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Geometrical Workspace Analysis of a Cable-Driven Redundant Parallel Manipulator: KNTU CDRPM

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***Abstract*— KNTU CDRPM is a cable driven redundant parallel manipulator, which is under investigation for possible high speed and large workspace applications. This newly developed mechanisms have several advantages compared to the conventional parallel mechanisms. Its rotational motion range is relatively large, its redundancy improves safety for failure in cables, and its design is suitable for long-time high acceleration motions. In this paper, collision-free workspace of the manipulator is derived by applying fast geometrical intersection detection method, which can be used for any fully parallel manipulator. Implementation of the algorithm on the Neuron design of the KNTU CDRPM leads to significant results, which introduce a new style of design of a spatial cable-driven parallel manipulators. The results are elaborated in three presentations; constant-orientation workspace, total orientation workspace and orientation workspace.**

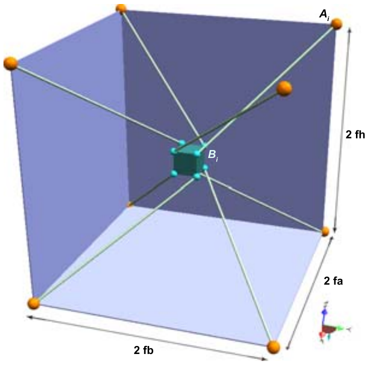

Fig. 1. The KNTU CDRPM, a perspective view

I. INTRODUCTION

Nowadays, *parallel manipulator* (PM)'s applications are significantly increasing. A closed chain kinematics between fixed and moving platforms, makes the end-effector's motions more stiff and high-accelerated by fully-constraining the end-effector[1]. In a parallel mechanism, each limb contributes in the movement of the payload. Thus, it can carry more payload to moving mass ratio which is suitable for special applications such as the popular Stewart-Gough platform in flight simulator [2]. On the other hand, a large motion of the linear actuator of the rigid links of a parallel manipulator leads to a small displacement of the end-effector. Thus, high precision is achieved relative to the serial manipulators [3]. However, additional to hardship of production [4] and control [5], there are some challenges to use PM structures in a wide range of applications. The main limitations of the PMs are limited workspace [6] and singularity regions within the workspace [7]. Using an electric powered cable-driven actuator, as an alternative for the massive and stroke-limited linear actuator, can extend the workspace of the manipulator inevitably large even within the size of a Football stadium [8], or a platform of large adaptive reflector with $2km^2$ footprint [9]. By locating the driver units on the fixed platform, only light-weight cables' mass is added to the mass of the end-effector. Therefore, manipulators such as a RoboCrane can carry large forces as the weight of a shipping cargo with the use of a CDRPM structure [10]. Moreover, CDRPM saves heredity of PMs about acceleration capabilities in addition to enlarged workspace. It makes CDRPM a suitable platform of virtual acceleration in virtual reality tasks [11]. However, a cable can only carry tension forces, and to guarantee that the cables are always under tension different solutions are advised. In some cases the end-effector is suspended from the cables and by use of the gravity force or any other passive force against the moving platform, this is ensured [9]. Another more applicable solution for high acceleration applications, is to use redundant actuators, and to resolve the redundancy to ensure positive tension in all the cables. This can be performed in a fully–constrained or over–constrained moving platform [12], but with more difficulties to analyse the geometrical or force feasible workspace.

The KNTU CDRPM is thus designed based on such structure with an 8 actuated 6 degrees of freedom cable driven redundant parallel manipulator. This manipulator is under investigation for possible high speed and wide workspace applications such as virtual acceleration generator in the K.N. Toosi University of Technology. This proposed design has significant advantages compared to the conventional parallel mechanisms. Its rotational motion range is relatively large, its 2 degrees of redundancy improves safety for failure in cables, and makes the design suitable for high acceleration motions. A special design for the KNTU CDRPM is suggested as shown in figure 1, which is called "Neuron" in this paper, that satisfies the possibility of tension forces in all the cables. The design and implementation of the KNTU CDRPM require deep investigation in various fields. The basis step is to analyze the geometrical workspace for the design, which is fully elaborated in this paper. Due to the importance of the workspace analysis, this issue is reported for various parallel manipulators in the literature. Nevertheless, there are only few research results reported on the cable driven parallel manipulators. In general, two important subjects of force feasibility, and geometrical collision avoidance is required to be carefully examined in the

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design of a CDRPM. In a few comprehensive studies the first subject is examined in detail [13],[14]. In this paper by using a special design, only geometrical workspace of CDRPMs is needed to be studied in detail.

In most of the reported research results only force feasibility in the workspace is analyzed, and through this analysis the boundaries of the cable driven robot workspace is determined [15]. In those cases the analysis of self collision is ignored, since such possibility is diminished through the design. Nevertheless, this is accomplished by enforcing stringent boundaries into the force feasible workspace. However, in this paper it is proposed to Design the KNTU CDRPM based on collision avoidance scheme, and by opposite positioning of the attachment points, force feasibility is achieved within the whole determined geometric workspace of the robot. This leads to a significant change in the structure of the KNTU CDRPM's compared to that to the other reported designs. Using two degrees of redundancy and inherent cross cabling and opposite positioning of cable attachment points result into a plausible force feasible regime in the entire workspace. On the other hand, the main analysis concerns is to maximize the collision free workspace of the robot, which is fully addressed in this paper. It should be mentioned that only few papers have reported primary methods for the determination of collisions in parallel manipulators [16]. Similar to our design, in this manipulator a novel and inventive idea is used in the attachment point arrangements, and workspace analysis is reduced to cable collision detection instead of force closure limitations.

In this paper the inverse kinematics of the proposed manipulator is derived and reported. Then, the conditions for collision detection are determined, which can be numerically evaluated by the defined algorithms within the geometrical limits of the cable driven parallel manipulator. Next, workspace of the KNTU CDRPM is examined and extended for different attachment points. The proposed method of analysis of the collision free workspace results into a larger workspace in terms of the reachable position and orientation. Tracking of such analysis introduces a new vision in design of cable driven parallel manipulators, which can be further examined and developed by other colleague researchers in the field of cable driven-parallel manipulator design.

II. KINEMATICS

A. Mechanism Description

The KNTU Cable Driven Redundant Parallel Manipulator is illustrated in figure 1. This figure shows a spatial six degrees of freedom manipulator with two degrees of redundancy actuated by eight identical cable limbs. The moving platform is illustrated as a rectangular box in here for simplicity, However, in the analysis the attachment points can be arbitrarily chosen. For the purpose of analysis, two cartesian coordinate systems $A(x, y, z)$ and $B(u, v, w)$ are attached to the fixed base and moving platform. Points $A_1, A_2, \cdots, A_8$ lie on the fixed cubic frame and $B_1, B_2, \cdots, B_8$ lie on the moving platform. The origin O of the fixed coordinate system is located at the centroid of the cubic fixed frame. Similarly, the origin G of the moving coordinate system is located at centroid of the cubic moving platform. The transformation from the moving platform to the fixed base can be described by a position vector $\overrightarrow{g} = \overrightarrow{OG}$ and a 3×3 rotation matrix ${}^AR_B$. Consider a_i and ${}^Bb_i$ be the position vectors of points A_i and B_i in the coordinate system A and B, respectively. Although in the analysis of the KNTU CDRPM, all the attachment points, can be arbitrarily chosen, the geometric parameters given in table I is used in the simulations.

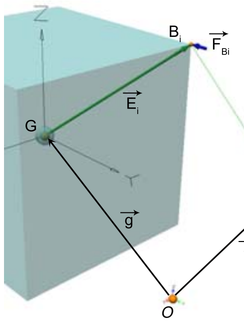

Fig. 2. The i'th attachment point on the moving platform and its related vectors

B. Inverse Kinematics

Similar to other parallel manipulators, CDRPM has a rather complicated forward kinematics [17]. Therefore, the collision free workspace cannot be studied just in the joint space which is more convenient to include the actuator limits. In this section, the kinematics of the system is studied in detail in order to determine the joint space parameters relations to the workspace parameters. As illustrated in figure 1, the B_i points lie at the vertexes of a cube. For inverse kinematic analysis of the cable driven parallel manipulator, it is assumed that the position and orientation of the moving platform $x = [x_G, y_G, z_G]^T$, ${}^AR_B$ is given and the problem is to find the joint variable of the CDRPM, $\boldsymbol{L} = [L_1, L_2, \ldots, L_8]^T$. From the geometry of the manipulator as illustrated in figure 2, the loop closure equation for each limb, $i = 1, 2, \ldots, 8$, can be written as,

$$ {}^A\overrightarrow{A_iB_i} + {}^A\overrightarrow{a_i} = {}^A\overrightarrow{g} + {}^A\boldsymbol{R}_B({}^B\overrightarrow{b_i}) \tag{1} $$

The length of the i'th limb is obtained through taking the square root of dot product of the vector $\overrightarrow{A_iB_i}$ with itself. Therefore, for $i = 1, 2, \ldots, 8$

$$ L_i = \left\{ [\boldsymbol{g} + \boldsymbol{E}_i - \boldsymbol{a}_i]^T [\boldsymbol{g} + \boldsymbol{E}_i - \boldsymbol{a}_i] \right\}^{\frac{1}{2}} \tag{2} $$

in which the corresponding vectors are shown in figure 2. If the solution of L_i becomes a complex number, then the location of the moving platform is not reachable. Along each cable, a unit vector is defined as below:

$$ \hat{\boldsymbol{S}}_i = \frac{\overrightarrow{A_iB_i}}{L_i} \tag{3} $$

III. COLLISION DETECTION METHODS

Main limitations in the design of CDRPM in Neuron structure is the cable to cable, cable to body and cable to workpiece collisions. In this section, all the above collisions are detected using the proposed collision detection algorithm. The i'th cable is simulated as a 3D line segment between A_i on the fixed frame and B_i on the moving frame attachment points. End-effector is simulated as a rectangular box with solid bodies, whose corners are the attachment points of the cables as shown in figure 1. The collision detection problem is converted to a geometrical search for collision. For a numerical solution of the problem, it is necessary to calculate efficiently the 6 dimensional position of the moving platform.

A. *Cable to Cable Collision*

A cable to cable collision occurs when two cables of the robot meet each other. If there exits an intersection of two straight cables' line segments, no further movement is plausible and robot looses its dexterity. To detect collision between two cables of CDRPM, we use a fast geometrical vector method that has been used in a real–time computer graphics solution.After examination of various solutions, in order to detect the interference, the distance between each two segments is calculated using the closest point approach [18]. As shown in figure 3, there exists a point like M_i on the A_iB_i line segment that has the shortest distance from another line, A_jB_j. Also, there exists a point like M_j on the A_jB_j line segment that is the closest point on the A_jB_j to A_iB_i:

$$d(A_iB_i, A_jB_j) = \min\ d(M_i, M_j), \{M_i \in A_iB_i, M_j \in A_jB_j\} \tag{4}$$

Where d denotes the distance function of the two cables. Cable to cable collision occurs when $\left\|\overrightarrow{M_iM_j}\right\| < 2\varepsilon$ in which, each cable has a circular cross section with ε radius. Furthermore, the closest points M_i and M_j can be evaluated from:

$$\boldsymbol{M}_i = \vec{a}_i + k_i.\hat{\boldsymbol{S}}_i\ ,\ \ \boldsymbol{M}_j = \vec{a}_j + k_j.\hat{\boldsymbol{S}}_j \tag{5}$$

in which, $\hat{\boldsymbol{S}}_i$ and $\hat{\boldsymbol{S}}_j$ are given in equation 3, and noting that the line containing M_iM_j will be uniquely perpendicular to both A_iB_i and A_jB_j lines. Thus, let $\overrightarrow{w}$ be the vector of $A_i - A_j$, as shown in figure 3, then for the perpendicular vectors we have:

$$(\hat{\boldsymbol{S}}_i.\hat{\boldsymbol{S}}_i)k_i - (\hat{\boldsymbol{S}}_i.\hat{\boldsymbol{S}}_j)k_j = -\hat{\boldsymbol{S}}_i.\overrightarrow{w} \tag{6}$$

$$(\hat{\boldsymbol{S}}_j.\hat{\boldsymbol{S}}_i)k_i - (\hat{\boldsymbol{S}}_j.\hat{\boldsymbol{S}}_j)k_j = -\hat{\boldsymbol{S}}_j.\overrightarrow{w} \tag{7}$$

TABLE I

GEOMETRIC PARAMETERS OF THE KNTU CDRPM

Description	Quantity
f_a: Fixed cube half length	1 m
f_b: Fixed cube half width	2 m
f_h: Fixed cube half height	1 m
a : Moving platform's half length (along x axes)	0.1 m
b : Moving platform's half width (along y axes)	0.1 m
h : Moving platform's half height (along z axes)	0.1 m

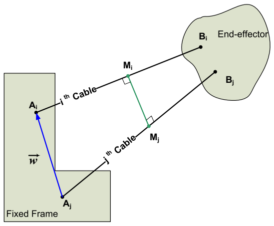

Fig. 3. The closest distance between a pair of the cables

Solving these equations, if the lines A_iB_i and A_jB_j are independent, unique values will be found for the parameters k_i and k_j. However, linear dependence of these equations means that the line segments are parallel. In this case, let $k_i = 0$ or $k_j = 0$ for the solution.If not $0 < k_i < L_i$ and $0 < k_j < L_j$, the points are on the line but out of the A_iB_i segment and can't make a collision.

B. *Cable to Body Collision*

Considering the attachment points lie in all corners of the moving platform, a computational algorithm is required to detect collision of the cables to the bodywhich depends on the cable's straight. There are various approaches to detect an intersection of a line segment and the rectangular box representing the body. One suitable approach is to extract the angle of a segmented line to the body by derivation of the line equation in 3D space. Another faster approach which is proposed in here, is to check the location of a number of point on the cable close to the attachment point, with respect to the body. As shown in figure 1, each cable must has an intersection (connection) with the body of the end-effector only at the moving attachment point, B_i. To check straight of the cable, relative position of a point on the cable is calculated instead of a time consuming connection angle extraction. If another point of the A_iB_i line segment exists inside of the body, a cable-body collision is occurred. Assume a point like D_i= $B_i - \delta\hat{\boldsymbol{S}}_i$ on the line segment, as shown in figure 4. Where the δ is the distance between D_i and B_i. Let D_i to get a neighborhood of the moving attachment point as:

$$0 < \delta < \min\left(2a, 2b, 2h\right) \tag{8}$$

Where $2a$, $2b$ and $2h$ are dimensions of the moving frame's box along x, y and z axes with respect to the moving coordinate (B). One of the states to satisfy the equation 8 is $\delta = \min\left(a, b, h\right)$. Now, if D_i point is inside of the rectangular box of the body as shown in figure 4 a collision

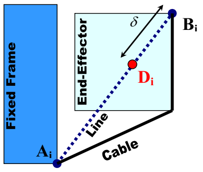

Fig. 4. Cable deflection when D_i is inside of the body

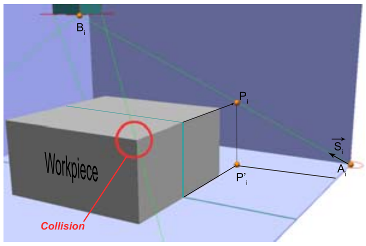

Fig. 5. Cable-workpiece collision

is occurred and such a situation can't be possible physically, because there exist a point of cable inside of another solid body. Thus, cable will lie on the surface of the end-effector's body and desired position is not accessible.

C. Cable to Workpiece Collision

The existence of any object in the fixed frame of the Neuron type CDRPM may decrease the workspace of the robot due to the collision of cables to the workpiece as shown in figure 5. Requiring over-constrained structure for the end-effector, cables must be attached in all directions to the end-effector. This type of design, adheres the actuators to be inside of the fixed frame. For some of applications such as accurate machining or forming processes, a workpiece must lie inside of the fixed frame. Thus, an analysis is required to obtain changes of the workspace, when a workpiece exists. For such a study, a box-based method is used [19] to detect workpiece to cable collisions. As shown in figure 5 the point P_i on the A_iB_i line segment is assumed to be the intersection point with the top surface plane of the workpiece. P_i is projected onto floor plane to find P_i'. If the P_i' is inside of the rectangular cross section of the workpiece, an interference has been occurred between the cable and the workpiece.

IV. WORKSPACE ANALYSIS

Generally it is impossible to derive the intersection constraints analytically for a given set of poses trajectories and workpiece geometries, [16]. However, in this section through numerical analysis of the workspace by using the above mentioned collision detection methods, the workspace of the Neuron design of the KNTU CDRPM is extracted. The main goal of this study is analyze the significance of the effective geometrical parameters of the design to obtain a large and collision free workspace for the KNTU CDRPM. The methods have to be implemented computationally efficient, since there are 44 collision detection steps required for each point in the fixed frame. It can be shown that by the above mentioned collision detection methods, for an n actuator manipulator, The number of collision detection check points are as follows:

$$k = R\left(\frac{n(n-1)}{2} + 2n\right) \tag{9}$$

Where R is the number of numerical grip points considered in the workspace analysis. A program is developed in Delphi compiler to analyze the workspace. For the KNTU CDRPM, the whole examination of the space takes at most 373 seconds of a 4.3GHz CPU time for 148000 points.

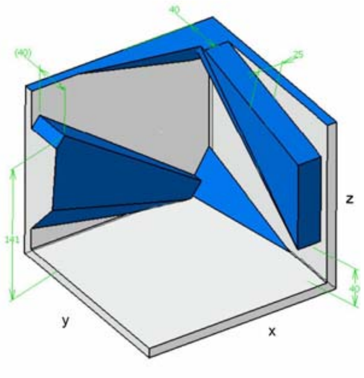

Fig. 6. Unreachable zones in the constant-orientation workspace ($\theta_x = 30°, \theta_y = 30°, \theta_z = 10°$)

A. Constant Orientation Workspace

The *constant orientation workspace* (COW) is defined as the three dimensional region that can be attained by the moving platform's centroid when it is kept at a constant orientation [20]. To extract valid points of the workspace, let's divide the fixed frame into a number of horizontal planes and lay a grid patterns with a resolution of $37000^{points}/_{m^3}$ for the whole space. The collision detection algorithms are implemented at each node of this grid pattern within the whole workspace. Validity of each node within the workspace is checked by the collision detection algorithms described in section III, while a set of fixed rotation angles are assumed for the end-effector . The number of valid points in the whole workspace grids are saved and counted as presented in table II. The table shows better results in comparison with the WARP CDRPM[21]. Figure 6 shows one of the resulted COWs when rotations about x, y and z axes are 30°, 30° and 10°, respectively. With no need to a desired orientation angle, all the points inside of the fixed frame are accessible with no collision. However, required orientations at the end-effector decreases the wide accessible points within the workspace. Fortunately, Unreachable points as shown in figure 6 are located at the corners of cubic workspace region. Thus, there exists *continues accessible space* in the middle of the fixed frame even with large end-effector orientations. Moreover, according to symmetric arrangement of the attachment points, a symmetric location of unreachable area is found and illustrated in figure 6. The condition of the collisions in $+z$ direction during clockwise rotation of the end-effector about z axes is similar to that of the collisions in $-z$ direction during counter-clockwise rotation of it about z axes. To have a quantitative measure for better judgment of COW performance at each CDRPM design, the following *accessible percentage* is defined as

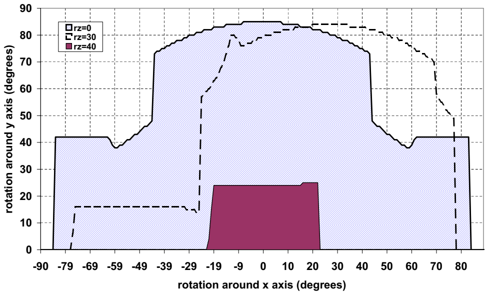

Fig. 7. Orientation workspace at the $[x=0, y=0, z=0]$ position

below:

$$\text{Accessible percentage} = \frac{\text{Number of accessible points}}{\text{Number of all points}} \times 100 \tag{10}$$

The results of COW for the Neuron design of the KNTU CDRPM is significantly better than other structures of 6 DOF CDRPMs, suggested before [21], [22].

B. Orientation Workspace

Orientation workspace is the set of all available rotation angles of the end-effector in a position inside of the workspace. Rigid linked parallel manipulator, specially the Stewart-Gogh platform is the most celebrated manipulator in the literature, whose workspace analysis is developed by researchers [23]. Looking into these researches, guides us to examine the boundaries of rotation around x axis while other rotations are fixed. Then, iterations of this algorithm can be performed for rotation around y axis. By this means one can recognize the orientation workspace and the effects of rotation around each axis on the other ones as shown in figure 7. Due to the symmetric shape of displayed workspace boundaries curves, the below part of r_x axis on the diagram is omitted. As it can be interpreted from this figure, when there is no rotation about z axis, all rotations provides a symmetric workspace. However, by increasing r_x the maximum rotation about y axis is limited due to the cable to body collision. Expanding rotation about z axis, a cable to cable collision occurs before cable to body collision. Thus, to obtain a wide range of rotations about x and y, we have to accept more limited r_z to increase other rotation angles in a fixed position.

C. Total Orientation Workspace

To achieve *total orientation workspace* (TOW) of the manipulator [24], let the end-effector rotate $\pm\theta_{m_x}$, $\pm\theta_{m_y}$ and $\pm\theta_{m_z}$ about the corresponding axes of the moving coordinate, and extract the COW of the rotation for θ_x , θ_y and θ_z by $|\theta_x| \leq \theta_{m_x}, |\theta_y| \leq \theta_{m_y}, |\theta_z| \leq \theta_{m_z}$. Intersection of accessible space of the COW of all rotations in this range creates the TOW of the CDRPM. Due to a symmetric arrangement of the attachment points, a symmetric behavior is expected in the COW analysis results. Noticing this fact makes the calculation of the total orientation workspace more efficient. Performing this task results in the generation of an irregular shaped space in 3D, in which all the inaccessible points are removed at each step of COW extraction. Thus, considering the whole accessible points which forms a continuum space and not considering the accessible points between the margins of accessible ones, a rectangular cube can be generated to show the minimum accessible TOW. Comparing the resulting COW workspace in the range of maximum angles, a cubic space can be defined by $1.2 \times 1.2 \times 1.4\ m^3$ while, $\theta_{m_x} = 30°$, $\theta_{m_y} = 30°$ and $\theta_{m_z} = 10°$ can be achieved.

TABLE II

ACCESSIBLE POINTS OF VARIOUS ROTATIONS OF END-EFFECTOR

θ_x	θ_y	θ_z	KNTU CDRPM	WARP
0°	0°	0°	100%	90%
30°	0°	0°	97%	30%
0°	30°	0°	97%	50%
0°	0°	30°	85%	10%
30°	30°	30°	71%	-

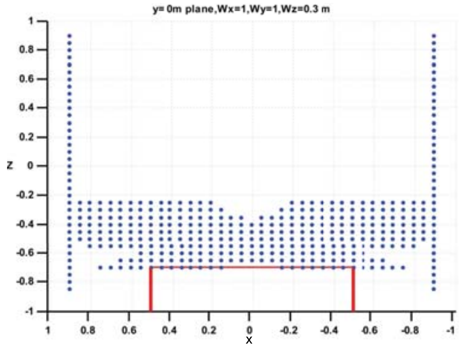

Fig. 8. Inaccessible area by cable to workpiece collision, with no rotation of the end-effector

D. Workpiece in the Workspace

To analyze the effects of putting a workpiece inside of the fixed frame in terms of the reachable workspace, a rectangular box workpiece is considered at the center of the fixed frame's floor, i.e., $z = -fh$ plane. The length, width and height of the workpiece are denoted by W_x, W_y and W_z, respectively. The effect of the existence of the workpiece, on the collision free zone inside of the fixed frame, is analyzed through examination of cable to workpiece collision on a vertical plane,$y = 0$. Collision free workspace decrease to %60 of that with no workpiece inside the fixed frame. Due to this limitation, the end effector cannot become near to the workpiece more than $0.5m$ in z direction. This is due to the fact that the cables will collide before the end-effector become close to the workspace. This condition gets worse if the end-effector has a rotation in either direction. Therefore, it is advised not to use this structure for the applications in which a task must be performed on a workpiece, and rather use Neuron structure, for a free 3D motion of the end-effector.

V. CONCLUSIONS

In this paper, the workspace analysis of the Neuron design of the KNTU CDRPM is studied in detail. This manipulator is a cable driven redundant parallel manipulator, which is under investigation for possible high speed application such as virtual reality, free 3D motion generators, or pick and place tasks. Workspace analysis is an essential step to design such manipulators in a way to accomplish a required large ranges of motions and it is useful for the classification of such robots. There are two main concerns in the design of CDRPMs: feasibility of motions using only tension forces of the cables and collision of the cables. The first and more popular method is to design a structure in which the cable collision is impossible within the whole workspace [11]. For such manipulators force feasible workspace boundaries are numerically determined for the end-user. The second solution, which is proposed in this paper is to design for a fully force feasible robot within the entire workspace and let the collisions determine the workspace boundaries. Therefore only collision detection is required during task planning, and therefore, the analysis becomes computationally less expensive and feasible for realtime purposes. It is shown that the collision avoidance of cables has an vital role in the enlargement of the workspace, provided that the force feasible workspace is large enough in the first step. Three main issues about the collision in the KNTU CDRPM are elaborated in this paper namely, cable to cable, cable to body and cable to workpiece collisions, and fast geometrical methods are proposed for the calculation of the intersections. Using these methods, a collision free workspace boundaries is generated for the robot, and the results are classified and presented graphically. The constant-orientation workspace (COW) is introduced and derived first, by the use of the collision detection methods. The dwtwermined COW for the special Neuron design of KNTU CDRPM is much wider than that for the other 8-6 CDRPMs introduced in the literature. This result not only explains the main advantages of a novel design for 8 actuated CDRPM, but also confirms the new way to extend ideas for CDRPM design process. This structure leads to save %90 of the positional workspace when end-effector has 30,30 and 10 degrees rotations about x, y and z axes, respectively. Comparing this results to the best reported design in the literature that more than %50 of the workspace is lost [21], when the end-effector has only a 30 degrees rotation about x axis. By repeating the COW analysis for the needed ranges of the rotations in Neuron, a collision-free sub-space is generated inside of the fixed frame. It is shown that a cube with %60 of each dimension of the fixed frame can be reached while $-30^\circ \leq \theta_x \leq 30^\circ, -30^\circ \leq \theta_y \leq 30^\circ, -30^\circ \leq \theta_z \leq 30^\circ$. The results of COW analysis with a workpiece inside of the fixed frame, shows an important feature about working on workpieces. The end-effector can't go near of a wide workpiece because of cable to workpiece collision. Thus, the Neuron design of the KNTU CDRPM can only be used in an object free space. Finally, having a wide range of end-effector's position and orientation, Neuron is a suitable platform for virtual reality applications such as virtual acceleration or gravity free test beds.

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