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Wrench Feasible Workspace Analysis of Cable-Driven Parallel Manipulators Using LMI Approach

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***Abstract*— Workspace analysis is one of the most important issues in robotic manipulator design. This paper introduces a systematic method of analysis the wrench feasible workspace for general redundant cable-driven parallel manipulators. In this method, wrench feasible workspace is formulated in term of linear matrix inequalities and projective method is used for solving them. This method is one of the most efficient interior-point methods with a polynomial-time complexity. Moreover, the notion of dexterous workspace is defined, which can be determined for redundant cable driven manipulators exerting a worst case external wrench at the end effector. A detailed case study of the wrench feasible workspace and dexterous workspace determination are included for a six DOF, eight actuated cable-driven redundant parallel manipulator.**

I. Introduction

PARALLEL manipulators consist of a number of limbs acting in-parallel on a mobile platform. In cable-driven redundant parallel manipulators (CDRPM), cables replace the rigid link limbs; therefore, high accelerations can be achieved due to the reduced mass of the limbs and enables CDRPM to perform in ultra-high-speed operations. In CDRPM, cables should be in tension which can be achieved by extra loading applied to the mobile platform. This loading can be obtained from a redundant cable [1], [2] or from another force-applying element such as a spring [3] or a pneumatic cylinder [4].

Cable robot has several attractive features and some advantage compared to conventional parallel manipulators. It has a comparatively large workspace with desirable stiffness. Cable robot provides a better balance between workspace and stiffness requirement than typical serial or parallel manipulators. It has high payload-to-weight ratios and low inertial properties due to its light moving parts (cables) and fixed heavy parts (motors and controllers), therefore, energy consumption is significantly reduced. It is easy to assemble/disassemble and reconfigure such mechanisms due to their flexible structure. They are reliable due to their simpler structure and relatively remote location of motors and controllers from the end-effector. This feature is very attractive when robots are needed to operate in stringent environment, like manipulating explosive objects.

With the above desirable characteristics, cable robots are very useful in many real world applications, such as heavy payload handling, manufacturing operations, haptic devices, remote/hazardous areas operation, and high-speed manipulation/positioning. Some typical applications have been developed during last decade. The NIST ROBOCRANE [5] is a six-cable, six DOF cable robot based on Stewart-Gough platform designed for tasks such as material handling, inspection, pipe/beam fitting and manufacturing operations such as welding, sawing and grinding [6],[7]. Cable robots have also been proposed for the use in transferring cargo to and from ships. One such system is the Automated All-Weather Cargo Transfer System (AACTS) [8], made by August Design.

In spite of many advantages and promising potentials, there are many challenging problems in the design and development of cable robot. Variations in the geometrical configuration of system and tasks may impose the potential of cables interfering with each other, which may greatly limit the potentially usable workspace and add obstacles in the control design. Variations in physical properties of different cable may cause different level of stretching, sagging and vibration, etc., which may greatly degrade the accuracy of the end-effector during operation. These problems are solved in the design of CDRPMs using an over constrained end-effector [2] or a passive force [9]. During the process of design, factors such as number of cables, shape of platform, location of attachment points, etc., may greatly limit important performance features such as singularity, stability, and wrench generation capability of the system. Workspace analysis can give analytical insights to designers before the implementation stage. However, factors like the unidirectional constraint and potential of interference imposed by cables make this kind of analysis more difficult than that of traditional robotic manipulators.

In literature, several different types of workspace have been addressed based on various definitions. One of the most general workspace definitions is referred to the workspace in which any wrench can be generated at the moving platform while cables are in tension. Gouttefarde and Gosselin term such workspace as Wrench-Closure Workspace (WCW) [10]. Hence, from a general design point of view, the WCW is of great interest. However, what should be pointed out is that this kind of workspace is too ideal in some extent, since the system is required to be able to generate arbitrary unbounded wrench set in such workspace. A number of researchers addressed the set of postures that the end-effector can statically attain while only taking gravity into account [11-14]. Since not all postures in reachable workspace are statically attainable, this is a subset of reachable workspace for cable robots. In most studies of this kind of workspace, numerical approaches are used to find out the corresponding workspace for specific system

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[15]. Moreover, some researchers addressed a set of postures when cable robots are needed to exert particularly required force/moment combinations to interact with the environment besides maintaining its own static equilibrium. Ebert-Uphoff et al. termed this type of workspace as wrench feasible workspace [16]. Few researchers proposed analytical approaches to this issue; Gouttefarde and Gosselin determined the boundaries of the workspace for planar 4-cable fully-constrained robots analytically [17], while assumed infinite upper tension limits. Another workspace that has been addressed is the dynamic workspace along with a set of wrench called pseudo-pyramid workspace, defined by Barrette and Gosselin as the set of all postures of the cable robot end-effector with specific acceleration requirement, and boundaries of this type of workspace are analytically formed for planar cable robots [18].

In all of the proposed methods, the worst case wrench that can be generated at the moving platform by tension cable force is not handled. In this paper, a new notation of workspace is defined, as its shape depends on the worst case external wrench and it determines a dexterous workspace of redundant cable-driven manipulator. This worst case wrench depends on the columns of the Jacobian matrix transpose. Also, a new systematic method of verifying the wrench feasible workspace for general redundant cable manipulators is developed. This method provides an easy to use approach to determine the wrench feasible workspace of cable manipulators. The proposed method is generally applicable to any cable manipulators with any redundant cables as long as its Jacobian matrix has full rank. In this method, wrench feasible workspace is formulated in term of linear matrix inequalities and projective method is used for solving them. This method is one of the most efficient interior-point methods with a polynomial-time complexity.

The remaining of the paper is organized as follows: the wrench feasible workspace analysis in cable driven redundant parallel manipulator is described in the next section, which is followed by introducing the systematic method of wrench feasible workspace analysis in section III. The determination of wrench feasible workspace is addressed along with its case study in section IV. Finally, section V provides the concluding remarks.

II. Wrench Projection Feasibility

The relationship between the tensions in the cables and external wrench acting on the moving platform is given by:

$$-\boldsymbol{J}^T\boldsymbol{F} = \boldsymbol{W} \tag{1}$$

In which, $\boldsymbol{F}$ denotes cable force, $\boldsymbol{W}$ denotes external wrench acting on the moving platform and $\boldsymbol{J}$ is the Jacobian matrix. Thus, to examine tension wrench feasibility of the robot, feasibility of projection of a wrench like $\boldsymbol{W}$ on the cable's tension force directions, $\boldsymbol{F}$, by the Jacobian matrix should be studied. The Jacobian matrix of a n DOF m fully parallel actuated manipulator like CDRPM can be written as following [19]:

$$\boldsymbol{J} = \begin{bmatrix} \widehat{\boldsymbol{S}}_1^T & (\boldsymbol{E}_1 \times \widehat{\boldsymbol{S}}_1)^T \\ \vdots & \vdots \\ \widehat{\boldsymbol{S}}_i^T & (\boldsymbol{E}_i \times \widehat{\boldsymbol{S}}_i)^T \\ \vdots & \vdots \\ \widehat{\boldsymbol{S}}_m^T & (\boldsymbol{E}_m \times \widehat{\boldsymbol{S}}_m)^T \end{bmatrix}_{m\times n} \tag{2}$$

As shown in figure 1, $\widehat{\boldsymbol{S}}_i$ and $\boldsymbol{E}_i$ are unit vector of $i'th$ cable and position vector of the moving attachment point of the $i'th$ limb, respectively.

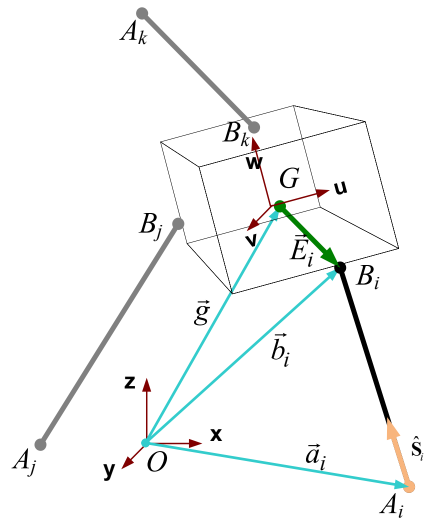

Fig. 1. The general structure of a cable-driven parallel manipulator and its vectors.

The results of feasibility analysis strongly depend on the wrench vector which will be projected by the Jacobian. The gravity force vector is used in the most of researches [13] is:

$$\boldsymbol{W} = \begin{bmatrix} \vec{\boldsymbol{G}} \\ \overrightarrow{\boldsymbol{e}_G} \times \vec{\boldsymbol{G}} \end{bmatrix}_{n\times 1} \tag{3}$$

Where $\vec{\boldsymbol{G}}$ is the gravity force and $\overrightarrow{\boldsymbol{e}_G}$ is eccentricity of mass center from origin of the moving coordinate and it may be a zero vector. Projection of this wrench may be a good measure for the CDRPMs with at most one degree of redundancy or under-actuated cable-driven manipulators. Hence, it is only useful to discover statically reachable force feasible workspace. Such a definition of wrench feasible workspace is similar to reachable workspace definition for serial and rigid-linked parallel manipulators [20]. However, effects of acceleration, deceleration, disturbances, and inertia forces or torques are not considered in the analyzed wrench feasibility. Therefore, dynamical behavior of the robot cannot be analyzed using just the gravity effects. On the other hand, a method proposed by Barrette and Gosselin [18] advises to include dynamical properties of the CDRPM instead of gravity. Nevertheless, a parallel manipulator like CDRPM has coupled dynamics in the equations of motion and force analysis cannot be studied for a constant Cartesian wrench vector or a set of constant Cartesian vectors, since the principle of superposition cannot be used here. For example, the robot may be able to move along x axis or y axis at a given pose, but it cannot move along both x, y axes simultaneously. Also, there is a great deal of variation among the role of each cable in motion along the same axis depending on the end–effector position. Thus, the fixed wrenches that used in the literature cannot be an appropriate vector to project on the cables while the proposed vector varies depending on the cable direction. Therefore, a more suitable problem is to warranty making any arbitrary resultant wrench by tension forces of the cables within the determined dexterous workspace.

Notice that the elements of the Jacobian matrix in (2) enlighten an important insight about the projection of wrenches (1) on the moving platform. The $i'th$ row of the Jacobian matrix corresponds to a force exerted on the center of mass and along the direction of the $i'th$ cable, $\boldsymbol{\widehat{S}_i}$ whose resulting torque can be determined by $\boldsymbol{E}_i \times \boldsymbol{\widehat{S}}_i$. Therefore, the worst condition for the wrench feasible workspace of a CDRPM is when such wrench is exerted on the end-effector, i.e. exactly in the direction of one of the cables at the center of mass. Such extreme wrench can be easily calculated by the $i'th$ row of the Jacobian matrix:

$$\boldsymbol{W}_i = \begin{bmatrix} \boldsymbol{\widehat{S}}_i \\ \boldsymbol{E}_i \times \boldsymbol{\widehat{S}}_i \end{bmatrix}_{n\times 1} \tag{4}$$

Repeating projection of $\boldsymbol{W}_i$ for each cable direction, $i = 1, 2, \cdots, m$ determines whether a pose a full-ranked Jacobian is wrench feasible or not. Feasibility of exerting a wrench by serial or rigid-linked parallel manipulators is studied in dexterity analysis. Dexterity depicts the ability to arbitrarily change its position and orientation, or apply wrench in arbitrary directions in the workspace [21]. According to this definition, traditional dexterity measures are not sufficient for CDRPMs. Therefore, wrench projection feasibility of the proposed wrench, $\boldsymbol{W}$ can be stated as the most important factor in the dexterity analysis of the CDRPMs instead of the typical dexterity measures. If exerting a force in the direction of $\boldsymbol{W}$ becomes impossible, there exists at least one inaccessible direction in motion. Such condition contradicts the dexterity of the robot, although the contemporary measures of dexterity or manipulability show a good condition for the Jacobian matrix. Therefore, having a good condition number alone is not sufficient to ensure the dexterity of a CDRPM. We propose to define *dexterous workspace* for a redundant cable-driven manipulator as the wrench feasible workspace when the worst case wrench is exerted on the manipulator. If such wrench is applied to the robot in the direction of a cable, that cable is not capable to produce any positive reaction to the end effector. Therefore, one degrees of redundancy of the robot is simply annihilated, and in order to achieve dexterous workspace, at least *two degrees of redundancy* is required. In order to show this claim mathematically, refer to equation (4), in which the proposed wrench can be defined as:

$$\boldsymbol{W_j} = J_j \ , \ j = 1, \dots, n \tag{5}$$

If there exists an optimal solution for (1) with this wrench, equation (5) can be defined as:

$$\boldsymbol{W_j} = -\sum_1^n \tau_i J_i^T \tag{6}$$

Where τ_i is assumed the optimal solution of force for (5) and J_i^T is $i'th$ column of the Jacobian transpose. According to (5) and (6), this problem is defined as:

$$J_j \ = -\sum_1^n \tau_i J_i^T \tag{7}$$

Therefore,

$$\left(1 + \tau_j\right) J_j^T \ = -\sum_{i \neq j} \tau_i J_i^T \tag{8}$$

Hence, when the τ_j has the minimum tension, we can obtain the minimum value of coefficient J_j^T. Therefore, the tension of $j'th$ cable is not capable to produce any positive reaction to the end effector and one degrees of redundancy of the robot is simply annihilated.

III. Optimization Method of Wrench Feasible Workspace

The wrench feasible workspace of an *n*-DOF cable-driven manipulator with *r* degrees of redundancy is the set of poses of the moving platform at which,

$$\forall \boldsymbol{W} \in W_d \ , \ \exists (\ \boldsymbol{F} > 0) \in \mathbb{R}^m \ \ni \ -\boldsymbol{J^T F} = \boldsymbol{W} \tag{9}$$

Where m denotes the number of cables and W_d is the set of desired wrenches. There exists the least square minimum norm solution to solve $-\boldsymbol{J^T F} = \boldsymbol{W}$. The least square minimum norm solution is obtained from:

$$\boldsymbol{F} = -\boldsymbol{J}^{\dagger} \boldsymbol{W} \tag{10}$$

Where $\boldsymbol{J}^{\dagger}$ is pseudo-inverse of $\boldsymbol{J}$ and can be computed as:

$$\boldsymbol{J}^{\dagger} = \boldsymbol{J^T} (J . \boldsymbol{J^T})^{-\mathbf{1}} \tag{11}$$

This solution does not guarantee that the all cables will be in tension. The general solution can be expressed in terms of least square minimum norm solution and the null space of matrix $\boldsymbol{A}$ in the following result [22]:

$$\boldsymbol{F} = \boldsymbol{A}^{\dagger} \boldsymbol{W} + \ (\boldsymbol{I} - \boldsymbol{A}^{\dagger} \boldsymbol{A}) \boldsymbol{h} \tag{12}$$

where $-\boldsymbol{A}$ is the Jacobian transpose, $\boldsymbol{I}$ is $m{\times}m$ identify matrix and $\boldsymbol{h}$ is an m-dimensional arbitrary vector. The first term of (12) corresponds to the least square minimum norm solution and the second term corresponds to the homogeneous solution that projection $\boldsymbol{h}$ to the null space of $\boldsymbol{A}$, [22]. The positivity of forces can be ensured by properly modulating the value of $\boldsymbol{h}$ distribution. According to the region of wrench feasible workspace in (9) and the general solution in (12), the wrench feasible workspace problem can be defined as:

$$\forall \boldsymbol{W} \in \mathbb{R}^n \ , \ \exists \ \boldsymbol{h} \in \mathbb{R}^m \ \ni \ \boldsymbol{A}^{\dagger} \boldsymbol{W} + \left(\boldsymbol{I} - \boldsymbol{A}^{\dagger} \boldsymbol{A}\right) \boldsymbol{h} > 0 \tag{13}$$

Equation (13) can be formulated in term of linear matrix inequalities (LMI). LMI approaches are formulations of convex optimization problem whose solutions are numerically tractable [23]. The strict feasible problem is one of the generic LMI problems. This problem is defined as

$$\boldsymbol{F}(\boldsymbol{x}) = \boldsymbol{L}_0 + \boldsymbol{L}\boldsymbol{x} > 0 \quad (14)$$

Where $\boldsymbol{x} \in \mathbb{R}^m$ is a variable and $\boldsymbol{L_0}$ and $\boldsymbol{L}$ are given constant symmetric real matrices. According to (13) and (14), wrench feasible workspace is formulated in term of an LMI problem as:

$$\boldsymbol{L_0} = \boldsymbol{A}^{\dagger}\boldsymbol{W} \ , \boldsymbol{L} = \left(\boldsymbol{I} - \boldsymbol{A}^{\dagger}\boldsymbol{A}\right), \boldsymbol{x} = \boldsymbol{h} \quad (15)$$

For solving feasibility problem, Projective method is used as an efficient interior-point method.

A. The Projective Method

As an interior-point method, the projective method generates a sequence of matrices that remain in the open cone *K*. to preserve positive definiteness, the optimization criterion includes a logarithmic barrier that is defined on *K* and tend to +∞ when approaching a boundary point of *K*. *K* denotes the open cone of positive definite matrices in space of symmetric matrices *S* of size *m* [23]. To find strictly feasible vector $\boldsymbol{x}$, the projective method relies on the following strategy. Given some *X* in the open cone *K*, test whether the Dikin ellipsoid centered at $\boldsymbol{x}$ intersects $\boldsymbol{E}$=Range ($\boldsymbol{F}$). If it does, this provides a strictly feasible point since $\Omega(X) \subset K$. Otherwise, update *X* to increase the chances that this intersection be nonempty [23].

Dikin ellipsoid is instrumental to the updating of the current solution. Given$X > 0$, consider the ellipsoid center at *X*:

$$\Omega(X) = \left\{ Y \middle| \ \left\| X^{-1/2} Y X^{-1/2} - I \right\|_{Fro} < 1 \right\} \quad (16)$$

Where $\|.\|_{Fro}$ is the Frobenius norm [23].

B. Flowchart of the Method

According to the pervious discussions, the flowchart of the method given in figure 2 reveals the details of the iterative method used to find the region of wrench closure workspace. As it is seen in this flowchart, for a grid of all positions and orientations of the end-effector, the values of *J*, Jacobian matrix, and *W* external force are calculated. Then the pseudo-inverse and the null space of negative transpose of Jacobian matrix are calculated. Furthermore, Projective method is solved, for strict feasibility of LMI approach `feasp` function of Matlab is used to solve this problem. If the LMI solution is feasible, this point is laid in the wrench feasible region.

IV. CASE STUDY

In this section through numerical analysis of the workspace using the proposed method, the workspace of the KNTU CDRPM is analyzed. The KNTU CDRPM is designed with an 8 actuated 6 degrees of freedom cable driven redundant parallel manipulator. This manipulator is under investigation for possible high speed and wide workspace applications in the K.N. Toosi University of Technology [24]. A special design for the KNTU CDRPM is suggested as shown in figure 3 in which, the fixed

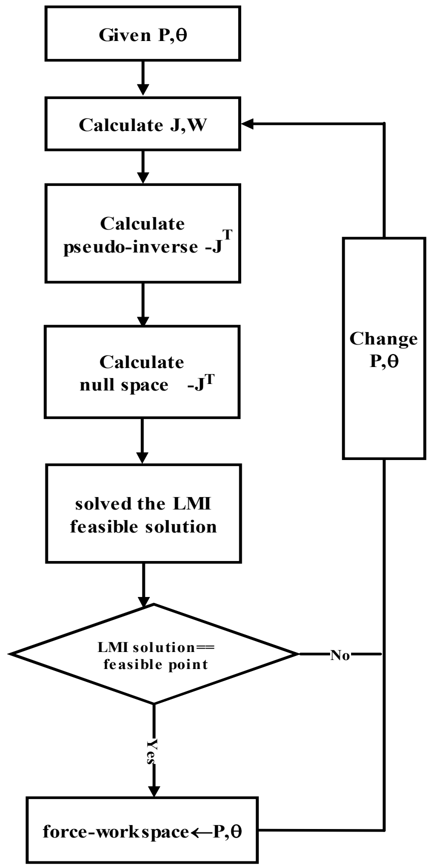

Fig. 2. Flowchart of represented method

attachment points, A_i are located on the corners of two rectangular plates at the top and bottom of the workspace. To avoid singularity, the top plate is rotated 10° around the z axis clockwise and the bottom one is rotated 10° counterclockwise. Although in the analysis of the KNTU CDRPM, all the attachment points can be arbitrarily chosen, the geometric parameters given in table I is used in the simulations.

For workspace analysis, constant orientation workspace of the manipulator is determined. The constant orientation workspace (COW) is defined as the three dimensional region that can be attained by the moving platform's centroid when it is kept at a constant orientation [25].

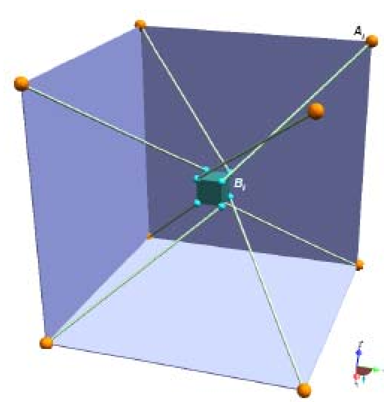

Fig. 3. The KNTU CDRPM, a perspective view

TABLE I
UNITS FOR MAGNETIC PROPERTIES

Description	Quantity
f_a: fixed cube half length	2 m
f_b: fixed cube half width	1 m
f_h: fixed cube half height	0.5 m
a: moving platform's half length	0.14 m
b: moving platform's half width	0.07 m
h: moving platform's half height	0.1 m
M: the moving platform's mass	5 Kg

According to this definition, the constant orientation workspace of KNTU CDRPM is computed for several external wrenches. We categorized the external wrenches into three types as gravity wrench (W_g), worst case wrench (W_J) and perturbed worst case wrench (W_Δ). Gravity wrench and worst case wrench are fully elaborated in pervious section. Perturbed worst case wrench is defined by Δ change of worst case wrench direction and it is obtained from:

$$\boldsymbol{W}_\Delta = \boldsymbol{W_J}(\boldsymbol{I} - \Delta) \quad (17)$$

Where $\boldsymbol{I}$ is an identify vector and Δ is perturbed vector. In this simulation Δ is set to [.01 .01 .04 .01 .01 .04]'.

Table II compares the results of these cases for various constant orientations of the moving platform. In this analysis, we first considered the collisions between cables and exclude these points from constant orientation force feasible workspace. The cable collision is determined according to the algorithm given in [24]. In the first column table II, several orientations are fixed, while in the second column, the percentage of COW which is wrench feasible is determined for gravity wrench. As it is observed from the obtained results, the percentage of COW is decreased by increasing the orientation. However, this decrease is not significant. Percentage of dexterous constant orientation workspace due to worst case wrench is determined using the algorithm elaborated in section III and is presented in third column of table II. It is seen that for the worst case wrench in KNTU CDRPM, the dexterous workspace is limited to only less than 7% of the reachable workspace for different orientations. Nonetheless, the Δ change in worst case wrench causes the workspace increase significantly (see 4th column of table II). This result verifies the claim proposed in this paper to define the dexterous workspace based on worst case wrench. As it is observed by a small perturbation in the direction of the applied wrench, the wrench feasible workspace increases significantly.

TABLE II
CONSTANT ORIENTATION WORKSPACE FOR SEVERAL EXTERNAL WRENCHES

$(\theta x, \theta y, \theta z)^0$	W_g	W_J	W_Δ
(0, 0, 0)	63.21%	5.83%	41.95%
(5, 0, 0)	62.08%	6.09%	34.89%
(0, 5, 0)	63.11%	6.13%	31.19%
(0, 0, 5)	60.16%	5.96%	38.74%
(-5,5,0)	61.18%	5.76%	30.8%
(10,-10,5)	50.69%	5.07%	28.53%

Figures 4 and 5 visualize the two obtained constant orientation wrench feasible workspace, in which the external wrench are gravity and worst case wrenches, respectively. As it can be seen from the overlap of these two figures, there exists a continuous accessible space within the wrench feasible workspace of the manipulator, in which the worst case wrench can be applied.

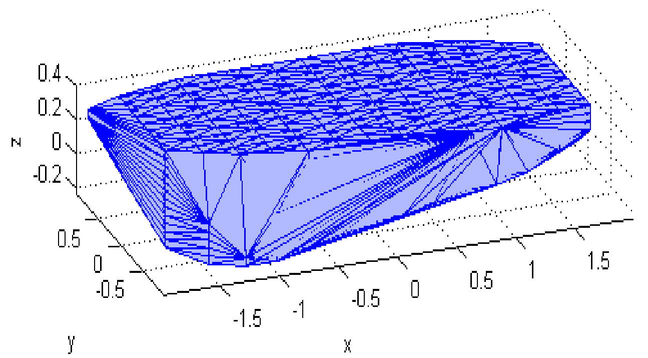

Fig. 4. The constant orientation wrench feasible workspace at the (θ_x=5°, θ_y=10°, θ_z=5°) for gravity wrench

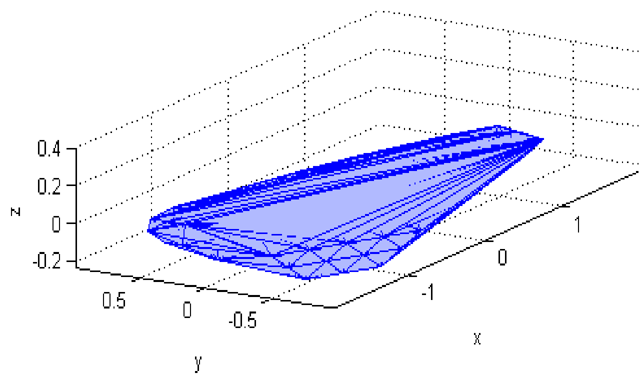

Fig. 5. The constant orientation wrench feasible workspace at the (θ_x=5°, θ_y=10°, θ_z=5°) for worst case wrench

In order to examine the sensitivity of the wrench feasible workspace to the small perturbation in the direction of the worst case wrench, figure 6 illustrates the variation of workspace for several Δ changes for various fixed rotation angles about x, y and z axes. As it is seen in the horizontal axis the variation of θ_x is between ±30 degrees, while each of the perturbed directions are considered in only one direction either in position or in orientation coordinates. For example, circle dashed line is the variation of the wrench feasible workspace for $\Delta_{\boldsymbol{\theta z}}$= [0 0 0 0 0 0.04]$'$. According to this figure, the dexterous workspace which is plotted with solid line is the smallest workspace compared to the perturbed changes in worst case wrench. Moreover, the force feasible workspace obtained by perturbation in θ_z has its maximum. Similarly, perturbation along z axis provides larger workspace than that in other directions This simulation results reveals in important fact in the design of the KNTU CDRPM that the force feasible workspace of this manipulator is more sensitive to the z and θ_z directions. This might be due to the special arrangement of the attachment points in this manipulator, and investigation to desensitize the manipulator workspace in this direction is underway.

In order to determine the computation costs in the determination of the force feasible workspace using LMI approach, it should be mentioned that for KNTU CDRPM, the computation time is at most 910 second of a core™ 2-1.8 GHz CPU for 64000 point, which is significantly faster than that of using convex optimization routines such as `fmincon` of MATLAB.

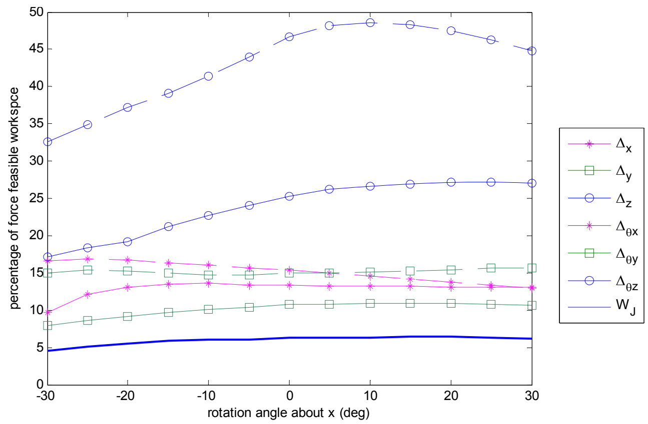

Fig. 6. The orientation wrench feasible workspace at the (θ_y=0°, θ_z=0°) for several Δ perturbed direction of worst case wrench.

V. CONCLUSIONS

In this paper a systematic method is proposed for verifying the wrench feasible condition of a general redundant cable manipulator. This method is an easy to use approach to determine the wrench feasible workspace of a cable manipulator. The proposed method is generally applicable to any cable manipulator with any redundant cables as long as its Jacobian matrix is full rank. In this method, force feasible workspace has been formulated in term of linear matrix inequalities which is numerically solved using projective method. The force feasible workspace has been analyzed by introducing a new concept of dexterous workspace and the worst case wrench at the end-effector. It is verified that only the cable driven manipulators with at least two degrees of redundancy can enjoy the benefits of dexterous workspace. It is shown that the Δ change in the direction of the worst case wrench can noticeably increase the wrench feasible workspace. Hence, the determination of a specific direction in the space which causes the robot force infeasible can be determined by the rows of the Jacobian transpose. Moreover, it is observed that the wrench feasible workspace of KNTU CDRPM is very sensitive to the z and θ_z directions. This might be caused by the structure of the design of this manipulator. Finally, the proposed dexterous workspace determination can be firstly used to optimize the structure of this manipulator in terms of the workspace, and secondly used for other cable driven manipulators.

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