Forward Kinematics of A Macro–Micro Parallel Manipulator

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Abstract— In this paper the kinematic analysis of a macro–micro parallel manipulator is studied in detail. The manipulator architecture is a simplified planar version adopted from the structure of Large Adaptive Reflector (LAR), the Canadian design of next generation giant radio telescopes. This structure is composed of two parallel and redundantly actuated manipulators at macro and micro level, which both are cable-driven. Inverse and forward kinematic analysis of this structure is presented in this paper. It is shown that unique closed form solution to the inverse kinematic problem of such structure exists. However, the forward kinematic solution is derived using numerical methods, and simulation results are reported to illustrate the integrity and accuracy of the solution.

Index Terms— Parallel manipulator, inverse kinematics, forward kinematics, macro-micro robot, redundancy.

I. Introduction

An international consortium of radio astronomers and engineers have agreed to develop technologies to build the Square Kilometer Array (SKA), a cm-to-m wave radio telescope for the next generation of cosmic phenomena investigations [4], [9], [14]. The Canadian proposal for the SKA design consists of an array of 30-50 individual antennas whose signals are combined to yield the resolution of a much larger antenna. Each of these antennas would use the Large Adaptive Reflector (LAR) concept put forward by a group led by the National Research Council of Canada and supported by university and industry collaborators [3], [5]. The LAR design is applicable to telescopes up to several hundred meters in diameter. However, design and construction of a 200-m LAR prototype is pursued by the National Research Council of Canada. Figure 1 is an artist's concept of a complete 200-m diameter LAR installation, which consists of two central components. The first is a 200 m diameter parabolic reflector with a focal length of 500 m, composed of actuated panels supported by the ground. The second component is the receiver package which is supported by a tension structure consisting of multiple long tethers and a helium filled aerostat.

The challenging problem in this system is accurately positioning of the feed (receiver) in the presence of wind disturbances. For the positioning structure of the receiver a macro–micro manipulator design is proposed, in which at both macro and micro levels two redundantly actuated cable-driven parallel manipulators are used. At the macro

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level the receiver is moved to various locations on a circular hemisphere and its positioning is controlled by changing the lengths of eight tethers with ground winches. The cable driven macro manipulator used in this design, which is called the Large Cable Mechanism (LCM), is in fact a 6DOF cable driven redundantly actuated manipulator. At micro level a Confluence Point Mechanism (CPM) is designed to perform the final small scale corrections at high frequencies. The CPM requires six degrees of freedom and is also a redundantly actuated cable driven parallel manipulator. CPM base is attached to the LCM structure, and its moving platform accurately positions the telescope feed (receiver). This design is intended to keep the moving platform of the CPM, and hence the feed, close to stationary and pointed toward the center of the reflector.

Since in the design of LCM/CPM, a macro–micro structure is proposed in which at each level a redundantly actuated parallel manipulator is used for extreme positioning accuracy, this paper is intended to study the kinematic analysis of such macro-micro structures in detail. In the LCM/CPM structure, two parallel manipulators with six degrees of freedom are used at macro and micro levels. However, in order to keep the analysis complexity at a managing level, while preserving all the important analysis elements, a simplified version of the macro-micro structure is considered in this paper. This structure is composed of two parallel 4RPR mechanisms, both actuated by cables. In this simplified structure, although a planar version of the mechanisms are considered, two important feature of the original design namely the actuator redundancy for each subsystem and the macro–micro structure of the original design are employed.
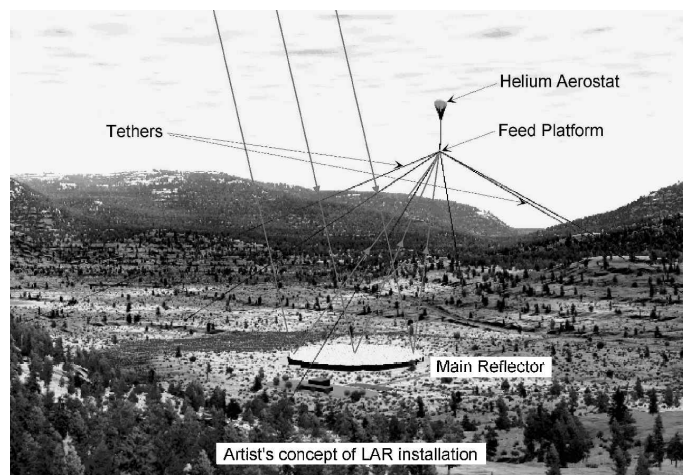

Fig. 1. An artists concept of a complete 200-m diameter LAR installation.

In the kinematic analysis of parallel manipulator literature, mostly 6 DOF parallel mechanisms based on the Stewart-Gough platform [1], [8], or Hexapods [15], are analyzed . For cable-driven manipulator at least one degree redundancy is necessary to make sure that in all configurations all the cables are in tension [6]. Complete kinematic modeling of such mechanisms have not received much attention so far, and is still regarded as a challenging problem in parallel robotics research. It is known that unlike serial manipulators, inverse position kinematics for parallel robots is usually simple and straight-forward. In most cases limb variables may be computed independently using the given pose of the moving platform, and the solution in most cases is uniquely determined. However, forward kinematics of parallel manipulators is generally very complicated, and its solution usually involves systems of nonlinear equations, which are highly coupled and in general have no closed form and unique solution. Different approaches are given in the literature to solve this problem either in general [11], or in special cases [10], [2]. Analysis of two such special 3 DOF constrained mechanisms have been studied in [7], [13]. In general, different solutions to the forward kinematics problem of parallel manipulators can be found using numerical [11] or analytical approaches [10]. Among the various research areas performed in parallel manipulators, very few have analyzed parallel manipulators with macro–micro structure [12], [16]. It is interesting to note that in [16] macro–micro structure is also proposed for a Chinese concept of LAR design. Due to the potential attraction of macro–micro structure in the LAR application, a thorough analysis on the kinematics of the described macro–micro parallel manipulator is presented in this paper.

II. MECHANISM DESCRIPTION

The architecture of the planar macro–micro $2\times$4RPR parallel manipulator considered for our studies is shown in figure 2. This manipulator consists of two similar 4RPR
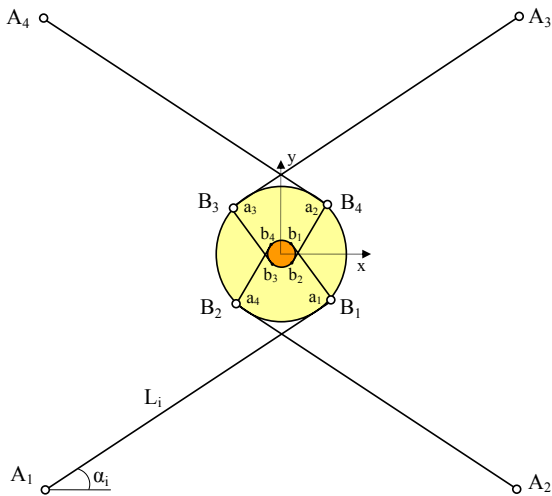

Fig. 2. The schematics of 2x4RPR mechanism employed for the analysis of LCM/CPM structure

parallel structure at macro and micro level. At each level the moving platform is supported by four limbs of identical kinematic structure. At macro level each limb connects the fixed base to the macro manipulator moving platform by a revolute joint (R) followed by a prismatic joint (P) and another revolute joint (R). The kinematic structure of a prismatic joint is used to model the elongation of each cable-driven limb. In order to avoid singularities at the central position of the manipulator at each level, the limbs are considered to be crossed. At micro level similar 4RPR structure is used, however, the base points of the macro manipulators are located on the moving platform of the macro manipulator. Angular positions of base and moving platform attachment points are given in table I, in which capital letters are used to describe the macro manipulator variables, while small letters are reserved for that of micro manipulator. In this representation, A_i denote the fixed base points of the limbs, B_i denote point of connection of the limbs on the moving platform, L_i denote the macro limb lengths, ℓ_i denote the micro limb lengths, α_i denotes the macro limb angles, and β_i denotes the absolute micro limb angles. The position of the center of the moving platform G, at macro level is denoted by $G = [x_G, y_G]$, and the final position of the micro manipulator g is denoted by $g = [x_g, y_g]$. The orientation of the macro manipulator moving platform is denoted by ϕ, and the orientation of the micro manipulator with respect to the fixed coordinate frame is denoted by ψ. The geometry of the fixed and moving platform attachment points, A_i, B_i, a_i and b_i's are considered to be arbitrary in the analysis, and they are not necessarily coincident. However, the parameter used in the simulations of the system is adopted from LCM/CPM design and is symmetrical as given in table I.

III. KINEMATIC ANALYSIS OF MACRO MANIPULATOR

In this section, the kinematic of the Macro manipulator is studied in detail. In this analysis first the inverse and forward kinematic analysis of the macro manipulator is elaborated in detail, and then the kinematics of the macro–micro assembly is analyzed. In order to verify the formulations, in section V the simulation results for computation of inverse kinematics and forward kinematics of the macro-micro manipulator is presented, and the accuracy of numerical solution to forward kinematics is verified.
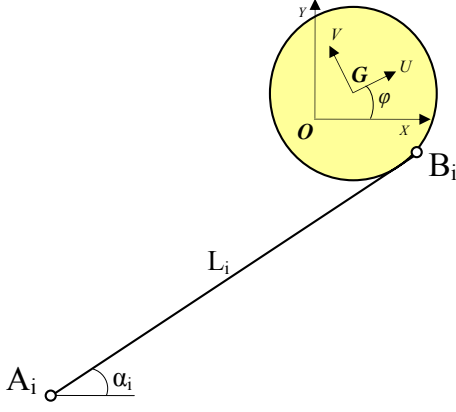
Fig. 3. Kinematic configuration of the macro manipulator

A. Inverse Kinematics

For inverse kinematic analysis of the macro manipulator, it is assumed that the position and orientation of the moving platform $\boldsymbol{X} = [x_G, y_G, \phi]^T$ is given and the problem is to find the joint variable of the macro manipulator, $\boldsymbol{L} = [L_1, L_2, L_3, L_4]^T$. For the purpose of analysis and as it is illustrated in figure 3, a fixed frame $O : xy$ is attached to the fixed base at the point O, the center of the base point circle which passes through $A_i's$, and another moving coordinate frame $G : UV$ is attached to the macro moving platform at point G. Furthermore, assume that the point A_i lie at the radial distance of R_A from point O, and the point B_i lie at the radial distance of R_B from point G in the xy plane, when the manipulator is at central location. In order to specify the geometry of the macro manipulator define θ_{Ai}, θ_{Bi} as the absolute angle of the points A_i and B_i at the central configuration of the macro manipulator, with respect to the fixed frame O. Moreover, notice that the moving platform position and orientation is denoted by the location vector $\boldsymbol{X} = [x_G, y_G, \phi]^T$ at each instant. Let's define the instantaneous orientation angle of B_i's as:

$$\phi_i = \phi + \theta_{Bi}. \tag{1}$$

For each limb, $i = 1, 2, \cdots, 4$, the position of the base points, A_i is given by,

$$A_i = [R_A \cos(\theta_{Ai}), R_A \sin(\theta_{Ai})]^T \tag{2}$$

From the geometry of the macro manipulator as illustrated in figure 3, the loop closure equation for each limb, $i = 1, 2, \cdots, 4$, can be written as,

$$\overrightarrow{A_iG} = \overrightarrow{A_iB_i} + \overrightarrow{B_iG}. \tag{3}$$

Rewriting the vector loop closure component–wise,

$$x_G - x_{Ai} = L_i \cos(\alpha_i) - R_B \cos(\phi_i) \tag{4}$$
$$y_G - y_{Ai} = L_i \sin(\alpha_i) - R_B \sin(\phi_i), \tag{5}$$

in which α_i are the absolute limb angles. To solve the inverse kinematic problem it is required to eliminate α_i's from the above equation and solve for L_i's. This can be

accomplished by reordering and adding the square of both sides of equations 4 and 5. Hence, the limb lengths are uniquely determined by:

$$L_i = \left[(x_G - x_{Ai} + R_B \cos(\phi_i))^2 + (y_G - y_{Ai} + R_B \sin(\phi_i))^2 \right]^{1/2} \tag{6}$$

Furthermore the limb angles α_i's can be determined by the following equation

$$\alpha_i = \operatorname{atan2} \left[(y_G - y_{Ai} + R_B \sin(\phi_i)), (x_G - x_{Ai} + R_B \cos(\phi_i)) \right] \tag{7}$$

Hence, corresponding to each given macro manipulator location vector $\boldsymbol{X} = [x_G, y_G, \phi]^T$, there is a unique solution for the limb length L_i's, and limb angles α_i's.

B. Forward Kinematics

For the forward kinematic problem, the joint variable L_i's are given and the position and orientation of the macro moving platform $\boldsymbol{X} = [x_G, y_G, \phi]^T$ are to be found. This can be accomplished by eliminating x_G, and y_G from equations 4 and 5 as follows. Let's define two intermediate variables $x_i = -x_{Ai} + R_B \cos(\phi_i)$ and $y_i = -y_{Ai} + R_B \sin(\phi_i)$ in order to simplify the calculations and consider the square of equation 6:

$$L_i^2 = (x_G + x_i)^2 + (y_G + y_i)^2 \tag{8}$$

We first try to solve for x_G and y_G. This can be accomplished by reordering the equation 8 into:

$$x_G^2 + y_G^2 + r_i x_G + s_i y_G + u_i = 0, \tag{9}$$

in which for $i = 1, 2, \cdots, 4$,

$$\begin{cases} r_i = 2x_i \\ s_i = 2y_i \\ u_i = x_i^2 + y_i^2 - L_i^2 \end{cases} \tag{10}$$

Equation 9 provides four quadratic relations for x_G and y_G for $i = 1, 2, \cdots, 4$. Subtracting each two equations from each other results into linear equation in terms of x_G and y_G.

$$\boldsymbol{A} \cdot \begin{bmatrix} x_G \\ y_G \end{bmatrix} = \boldsymbol{b}, \tag{11}$$

in which,

$$\boldsymbol{A} = \begin{bmatrix} r_1 - r_2 & s_1 - s_2 \\ r_2 - r_3 & s_2 - s_3 \\ r_3 - r_4 & s_3 - s_4 \\ r_4 - r_1 & s_4 - s_1 \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} u_2 - u_1 \\ u_3 - u_2 \\ u_4 - u_3 \\ u_1 - u_4 \end{bmatrix}. \tag{12}$$

The components of $\boldsymbol{A}$ and $\boldsymbol{b}$ are all function of ϕ. Note that only two equation of the above could be used to evaluate x_G and y_G in terms of ϕ, but all possible four equations are used in here to have tractable solutions even in case of configuration singularities in the manipulator. Over-determined equation 11 can be solved using pseudo-inverse,

$$\begin{bmatrix} x_G \\ y_G \end{bmatrix} = \boldsymbol{A}^\dagger \cdot \boldsymbol{b}, \tag{13}$$

in which, $A^\dagger$ denotes the pseudo-inverse solution of A, and for an over–determined set of equations is calculated from,

$$A^\dagger = (A^T A)^{-1} A^T. \tag{14}$$
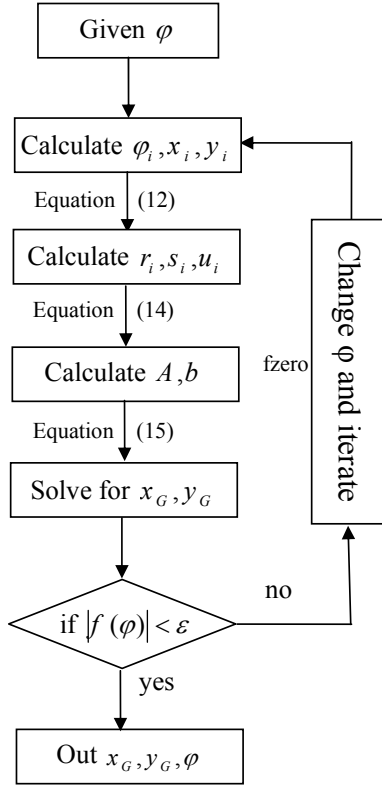
Fig. 4. Flowchart of iterative routine used to solve the forward kinematics of the macro manipulator

Flowchart text:

Given φ

Calculate φ_i, x_i, y_i

Equation (12)

Calculate r_i, s_i, u_i

Equation (14)

Calculate A, b

Equation (15)

Solve for x_G, y_G

if $|f(\varphi)| < \varepsilon$

no → Change φ and iterate → fzero

yes

Out x_G, y_G, φ

There exist tractable numerical methods such as Householder reflection, or Choleski decomposition to evaluate the pseudo–inverse. Note that the components of A and b are all function of ϕ, and for a given ϕ equation 13 gives the least square solution for x_G and y_G. In order to solve for ϕ this solution can be substituted in equation 9, and a function of the only unknown variable ϕ is obtained:

$$f_i(\phi) = x_G^2 + y_G^2 + r_i x_G + s_i y_G + u_i \quad (15)$$

$$f(\phi) = \sum_{i=1}^{4} f_i(\phi) \quad (16)$$

Numerical methods using iterative search routines[1] can be used to find the final solution of $f(\phi) = 0$. Notice that any function $f_i(\phi) = 0$ could be used to find ϕ which solves the forward kinematics, however, the summation of all four possible equations are used in here to have tractable solution even in case of singularity in manipulator configuration. The flowchart given in figure 4 reveals the details of the iterative method used to find the forward kinematic solution.

Multiple solution may exist for the equation $f(\phi) = 0$, and in order to avoid jumps in the forward kinematic solutions, in the numerical routine the solution at previous iteration is used for the search of the next solution. Simulation results detailed in section V illustrate the effectiveness, and accuracy of the numerical routines used to solve forward kinematics.
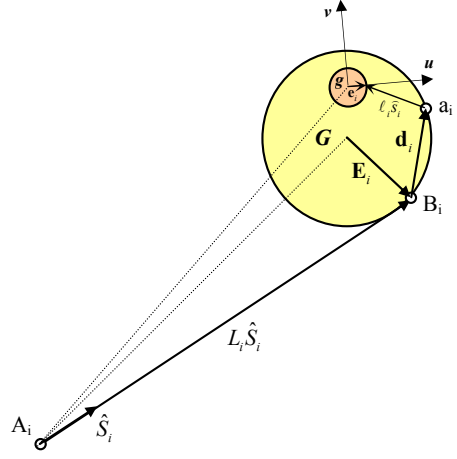
[1] fzero function of Matlab is used

Fig. 5. Vectors definitions of the macro–micro manipulator

If the limb angles α_i's are needed to be found, equation 7 can be used directly, by substituting x_G, y_G and ϕ.

IV. MACRO–MICRO KINEMATICS

A. Inverse Kinematics

For inverse kinematic analysis of the macro–micro manipulator, it is assumed that the location vector of the macro moving platform $\boldsymbol{X} = [x_G, y_G, \phi]^T$ and that of the micro moving platform $\boldsymbol{x} = [x_g, y_g, \psi]^T$ is given. The problem is then to find the joint variable of the macro manipulator, $\boldsymbol{L} = [L_1, L_2, L_3, L_4]^T$ and and that of the micro manipulator $\boldsymbol{\ell} = [\ell_1, \ell_2, \ell_3, \ell_4]^T$, respectively. As explained before capital letters are reserved for the macro manipulator variables, while small letters are used to denote micro manipulator ones.

For the purpose of analysis and as it is illustrated in figure 5, consider micro moving coordinate frame $g : uv$ which is attached to the micro manipulator moving platform at point g, the center of the micro moving platform. Furthermore, assume that the point a_i lie at the radial distance of R_a from point G, and the point b_i lie at the radial distance of R_b from point g in the uv plane. Similar to macro manipulator, define θ_{ai}, θ_{bi} as the absolute angle of the points a_i and b_i at the central configuration of the macro manipulator, with respect to the fixed frame O. Although in the analysis arbitrary angles are used for the macro and micro attachment points, in the simulations a symmetrical design is considered for this manipulator and which are given in table I.

Since the structure of macro and micro manipulators are the same, the inverse kinematic solution of the macro–micro system is similar to that of the macro manipulator, and can be found in the following sequence. Starting with the macro manipulator inverse kinematic solution, for a given $\boldsymbol{X} = [x_G, y_G, \phi]^T$, the macro joint variables L_i's, and α_i's are determined from equations 6 and 7, respectively. Then the macro moving platform points coordinate B_i's and a_i's are determined from the following geometrical relation:

$$B_i = [x_G + R_B \cos(\theta_{Bi} + \phi), y_G + R_B \sin(\theta_{Bi} + \phi)]^T \quad (17)$$

$$a_i = [x_G + R_a \cos(\theta_{ai} + \phi), y_G + R_a \sin(\theta_{ai} + \phi)]^T \quad (18)$$

Finally, assuming that the micro manipulator position and orientation $[x_g, y_g, \psi]^T$ is given the micro manipulator joint variables can be determined by the following equations, similar to that to the macro manipulator equations 6, and 7.

$$\ell_i = \left[(x_g - x_{ai} + R_b \cos(\psi_i))^2 + (y_g - y_{ai} + R_b \sin(\psi_i))^2 \right]^{1/2} \tag{19}$$

in which,

$$\psi_i = \psi + \theta_{bi}. \tag{20}$$

Furthermore the micro manipulator limb angles β_i's can be determined from the following equation.

$$\beta_i = \operatorname{atan2} \left[(y_g - y_{ai} + R_b \sin(\psi_i)), (x_g - x_{ai} + R_b \cos(\psi_i)) \right] \tag{21}$$

B. Forward Kinematics

Forward kinematic solution of the macro–micro manipulator can be solved by parallel computation, too. For forward kinematic problem the joint variable L_i's and ℓ_i's are given, and the position and orientation of the macro moving platform $X = [x_G, y_G, \phi]^T$, and that of micro manipulator $x = [x_g, y_g, \psi]^T$ are to be found. Similar to equations 4 and 5 for micro manipulator the vector loop closure results into:

$$\ell_i \cos(\beta_i) = x_g - x_{ai} + R_b \cos(\psi_i) \tag{22}$$

$$\ell_i \sin(\beta_i) = y_g - y_{ai} + R_b \sin(\psi_i), \tag{23}$$

Hence similar numerical solution for the macro manipulator can be determined with the identical equation used for macro manipulator, namely equations 11 and 16, replacing the following variables:

$$\begin{cases} L_i, \alpha_i & \longrightarrow & \ell_i, \beta_i \\ \phi, \phi_i & \longrightarrow & \psi, \psi_i \\ x_G, y_G & \longrightarrow & x_g, y_g \\ x_A, y_A & \longrightarrow & x_a, y_a \\ x_B, y_B & \longrightarrow & x_b, y_b \\ R_A, R_B & \longrightarrow & R_a, R_b \end{cases} \tag{24}$$

The sequence of macro–micro forward kinematic solution is to first solve macro manipulator forward kinematic. This can be accomplished as illustrated in the flowchart of figure 4. Similarly ψ is numerically calculated solving equation 16, by substituting variables in 24. Finally x_g and y_g can be solved using equation 13 whose variables are substituted by 24. It is noticeable, that due to the similar structure of macro and micro manipulator, complete kinematic solution of the system can be calculated in parallel for macro and micro manipulators. Moreover, identical subroutines can be used to solve the individual kinematic problems more efficiently.

V. KINEMATICS VERIFICATION

In order to verify the accuracy and integrity of the numerical solutions, a typical desired trajectory for the macro–micro manipulator is generated and is shown in figure 6. Identical trajectories for macro and micro manipulator moving platform is considered in this case $x_D = x_d$, to examine the minimal motion of the micro manipulator. The inverse kinematic solution of the macro–micro manipulator is found for this trajectory by equations 6 and 19, and the
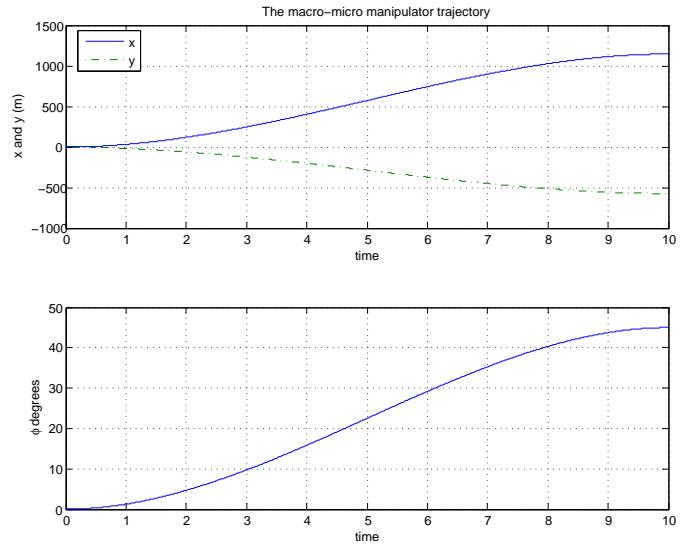

Fig. 6. Desired trajectory for macro and micro absolute position and orientation

macro manipulator limb lengths are uniquely derived and illustrated in figure 7. As it is seen in this figure there is no relative motion in the micro manipulator, as expected due to the minimal micro motion criteria used in the desired trajectory generation. Macro limb length change however, according to the desired trajectory variation. In order to verify the accuracy and integrity of the numerical solutions the forward kinematic solution for the macro and micro manipulators are applied for the obtained limb lengths, and by the sequence depicted in flowchart figure 4. The difference between the numerical solutions and the desired motion is illustrated in figure 8. It is observed that the final forward kinematic solution are completely matching the given desired trajectories to the accuracy of 10^{-13} for macro manipulator and 10^{-12} for the micro manipulator. It should be noted
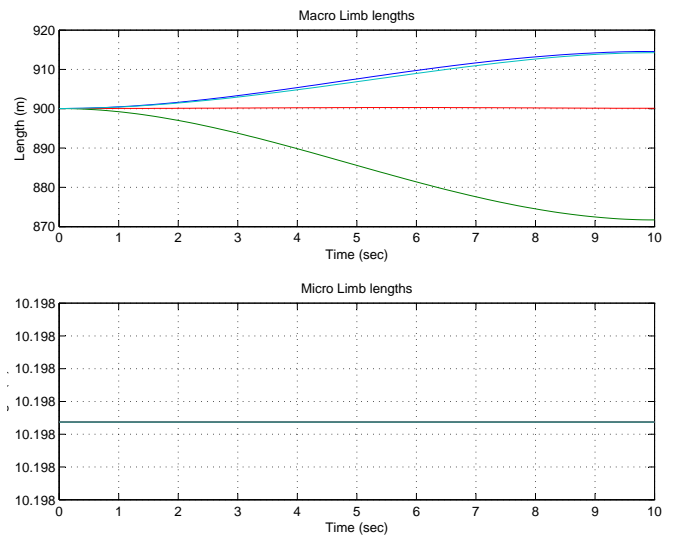

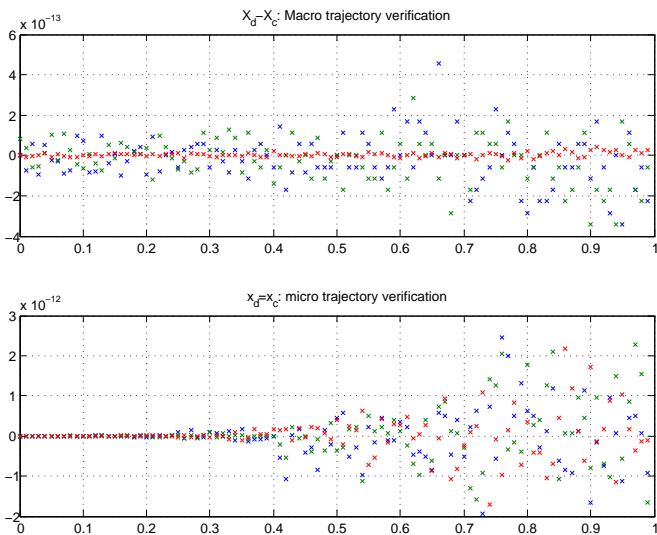Fig. 7. Inverse kinematic solution for the given trajectory

Fig. 8. Forward kinematics verification for macro and micro manipulators

that the forward kinematic solutions are not unique, and to avoid converging to the other solutions at each step of time the last step forward kinematic solution is used as the initial guess for the next step iteration. By this means the numerical solution converge to the right solution in all the examined points. Moreover, it is noticeable, that due to the similar structure of macro and micro manipulator, complete kinematic solution of the system can be calculated in parallel for macro and micro manipulators and identical subroutines are used to solve the individual kinematic problems very efficiently.

VI. Conclusions

In this paper the kinematic analysis of a macro–micro redundantly actuated parallel manipulator is studied in detail. The analyzed manipulator is a planar version adopted from the structure of Large Adaptive Reflector (LAR), the Canadian design of next generation giant radio telescopes. In the LAR design the telescope receiver package is supported by a tension structure consisting of multiple long tethers and a helium filled aerostat. The positioning structure of the receiver is designed as a macro–micro manipulator, in which at both macro and micro levels two redundantly actuated cable-driven parallel manipulators are used and both manipulators experience 6DOF motion in the space. The planar structure used in this paper, is a simplified version of LAR design, in which the two important feature of the main mechanism, namely macro–micro structure, and actuator redundancy are preserved in the planar structure. This structure is composed of two 3DOF parallel redundant manipulators at macro and micro level, both actuated by cables. In this paper the kinematic analysis of this system is presented. It is shown that unique closed form solution to the inverse kinematic problem of such structure exists. Moreover, the forward kinematic solution is derived using numerical methods. Some simulations are presented for the macro–micro manipulators, by which the integrity and accuracy of numerical approaches used to derive the forward kinematic solution for this manipulator is presented.

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