mathbf{S}}\bar{y}\\ &= 2\bar{x}^T\mathbf{P}[\mathbf{A}\bar{x}+\mathbf{B}\Delta\mathbf{A}] + \bar{x}^T\dot{\mathbf{P}}\bar{x}\\ &\quad + 2\bar{x}^T\mathbf{PC}[\mathbf{I}\quad \mathbf{0}]\bar{y} + \bar{y}^T[\mathbf{A_f}^T\mathbf{S}+\mathbf{S}\mathbf{A_f}]\bar{y} + \bar{y}^T\dot{\mathbf{S}}\bar{y}\end{aligned} \qquad (47)$$

the following lemmas will be used to prove that V is a Lyaponov function.

Lemma 3: For the matrices $\mathbf{P}$, $\mathbf{A}$, $\mathbf{B}$ and $\Delta\mathbf{A}$ defined previously, the following inequality holds

$$2\bar{x}^T\mathbf{P}[\mathbf{A}\bar{x}+\mathbf{B}\Delta\mathbf{A}] + \bar{x}^T\dot{\mathbf{P}}\bar{x} \le \|\bar{x}\|(\varepsilon_0 - \varepsilon_1\|\bar{x}\| + \varepsilon_2\|\bar{x}\|^2) \qquad (48)$$

In this inequality ε_0, ε_1 and ε_2 are real positive constants that depend only on α_1, α_2 and the uncertainty bounds introduced in equations (4) to (6) as follows:

$$\varepsilon_0 = \lambda_1(\beta_0 + \bar{m}\lambda_3) \qquad (49)$$

$$\varepsilon_1 = \gamma - \lambda_1\beta_3 - \bar{m}\lambda_2 - \lambda_1\beta_1 \qquad (50)$$

$$\varepsilon_2 = \lambda_1\beta_4 + \lambda_1\beta_2 \qquad (51)$$

in which

$$\lambda_1 = \bar{\lambda}(\mathbf{R}_1),\ \lambda_2 = \bar{\lambda}(\mathbf{R}_2)$$
$$\lambda_3 = \|\ddot{q}_d(t)\|_\infty,\quad \gamma = \underline{\lambda}(\mathbf{Q}) \qquad (52)$$

where

$$\mathbf{Q} = \begin{bmatrix}\alpha_2k_I\mathbf{I} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & (\alpha_1k_P - \alpha_2k_D - k_I)\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & k_D\mathbf{I}\end{bmatrix} \qquad (53)$$

$$\mathbf{R}_1 = \begin{bmatrix}\alpha_2^2\mathbf{I} & \alpha_1\alpha_2\mathbf{I} & \alpha_2\mathbf{I}\\ \alpha_1\alpha_2\mathbf{I} & \alpha_1^2\mathbf{I} & \alpha_1\mathbf{I}\\ \alpha_2\mathbf{I} & \alpha_1\mathbf{I} & \mathbf{I}\end{bmatrix} \qquad (54)$$

$$\mathbf{R}_2 = \begin{bmatrix}\mathbf{0} & \alpha_2^2\mathbf{I} & \alpha_1\alpha_2\mathbf{I}\\ \alpha_2\mathbf{I} & 2\alpha_1\alpha_2\mathbf{I} & (\alpha_1^2+\alpha_2)\mathbf{I}\\ \alpha_1\alpha_2\mathbf{I} & (\alpha_1^2+\alpha_2)\mathbf{I} & \alpha_1\mathbf{I}\end{bmatrix} \qquad (55)$$

Proof is very simple and is done by performing some matrix manipulations and taking into account the fact that for robot manipulators the following equation holds for any vector $\bar{v}$ [16]

$$\bar{v}^T \dot{\mathbf{M}}(\bar{q})\bar{v} = 2\bar{v}^T \mathbf{V_m}(\bar{q},\dot{\bar{q}})\bar{v} \qquad (56)$$

■

Lemma 4: For the matrix **C** defined previously the following inequality holds

$$2\bar{x}^T \mathbf{PC}[\mathbf{I} \quad \mathbf{0}]\bar{y} \le 2\|\bar{x}\|\bar{\lambda}(\mathbf{P})\bar{\lambda}(\mathbf{M}^{-1})\|\bar{y}\| \qquad (57)$$

Proof is straightforward [15].

■

Lemma 5: Suppose that the Lyaponov function of a dynamic system has the following properties

$$\dot{V}(X) \le \|X\|(\phi_0 - \phi_1\|X\| + \phi_2\|X\|^2) \qquad (58)$$

and

$$\underline{\lambda}\|X\|^2 \le V(X) \le \bar{\lambda}\|X\|^2 \qquad (59)$$

where $\underline{\lambda},\bar{\lambda}$ and ϕ_is are constants. Given that

$$d = \frac{2\phi_0}{\phi_1 + \sqrt{\phi_1^2 - 4\phi_0\phi_2}} \times \sqrt{\frac{\bar{\lambda}}{\underline{\lambda}}} \qquad (60)$$

then the system is UUB stable with respect to **B**(0,*d*), provided that

$$\phi_1 > 2\sqrt{\phi_0\phi_2} \qquad (61)$$

$$\phi_1\left[\phi_1 + \sqrt{\phi_1^2 - 4\phi_0\phi_2}\right] > 2\phi_0\phi_2\left(1 + \sqrt{\frac{\bar{\lambda}}{\underline{\lambda}}}\right) \qquad (62)$$

$$\phi_1 + \sqrt{\phi_1^2 - 4\phi_0\phi_2} > 2\phi_2\|X_0\|\sqrt{\frac{\bar{\lambda}}{\underline{\lambda}}} \qquad (63)$$

where $\|X_0\|$ denotes the initial condition. Proof can be found in [11] under proof of its lemma 3.5.

■

4.3. Stability of the complete system

In this subsection we present the main result. To be compact we simply refer to the equations by their numbers in the body of the theorem and then to facilitate understanding and using the conditions we will present the design procedure in the next subsection.

Theorem 2: Consider the flexible joint manipulator of equations (15) and (16) with the composite controller structure of equations (17) and (18), under supervisory loop. The overall closed loop system with governing equations of motion (24) and (25) is UUB stable and the state variables converge to the origin under conditions of theorem 1 and lemma 2 and some new certain limits imposed on the fast (PD) and slow (PID) controller gains which will be .

Proof: The Lyapunov function candidate V introduced in (41) has been shown to be positive. This imposes conditions of lemma 2, equations (43) to (45), to be satisfied. In order to study $\dot{V}(\bar{x},\bar{y})$ to see if it is negative or not consider (47),(48), (57) and (40) which yields

$$\begin{aligned}\dot{V}(\bar{x},\bar{y}) &\le \|\bar{x}\|(\varepsilon_0 - \varepsilon_1\|\bar{x}\| + \varepsilon_2\|\bar{x}\|^2) \\ &\quad + 2\|\bar{x}\|\bar{\lambda}(\mathbf{P})\bar{\lambda}(\mathbf{M}^{-1})\|\bar{y}\| - \bar{\lambda}(\mathbf{W})\|\bar{y}\|^2 \qquad (64) \\ &= [\|\bar{x}\| \quad \|\bar{y}\|]\begin{bmatrix} -\varepsilon_1 & \bar{\lambda}(\mathbf{P})\bar{\lambda}(\mathbf{M}^{-1}) \\ \bar{\lambda}(\mathbf{P})\bar{\lambda}(\mathbf{M}^{-1}) & -\bar{\lambda}(\mathbf{W})\end{bmatrix}\begin{bmatrix}\|\bar{x}\| \\ \|\bar{y}\|\end{bmatrix} \\ &\quad + \varepsilon_0\|\bar{x}\| + \varepsilon_2\|\bar{x}\|^3\end{aligned}$$

or

$$\dot{V}(\bar{x},\bar{y}) \le -\bar{z}_1^T\mathbf{R}\bar{z}_1 + \varepsilon_0\|\bar{x}\| + \varepsilon_2\|\bar{x}\|^3 \qquad (65)$$

in which

$$\bar{z}_1 = \begin{bmatrix}\|\bar{x}\| \\ \|\bar{y}\|\end{bmatrix}, \quad \mathbf{R} = -\begin{bmatrix} -\varepsilon_1 & \bar{\lambda}(\mathbf{P})\bar{\lambda}(\mathbf{M}^{-1}) \\ \bar{\lambda}(\mathbf{P})\bar{\lambda}(\mathbf{M}^{-1}) & -\bar{\lambda}(\mathbf{W})\end{bmatrix} \qquad (66)$$

here we see that matrix **W** must be p.d. thus condition (33) or (34) must be met too. Now if we define

$$\bar{z} = \begin{bmatrix}\bar{x} \\ \bar{y}\end{bmatrix} \qquad (67)$$

we have

$$\|\bar{z}\| = \|\bar{z}_1\|, \ \|\bar{z}\| \ge \|\bar{x}\| \qquad (68)$$

thus

$$\dot{V}(\bar{x},\bar{y}) \le \|\bar{z}\|(\varepsilon_0 - \bar{\lambda}(\mathbf{R})\|\bar{z}\| + \varepsilon_2\|\bar{z}\|^2) \qquad (69)$$

Now apply the Railey Ritz inequality which reads

$$\begin{aligned}\underline{\lambda}(\mathbf{P})\|\bar{x}\|^2 &\le \bar{x}^T\mathbf{P}\bar{x} \le \bar{\lambda}(\mathbf{P})\|\bar{x}\|^2 \\ \underline{\lambda}(\mathbf{S})\|\bar{y}\|^2 &\le \bar{y}^T\mathbf{S}\bar{y} \le \bar{\lambda}(\mathbf{S})\|\bar{y}\|^2\end{aligned} \qquad (70)$$

adding these two equations yields

$$\bar{z}_1^T\begin{bmatrix}\underline{\lambda}(\mathbf{P}) & 0 \\ 0 & \underline{\lambda}(\mathbf{S})\end{bmatrix}\bar{z}_1 \le V(\bar{z}) \le \bar{z}_1^T\begin{bmatrix}\bar{\lambda}(\mathbf{P}) & 0 \\ 0 & \bar{\lambda}(\mathbf{S})\end{bmatrix}\bar{z}_1 \qquad (71)$$

in other words

$$\underline{\lambda}\|\bar{z}\|^2 \le V(\bar{z}) \le \bar{\lambda}\|\bar{z}\|^2 \qquad (72)$$

in which

$$\begin{aligned}\bar{\lambda} &= \max\{\bar{\lambda}(\mathbf{P}),\bar{\lambda}(\mathbf{S})\} \\ \underline{\lambda} &= \min\{\underline{\lambda}(\mathbf{P}),\underline{\lambda}(\mathbf{S})\}\end{aligned} \qquad (73)$$

Now from (69) and (72) and by lemma 5 we can state that given that

$$d = \frac{2\varepsilon_0}{\bar{\lambda}(\mathbf{R}) + \sqrt{\bar{\lambda}(\mathbf{R})^2 - 4\varepsilon_0\varepsilon_2}} \times \sqrt{\frac{\bar{\lambda}}{\underline{\lambda}}} \qquad (74)$$

the system is UUB stable with respect to **B**(0,*d*), provided the following stability conditions are satisfied

$$\bar{\lambda}(\mathbf{R}) > 2\sqrt{\varepsilon_0\varepsilon_2} \qquad (75)$$

$$\bar{\lambda}(\mathbf{R})\left[\bar{\lambda}(\mathbf{R}) + \sqrt{\bar{\lambda}(\mathbf{R})^2 - 4\varepsilon_0\varepsilon_2}\right] > 2\varepsilon_0\varepsilon_2\left(1 + \sqrt{\frac{\bar{\lambda}}{\underline{\lambda}}}\right) \qquad (76)$$

$$\bar{\lambda}(\mathbf{R}) + \sqrt{\bar{\lambda}(\mathbf{R})^2 - 4\varepsilon_0\varepsilon_2} > 2\varepsilon_2\|z_0\|\sqrt{\frac{\bar{\lambda}}{\underline{\lambda}}} \qquad (77)$$

where $\|z_0\|$ denotes the initial condition.

■

This proof reveals an important aspect of the supervisory loop dynamics included in the proposed controller law. Due to its nature that only multiplies a gain λ(t) to the controller gains and it does not disturb the main control structure, it will not disturb the stability of the system that has been proven for the system without supervisor. This general idea which was observed through various simulations of implemented supervisory loop for different systems is technically proven here. Another aspect that can be concluded from this analysis is the robustness property of the stability, in presence of modeling uncertainty. Since the unmodeled but bounded dynamics of the system is systematically encapsulated in the system model (as stated in Equations (3) to (5)), the only influence this will impose on the stability is the respective controller gains bound depicted in mentioned conditions.

5. Design procedure

To facilitate use of the nested set of equations, in this section we summarize the design procedure. Note that there are three sets of conditions which should be satisfied. The first set is condition (33) or (34) on PD gains and the supervisor gain, λ(t), which should be met in order to make **W** p.d. The second set is the set of equations (43) to (45) on PID gains in order to make **P** p.d. The last and the most important and the most complex set is the set of equations (75) to (77) which is indirectly dependent to all design parameters.

6. Conclusions

In this paper the problem of implementable controllers for flexible joint robots in presence of actuator saturation is considered in detail. The singularly perturbed model of the system is first introduced briefly, and a composite controller structure is proposed for the system. The proposed composite control consists of a robust PID term for rigid (slow) and a PD controller for flexible (fast) dynamics. In order to remedy the limitations caused by actuator bounds, a supervisory loop is proposed, and it is shown that a model free fuzzy supervisory loop makes it possible to reduce the minimum acceptable saturation limit, without great loss in performance. The supervisor will affect the signals *in prior to* the controller, and therefore, affecting the controller states. This is on contrary to the static saturation block which will be placed *after* the controller. It is shown through a Lyapunov based stability analysis, that due to the structure of the supervisory loop, and regardless of the logic it uses, since the controller adaptation gain is bounded and the overall variation of the system energy is dissipative, the stability condition of the composite controller remains unchanged. The detail stability analysis of the overall closed loop system is performed using Lyapunov direct method and the robust stability conditions are derived, respectively. These all considerations have been enabled us to offer a safe and implementable controller with guaranteed robust stability which is an essential requirement for susceptible applications such as space robotics.

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