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Nonparametric Identification and Robust H∞ Controller Synthesis for a Rotational/Translational Actuator

F.Bagheri, S. Purazarm, and H. D. Taghirad[1], Member, IEEE

Abstract— **In this paper an H∞ methodology based on nonparametric identification is developed for RTAC benchmark problem. In this nonlinear system, it is required to design a controller to satisfy stabilization and disturbance rejection objectives in spite of limited control effort. In order to design an H∞ controller, first, the nonlinear system is estimated as a nominal linear system. Moreover, the deviation of the system from the model, which involves nonlinearities, uncertainties, and disturbances, is encapsulated by a multiplicative uncertainty. Then, a robust controller is designed by developing a mixed-sensitivity problem to satisfy all performance requirements. Simulation results illustrate the tracking performance achievements.**

I. INTRODUCTION

THE Rotational/Translational Actuator experiment has originally been studied as a simplified model of a dual-spin spacecraft to investigate the resonance capture phenomena [1]. It is shown that this system is mathematically and qualitatively equivalent to a dual-spin spacecraft, i.e. they have similar averaged behavior and exhibit similar dynamic behaviors. The RTAC system has been studied to investigate the usefulness of a rotational proof mass for stabilizing translational motion [2].

Consider the unbalanced oscillator shown in Figure 1. The RTAC is built with a card (weigh M) fixed to the wall by a linear spring (stiffness k) and constrained to have one dimensional travel. An embedded proof mass actuator (mass m, moment of inertia I) is attached to the center of the card and can rotate in the horizontal plane. The radius of rotation is e. The card is submitted to the disturbance F and a control torque N is applied to the proof mass. The control objectives such as internal stability, fast settling time for a class of initial condition and good disturbance rejection for some signals in spite of limited control effort must be satisfied by designing a controller.

Several approaches of the RTAC problem have been proposed in the past few years. Various stabilization strategies are compared in [3]. Partial feedback linearization and integrator back-stepping are studied in [4]. Adequate

Manuscript received on February 1, 2006. Authors are with the K.N. Toosi University of Technology, Electrical Engineering Department, Advanced Robotics and Automated System (ARAS), P.O. Box 16315-1355, Tehran, IRAN. (Phone: +98-21-8846-8094; fax: +98-21-8846-2066; e-mail: Taghirad@kntu.ac.ir).

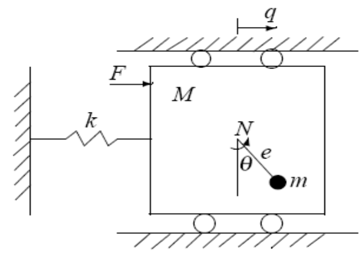

Figure.1. Model of the RTAC Experiment.

Lyapunov functions lead to a back-stepping controller in [5] and to a passive nonlinear in [6]. State-feedback nonlinear controller is obtained in [7]. Most of these techniques do not address multi-objective problems directly. Furthermore, they may lead to equations that are hard to solve numerically, in particular when output-feedback robust synthesis is involved. Finally, most of these approaches do not take into account uncertainty, or multiple specifications, explicitly. An H∞ methodology proposed here, that guarantees the various design constraints simultaneously.

II. SYSTEM MODELING

Let θ and $\dot{\theta}$ denote the angular position and velocity of the proof mass, and let q and $\dot{q}$ denote the translational position from its equilibrium position and the velocity of the card. The nonlinear equations of the motion are given by:

$$\begin{aligned}(M+m)\ddot{q}+kq &= -me(\ddot{\theta}\cos\theta-\dot{\theta}^2\sin\theta)+F \\ (I+me^2)\ddot{\theta} &= -me\ddot{q}\cos\theta+N\end{aligned} \tag{1}$$

With following transformation [8]

$$\zeta=\sqrt{\frac{M+m}{I+me^2}}q\,,\ \tau=\sqrt{\frac{k}{M+m}}t\,,\ v=\sqrt{\frac{M+m}{k(I+me^{2})}}N$$

$$D=\frac{1}{k}\sqrt{\frac{M+m}{I+me^2}}F\ ,\ \varepsilon=\frac{me}{(I+me^2)(M+m)} \tag{2}$$

the normalized nonlinear equations of motion of the card are

$$\begin{aligned}\ddot{\zeta}+\zeta &= \varepsilon(\dot{\theta}^2\sin\theta-\ddot{\theta}\cos\theta)-\frac{b+c}{\sqrt{k(M+m)}}\zeta+D \\ \ddot{\theta} &= -\varepsilon\zeta\cos\theta+V\end{aligned} \tag{3}$$

Values of the parameters for the RTAC equations are given in Table I.

TABLE I: RTAC PHYSICAL PARAMETERS

Cart Mass	$M = 1Kg$
Arm Mass	$m = 0.1Kg$
Spring Constant	$k = 186N/m$
Eccentricity	$e = 0.06m$
Arm Inertia	$I = 2.2e-4$
Viscous friction constant	$b + c = 0.01$

III. NON-PARAMETRIC SYSTEM IDENTIFICATION

Several approaches of nonlinear control design were developed for RTAC problem based on the ideal non-dissipative model of the nonlinear system given by Equation (1) and (2). The basic problem we consider in this paper is to find a linear controller for the given nonlinear system such that the closed-loop system meets a number of specifications. We design a linear H_∞ controller while the nonlinear system is estimated as a nominal linear system and unmodeled dynamics and derivations from this nonlinear model are taken into account as an uncertainty model.

The basic technique is to model the plant as belonging to an unstructured set $\mathcal{P}$ of linear plant. Unstructured set is used for two reasons. First we believe that all models used in feedback design should include some unstructured uncertainty to cover unmodeled dynamics, particularly at high frequency. Secondly, for a specific type of unstructured uncertainty, we can develop a simple and general analysis method.

Suppose that the nominal plant transfer function is $P_0(s)$ and consider the perturbed plant transfer functions of the form $P(s) = P_0(s)(1 + \Delta(s)W(s))$, where W is a fixed stable transfer function and Δ is a variable transfer function satisfying $\|\Delta\|_\infty < 1$. Furthermore, it is assumed that no unstable poles of P are canceled in forming P_0. Hence if $\|\Delta\|_\infty < 1$, then

$$\left|\frac{P(j\omega)}{P_0(j\omega)} - 1\right| \le |W(j\omega)|, \qquad \forall \omega$$

By plotting the system variations $\left|\frac{P(j\omega)}{P_0(j\omega)} - 1\right|$, for all experimental frequency response estimates of the system $P(j\omega)$, and estimate an upper bound to those variations as a transfer function, the multiplicative uncertainty weighting function $W(s)$ is obtained.

If a set of identification experiments is performed on the nonlinear system, under different operating condition, the linear frequency response estimate results $P(j\omega)$, will form the set $\mathcal{P}$. By finding a nominal fit $P_0(j\omega)$ through these frequency response estimates, the nonlinearities, disturbances and uncertainties of the system will be

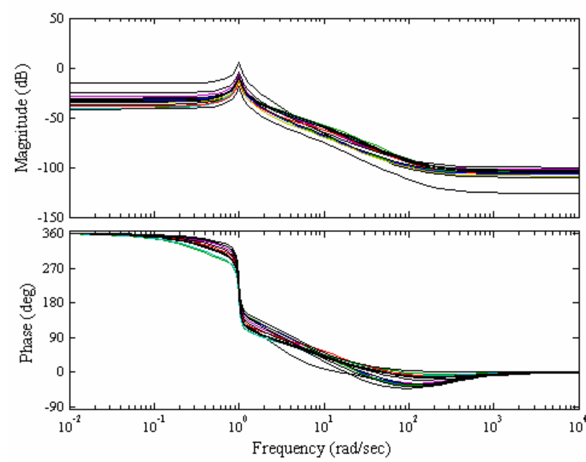

Figure. 2-a. Frequency responses of the estimated system.

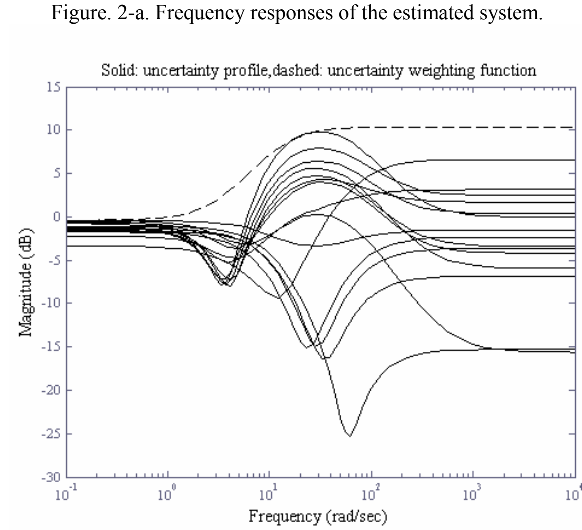

Figure. 2-b. The uncertainty profile and the uncertainty weighting function.

condensed into the perturbation ΔW. It should be noted that the selected nominal system should attain the minimum uncertainty weighting function at least in lower frequencies.

For identification tests, a chirp function generator is used as the excitation input. The reason is that a chirp signal demonstrates an almost flat spectrum over the frequency range of interest. It is also highly persistent excitation. Identification tests are done for different initial arm angles such that the estimated transfer functions are valid representatives of the nonlinear system all over its operating range. The amplitudes of the chirp range from 50% to 100% of their maximum value.

To obtain a lower uncertainty weighting function, it is necessary to control the angular position during the identification experiments, therefore, a simple negative PD feedback of θ is used.

The nominal linear model has been obtained through frequency response estimates depicted in Figure 2-a. The nominal transfer function ζ / V is given by

$$P_0(s) = 3.18 \times 10^{-6} \frac{(s - 65.8)(s - 3.6)(s + 6.7)(s + 200)}{(s + 3.54)(s + 1.73)(s^2 + 0.086s + 1)}$$

which is a stable and non-minimum phase transfer function. It has two slow modes and two fast modes. Having these two types of modes is one of the difficulties of the control design. Another difficulty is the presence of unstable zeroes.

The uncertainty profile $\left|\dfrac{P(j\omega)}{P_0(j\omega)}-1\right|$ and the uncertainty weighting function of this system are depicted in Figure 2-b. It shows that the selected function falls a little lower than the uncertainty level over low frequency. $W(s)$ is

$$W=\frac{2.2s+11}{s+11.5}.$$

The nominal transfer function and uncertainty weighting function of $\dfrac{\zeta}{D}$ have been obtained as follows.

$$P_d(s)=-1.47\times10^{-5}\,\frac{(s-2686)(s+8.1)(s+200)(s+2606)}{(s+1961)(s+8.57)(s^2+0.086s+1)}$$

$$W_d=\frac{5(s+1)}{100}$$

IV. Robust H∞ Control

The control objectives can be defined as robust stability of the closed loop system as well as a satisfactory disturbance rejection and oscillation suppression of the outputs prone to the dominant oscillatory modes while avoiding actuator saturation. Robust controller $C(s)$, will be designed by developing the so-called mixed-sensitivity problem:

$$\gamma_{opt}=\min\left\|\begin{matrix}W_sS\\W_TT\\W_uu\end{matrix}\right\|_\infty<1$$

where $S=\dfrac{p_d(s)}{1+p_0(s)C(s)}$ and $T=\dfrac{P_0(s)C(s)}{1+P_0(s)C(s)}$.

Figure 3 illustrates the block diagram of the setup using multiplicative uncertainty representation, which formulates the abovementioned problem as a standard H∞ problem.

Replacing P_d, sensitivity function will change into $S=1/(1+p_0(s)C(s))$ which is desired, therefore $W_s(s)$ should be chosen in the form of W_1P_d.

More specifically, referring to Figure 3, we would like to design a controller to minimize the following norm:

$$\gamma_{opt}=\min\left\|\begin{matrix}W_TT\\W_1P_dS\\W_uu\end{matrix}\right\|_\infty$$

where W_T is the uncertainty weighting function. The robust stability is guaranteed by minimizing the infinity norm of weighted transfer function from d to z3, which is equivalent to the weighted complementary sensitivity function $\|W_TT\|_\infty<1$ (Small-gain theorem). Weighting functions Ws

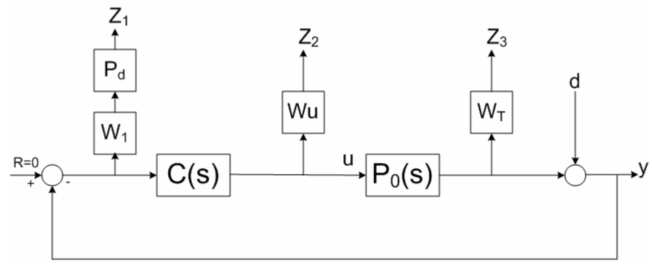

Figure. 3. Closed-loop system as a generalized regulator problem

and W_u are also considered to normalized and assign frequency content of the performance objectives. Response speed and disturbance attenuation is obtained by minimizing the infinity norm of d to z1, or the weighted sensitivity function $\|W_SS\|_\infty<1$. Finally, the infinity norm of d to z2, or weighted control effort transfer function penalizes controllers with high control effort, and provides a media in our optimization to include directly the control effort limitations into the controller synthesis.

Hence, by simultaneously optimization of the infinity norm of the $\gamma_{opt}<1$, all the objectives are satisfied. The problem has been solved for the represented RTAC with an upper bound for control effort corresponding to $W_{u1}=10^{-6}$. The sensitivity weighting function is determined in order to have solution for the mixed sensitivity problem as well as to have maximum reachable bandwidth as following:

$$W_S=\frac{3P_d}{s+10}$$

LMI approach has been employed in the design of the H∞ solution in this problem.

Figure 4 shows magnitude diagram of the sensitivity function by implementing H∞ controller. Since the system dominant oscillatory modes are at $\omega=1rad/\sec$, by decreasing the sensitivity function magnitude in that frequency, the settling performance has been improved.

To analyze the performance of the closed-loop system, the nonlinear model is used in simulations. Closed-loop response is illustrated as dashed line in Figure 6. The result illustrates the effectiveness of the proposed H∞ controller to provide a fast settling time. Figure 7 shows the control torque. Dashed line illustrates large picks in control effort that can damage actuators.

We would like to design a controller to trade off between increasing the settling time and decreasing the control effort.

As mentioned before, the system dominant oscillatory modes are at $\omega=1rad/\sec$. Therefore it is enough to have maximum control effort only around this range. By shaping W_u in order to minimize this transfer function around $\omega=1rad/\sec$, the maximum magnitude of the control effort can be equal to $W_u{}^{-1}$ and it can be limited at the other frequencies. W_{u2} is determined according to

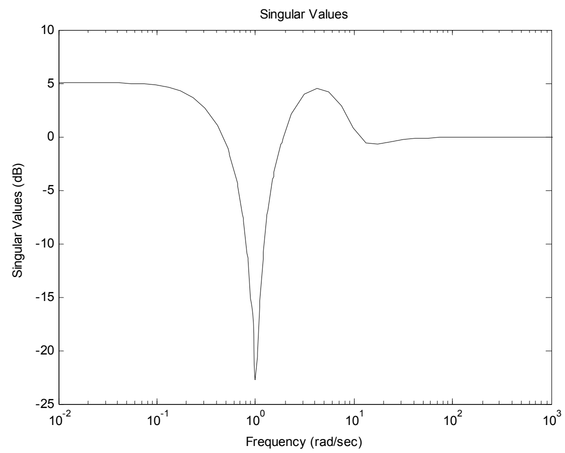

Figure. 4. Singular value plot of sensitivity function.

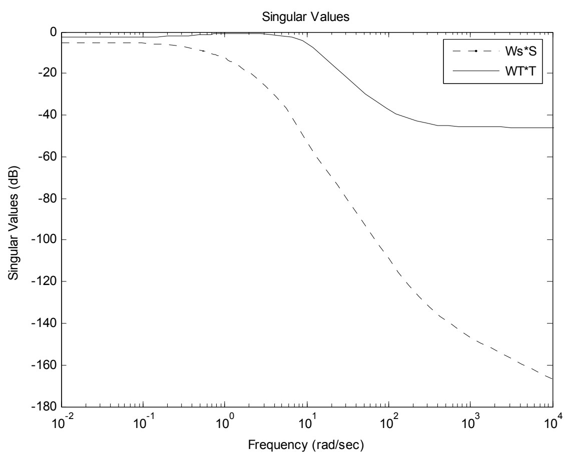

Figure. 5. Singular value plot of $W_T T, W_S S$.

$$W_{u2} = \frac{0.3047s^3 + 0.1807s^2 + 0.3554s + 0.0019}{s^3 + 59.24s^2 + 13.68s + 0.2224}.$$

Figure 8 shows the magnitude diagram of W_{u2}.

Closed-loop response is illustrated as solid line in Figure 6 and control torque is illustrated as solid line in Figure 7. It shows a smooth pattern in control torque in spite of a slow settling time in system response. These results can be obtained out of Table II. It gives the 2-norm of the control effort and settling time of two closed-loop systems which are discussed above.

TABLE II: TWO- NORM OF CONTROL EFFORT AND SETTLING TIME

Controller	$\|u\|_2$	Settling time (sec)
First (W_{u1})	271	13
Second (W_{u2})	141	40

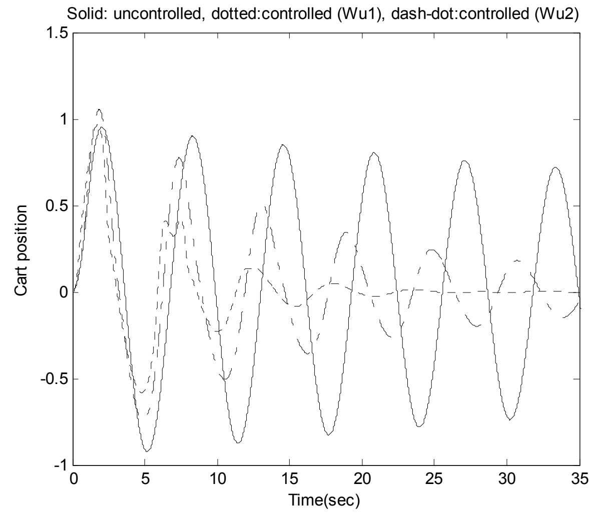

Figure. 6. Free responses of open-loop system (solid), closed-loop system with W_{u1} (dotted) and closed-loop system with W_{u2} (dash-dot).

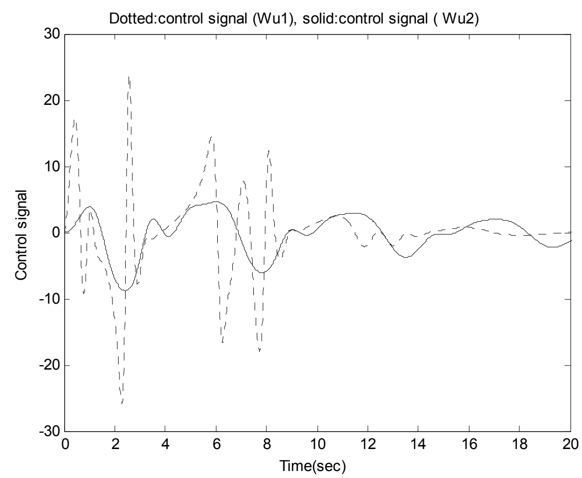

Figure 7. Control torque when W_{u1} is used (dotted), control torque when W_{u2} is used (solid).

Now consider the closed-loop systems in presence of a random measurement noise. Figure 9 shows the closed- loop response in this situation. Although noise does not influence output response, it affects control effort considerably. Table III compares the 2-norm of the control effort in both closed-loop systems.

TABLE III: TWO- NORM OF CONTROL EFFORT AND SETTLING TIME IN PRESENCE OF NOISE

Controller	$\|u\|_2$	Settling time (sec)
First (W_{u1})	2816	13
Second (W_{u2})	200	40

It is evident that a reasonable choice for W_u, effectively limits control effort in presence of measurement noise.

Satisfaction of the performance requirement by the

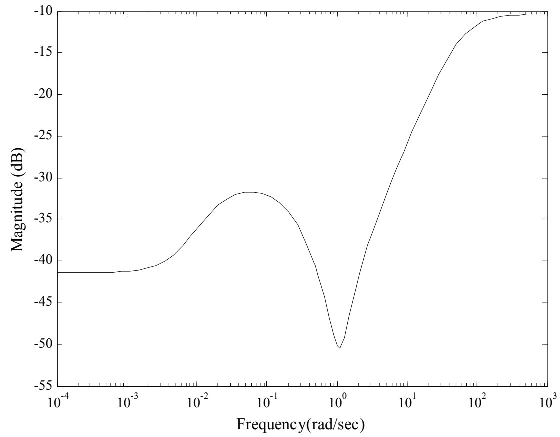

Figure 8. Magnitude diagram of W_{u2}

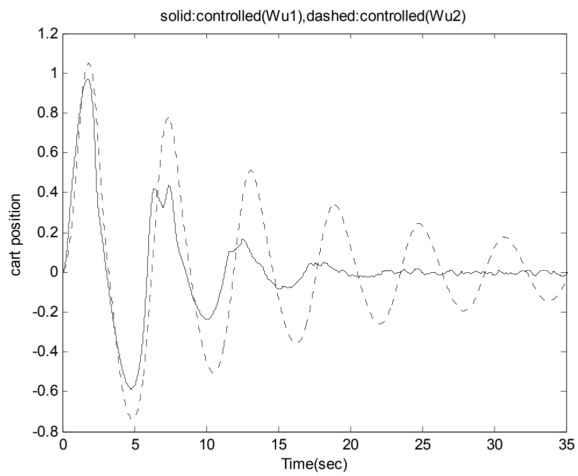

Figure. 9. Closed-loop system response in presence of noise

consequent H$_\infty$ controller is illustrated in frequency and time domain simulations. According to this simulation study, the robust control design is entirely successful in satisfying the required control objectives with a limited control effort.

V. Conclusions

Several nonlinear control design approaches for the RTAC problem have been proposed. Most of these techniques do not address multi-objective solutions directly. Furthermore, they may lead to equations that are hard to solve numerically, and most of these approaches do not explicitly take into account modeling uncertainty.

In this paper an H$_\infty$ controller was developed for this nonlinear problem to tackle the above objectives. Use of a rotational proof mass actuator for stabilizing translational motion was investigated by implementing an H$_\infty$ controller.

The basic method proposed in this paper is how to find a linear controller for the given nonlinear system such that the closed-loop system meets a number of specifications. Then an appropriate controller was designed, while the nonlinear system was estimated as a nominal linear system and unmodeled dynamics and derivations of the nonlinear system from the nominal model were encapsulated in the model uncertainty. The nominal model was obtained through an unstructured set of linear frequency response estimates by performing identification experiments on the nonlinear system under different operating condition. This representation provided sufficient information to build a robust controller for the RTAC system. The H$_\infty$ controller was designed by solving the mixed-sensitivity problem for a fast settling time and disturbance attenuation objectives, respecting the actuator saturation limits.

The closed loop performance of the system in frequency and time domain simulations were examined and the performance of the closed-loop system was evaluated. It was illustrated that the closed-loop system retains robust stability, while improving the desirable performance exceptionally well.

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