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Kinematics and Jacobian Analysis of the KNTU CDRPM: A Cable Driven Redundant Parallel Manipulator

Mohammad M. Aref, Reza Oftadeh and Hamid D. Taghirad

***Abstract*— KNTU CDRPM is a cable driven redundant parallel manipulator, which is under investigation for possible implementation in large workspace applications. This type newly developed manipulator has numerous advantages compared to that of the conventional parable mechanisms. The rotational motion range is relatively large, the inherent redundancy improves dexterity of the manipulator, and the light weight structure makes the robot more energy efficient and significantly fast. However, there exist some challenging issues in the over-constrained mechanism like KNTU CDRPM. Collision avoidance, force feasibility, and linear independency of the cables are the main issues being under study in the design of such manipulators. In this paper, singularity of the KNTU CDRPM is studied in detail. To extract kinematic properties of the robot, the inverse and forward kinematics are analyzed. It is shown that singularity analysis can well describe the characteristics of the design and provide the sufficient means to the designer to improve these characteristics. Finally, a suitable design strategy is proposed to significantly reduce the singularity of the manipulator within its whole workspace. The outcomes of this strategy implemented on KNTU CDRPM result in a significant improvement of the singular free workspace of the proposed design compared to that of the latest parallel manipulators.**

***Index Terms*— Cable driven, Parallel manipulator, Kinematic analysis, Wire suspended robot, Singularity.**

I. Introduction

Nowadays, *parallel manipulator* (PM)'s applications are significantly increasing. A closed chain kinematics between fixed and moving platforms, makes the end-effector's motions more stiff and high-accelerated by fully-constraining the end-effector[1]. In a parallel mechanism, each limb contributes in the movement of the payload. Thus, it can carry more payload to moving mass ratio which is suitable for special applications such as the popular Stewart-Gough platform in flight simulator [2]. On the other hand, a large motion of the linear actuator of the rigid links of a parallel manipulator leads to a small displacement of the end-effector. Thus, high precision is achieved relative to the serial manipulators. However, additional to hardship of

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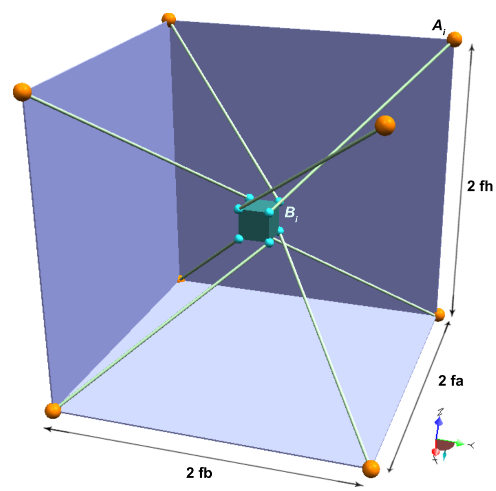

Fig. 1. The KNTU CDRPM, a perspective view

production [3] and control [4], there are some challenges to use PM structures in a wide range of applications. The main limitations of the PMs are limited workspace [5] and singularity regions within the workspace [6]. Using an electric powered cable-driven actuator, as an alternative for the massive and stroke-limited linear actuator, can extend the workspace of the manipulator inevitably large even within the size of a Football stadium [7], or a platform of large adaptive reflector with $2km^2$ footprint [8]. By locating the driver units on the fixed platform, only light-weight cables' mass is added to the mass of the end-effector. Therefore, manipulators such as a RoboCrane can carry large forces as the weight of a shipping cargo with the use of a CDRPM structure [9]. Moreover, CDRPM saves heredity of PMs about acceleration capabilities in addition to enlarged workspace. It makes CDRPM a suitable platform of virtual acceleration in virtual reality tasks [10]. However, a cable can only carry tension forces, and to guarantee that the cables are always under tension different solutions are advised. In some cases the end-effector is suspended from the cables and by use of the gravity force or any other passive force against the moving platform, this is ensured [8]. Another more applicable solution for high acceleration applications, is to use redundant actuators, and to resolve the redundancy to ensure positive tension in all the cables. This can be performed in a fully–constrained or over–constrained moving platform [11], but with more difficulties to analyze the geometrical[12] or force feasible workspace [13].

The KNTU CDRPM is thus designed based on such structure with an 8 actuated 6 degrees of freedom cable driven redundant parallel manipulator. This manipulator is under investigation for possible high speed and wide workspace applications such as virtual acceleration generator in the K.N. Toosi University of Technology. This proposed design has significant advantages compared to the conventional parallel mechanisms. Its rotational motion range is relatively large, its 2 degrees of redundancy improves safety for failure in cables, and makes the design suitable for high acceleration motions. A special design for the KNTU CDRPM is suggested as shown in figure 1, which is called *"Neuron"* in this paper, that satisfies the possibility of tension forces in all the cables. The design and implementation of the KNTU CDRPM require deep investigation in various fields.

In this paper, inverse kinematics and forward kinematics are derived. Then, the 8×6 Jacobian matrix of the manipulator is extracted by velocity formulation. Furthermore, singularity of the Jacobian is analyzed to determine condition of the manipulator. Next, effects of design parameters on the manipulator condition are examined. The examination explains that how design parameters variation repairs singularity. Finally, a combination of the methods proposed to determine kinematic properties of the robot in a given position and its accessibility. The results show that the improvement has significantly increase manipulator dexterity.

II. Kinematics

A. Mechanism Description

The KNTU Cable Driven Redundant Parallel Manipulator is illustrated in figure 1. This figure shows a spatial six degrees of freedom manipulator with two degrees of redundancy. This robot has eight identical cable limbs. The cable driven limbs are modeled as spherical-prismatic-spherical(SPS) joints, for cables can only bear tension force and not radial or bending force. Two cartesian coordinate systems $A(x, y, z)$ and $B(u, v, w)$ are attached to the fixed base and moving platform. Points $A_1, A_2, \ldots, A_8$ lie on the fixed cubic frame and $B_1, B_2, \ldots, B_8$ lie on the moving platform. The origin O of the fixed coordinate system is located at the centroid of the cubic frame. Similarly, the origin G of the moving coordinate system is located at centroid of the cubic moving platform. The transformation

TABLE I

Geometric and Inertial Parameters of the KNTU CDRPM

Description	Quantity
f_a: Fixed cube half length	$1\ m$
f_b: Fixed cube half width	$2\ m$
f_h: Fixed cube half height	$1\ m$
C : Cubic moving platform half dimension	$0.1\ m$

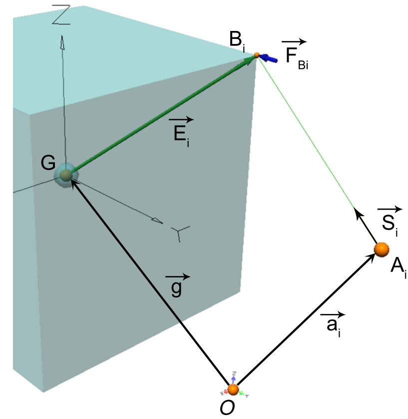

Fig. 2. ith Attachment point on the moving platform and related vectors

from the moving platform to the fixed base can be described by a position vector $\overrightarrow{g} = \overrightarrow{OG}$ and a 3×3 rotation matrix ${}^{A}R_{B}$. Consider a_i and ${}^{B}b_i$ denote the position vectors of points A_i and B_i in the coordinate system A and B, respectively. Although in the analysis of the KNTU CDRPM, all the attachment points, are considered to be arbitrary, the geometric parameters given in table I are used in the simulations.

B. Inverse Kinematics

The inverse kinematic analysis is first and simplest step in the kinematics of PMs. Since the *forward kinematic* (FK)is complicated, the inverse kinematic is very useful in the dynamic analysis[14] and control [15]. For inverse kinematic analysis of the CDRPM, it is assumed that the position and orientation of the moving platform $x = [x_G, y_G, z_G]^T$, ${}^{A}R_B$ are given and the problem is to find the joint variable of the CDRPM, $\boldsymbol{L} = [L_1, L_2, \ldots, L_8]^T$. From the geometry of the manipulator as illustrated in figure 2 the following vector loops can be derived:

$$ {}^{A}\overrightarrow{A_iB_i} + {}^{A}\overrightarrow{\boldsymbol{a}_i} = {}^{A}\overrightarrow{\boldsymbol{g}} + \boldsymbol{E}_i \quad (1) $$

in which, the vectors $\boldsymbol{g}, \boldsymbol{E}_i$, and $\boldsymbol{a}_i$ are illustrated in figure 2. The length of the i'th limb is obtained through taking the dot product of the vector $\overrightarrow{A_iB_i}$ with itself. Therefore, for $i = 1, 2, \ldots, 8$:

$$ L_i = \left\{ [\boldsymbol{g} + \boldsymbol{E}_i - \boldsymbol{a}_i]^T [\boldsymbol{g} + \boldsymbol{E}_i - \boldsymbol{a}_i] \right\}^{\frac{1}{2}} . \quad (2) $$

C. Forward Kinematics

In the CDRPMs, forward kinematics defines the problem of finding the pose of the moving platform as a function of the cable lengths of the manipulator. KNTU CDRPM has eight actuator variables and forward kinematic solution should calculate the position and orientation of the end-effector in a six dimensional workspace as a function of these eight variables. However, most of the studies in the literature focused on the fully actuated six DOF Stewart–Gough platform. Forward kinematic solution is known as a challenging problem in most of the parallel manipulators[16]. It is shown that the forward kinematic solution of the Stewart-Gough platform is not unique [17]. Moreover, many other solutions for this problem was proposed such as positioning of additional sensors for the passive joints [18] or solving the problem as an optimization problem [19]. Another solution of the forward kinematic equations is to simplify relations between coordinates of the attachment points and inverse kinematic by means of a closed form solution[20]. This solution can be fused with a linear sensor data in order to precise the forward kinematic solution[21]. In another research the nonlinear equations of forward kinematics is converted to two groups of linear matrix equations[22]. In the latest researches it is proposed to solve algebraic polynomial relations for the attachment points instead of solving complicated trigonometric equations[18]. However, most of above mentioned solutions depend on the flat shape end-effectors, fixed frames and the attachment points location. Therefore, they cannot be used in the F.K. of the KNTU CDRPM, and it is necessary to generate a F.K. solution for the KNTU CDRPM. In this paper a numerical method is used here to satisfy the required performance for the purpose of kinematics and workspace analysis. For the numerical solution of the forward kinematics problem, assume that the limb lengths $\boldsymbol{L} = [l_1, l_2, l_3, l_4, l_5, l_6, l_7, l_8]^T$ are given and the problem is to find the position ${}^A\boldsymbol{x} = [x, y, z]^T$, and the orientation of the moving platform represented by moving x, y, z, euler angles. Therefore, the six unknown variables can be encapsulated by the vector x as

$$\boldsymbol{x} = [x, y, z, \theta_x, \theta_y, \theta_z]^T \tag{3}$$

On the other hand, equation 2 provides eight nonlinear equations, for i = 1, 2, . . . , 8, to be solved simultaneously. The numerical solution consists of iteratively finding the

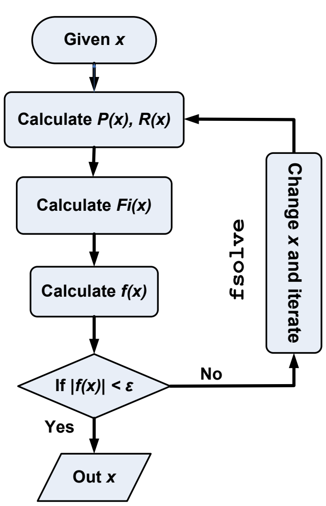

Fig. 3. Flowchart of iterative routine used to solve the forward kinematics of the KNTU CDRPM manipulator

solution to the following eight nonlinear equations:

$$F_i(x) = [\boldsymbol{P}(x) + \boldsymbol{R}(x)\boldsymbol{E}_i - \boldsymbol{a}_i]^T [\boldsymbol{P}(x) + \boldsymbol{R}(x)\boldsymbol{E}_i - \boldsymbol{a}_i] - L_i^2 \tag{4}$$

Numerical methods using nonlinear least-square optimization routines , can be used to find a solution to $F(x) = 0$. In a least-square problem, the functional $f(x) = \frac{1}{2}\sum_i F_i(x)^2$ is minimized over $\boldsymbol{x} \in \mathcal{R}^{\mathrm{n}}$. The Gauss–Newton and the Levenberg–Marquardt methods [23], are the two main search routines used to solve the nonlinear least-square problem [1]. The flowchart given in figure 3 reveals the details of the iterative method used to find the forward kinematics solution. As it is seen in this flowchart, for a given x, the values of $\boldsymbol{P}(x)$ and $\boldsymbol{R}(x)$. Then $F_i(x)$ for $i = 1, 2, \ldots, 8$ are calculated. Then the value of $f(x) = \frac{1}{2}\sum_i F_i(x)^2$ is calculated, and if it is not very close to zero, an optimal search routine is used to recalculate a new value for $\boldsymbol{x}$. This iteration is followed to reach to a solution to $f(x) = 0$ with an accuracy of $\varepsilon \ll 1$. Multiple solution may exist for the equation $F(x) = 0$, and in order to avoid jumps in the forward kinematics solutions, in the numerical routine the solution at previous iteration is used for the search of the next solution. Simulation results detailed in figure 4 illustrate the effectiveness, and accuracy of the numerical routines used to solve forward kinematics in a trajectory like 5. Furthermore, comparing F.K method results with inverse one can be used for detecting whether F.K is accessible in the given point or not.

III. Jacobian Analysis

Jacobian analysis plays a vital role in the study of robotic manipulators [24]. Let the actuated joint variable be denoted by a vector $\boldsymbol{L}$ and the location of the moving platform be described by a vector $\boldsymbol{x}$. Then the kinematic constrains imposed by the limbs can be written in the general form $\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{L}) = 0$ by differentiating with respect to time, we obtain a relationship between the input joint rates and the end-effector output velocity as follows :

$$J_x\dot{\boldsymbol{x}} = J_L\dot{\boldsymbol{L}} \tag{5}$$

where $J_{\boldsymbol{x}} = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}$ and $J_{\boldsymbol{L}} = -\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{L}}$. The derivation above leads to two separate Jacobian matrices Hence the overall

[1] `fsolve` function of Matlab®

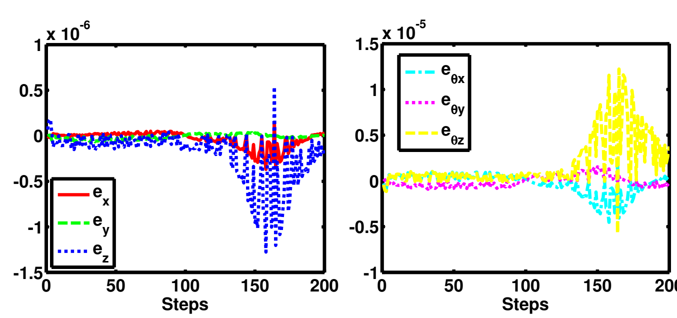

Fig. 4. Orientation errors of the forward kinematics solution

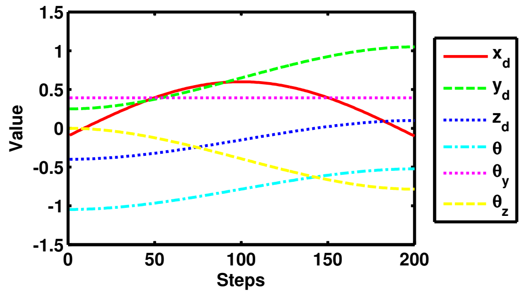

Fig. 5. Given 6D trajectory for the forward kinematics solution

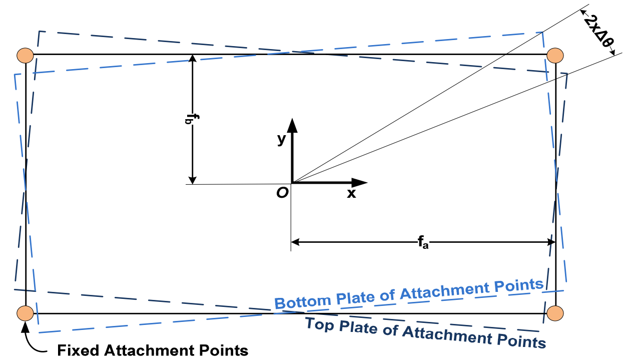

Fig. 6. Changed design parameters

Jacobian matrix J can be written as:

$$\dot{\boldsymbol{L}} = \boldsymbol{J} \cdot \dot{\boldsymbol{x}} \tag{6}$$

where $\boldsymbol{J} = \boldsymbol{J}_L^{-1} \boldsymbol{J}_{\boldsymbol{x}}$.

A. Jacobian Formulation

The Jacobian matrix not only reveals the relation between the joint velocities $\dot{\boldsymbol{L}}$ and the moving platform velocities $\dot{\boldsymbol{x}}$, but also constructs the transformation needed to find the actuator forces $\boldsymbol{\tau}$ from the forces acting on the moving platform $\boldsymbol{F}$. When J_L is singular and the null space of J_L is not empty, there exist some nonzero $\dot{\boldsymbol{L}}$ vectors that result zero $\dot{\boldsymbol{x}}$ vectors which called serial type singularity and when J_x becomes singular, there will be a non-zero twist $\dot{\boldsymbol{x}}$ for which the active joint velocities are zero. This singularity is called parallel type singularity [1]. In this section we investigate the Jacobian of the CDRPM platform shown in figure 1. For this manipulator, the input vector is given by $\boldsymbol{L} = [L_1, L_2, \ldots, L_8]^T$, and the output vector can be described by the velocity of the centroid G and the angular velocity of the moving platform as follows :

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \boldsymbol{V}_G \\ \boldsymbol{\omega}_G \end{bmatrix} \tag{7}$$

The Jacobian matrix can be derived by formulating a velocity loop-closure equation for each limb.

$$\boldsymbol{V_G} + \boldsymbol{\omega}_G \times \boldsymbol{E_i} = \dot{L}_i \hat{\boldsymbol{S}}_i + L_i \left(\boldsymbol{\omega}_i \times \hat{\boldsymbol{S}}_i\right) \tag{8}$$

where, the vector definitions $\hat{S}_i$ and $\overrightarrow{E_i}$ are illustrated in figure 2. Furthermore $\boldsymbol{\omega}_i$ denotes the angular velocity of i'th limb with respect to the fixed frame A. To eliminate $\boldsymbol{\omega}_i$, dot-multiply both sides of equation 8 by $\hat{\boldsymbol{S}}_i$.

$$\dot{L}_i = \hat{\boldsymbol{S}}_i \boldsymbol{V}_G + \left(\boldsymbol{E}_i \times \hat{\boldsymbol{S}}_i\right) \boldsymbol{\omega}_G \tag{9}$$

. Using a matrix form of equation 9 for $i = 1, 2, \ldots, 8$, the CDRPM Jacobian matrix J is derived as following.

$$\boldsymbol{J} = \left[\begin{array}{c|c} \hat{\boldsymbol{S}}_1^T & (\boldsymbol{E}_1 \times \hat{\boldsymbol{S}}_1)^T \\ \hat{\boldsymbol{S}}_2^T & (\boldsymbol{E}_2 \times \hat{\boldsymbol{S}}_2)^T \\ \vdots & \vdots \\ \hat{\boldsymbol{S}}_8^T & (\boldsymbol{E}_8 \times \hat{\boldsymbol{S}}_8)^T \end{array}\right] \tag{10}$$

Note that the CDRPM Jacobian matrix $\boldsymbol{J}$ is a non–square 8×6 matrix, since the manipulator is a redundant manipulator.

B. Singularity Analysis

In the robotic literature many dexterity and singularity measures have been introduced which have helped the designers of serial and parallel robots. Some well–defined dexterity measures have been introduced for serial [25] and parallel manipulators [26]. Moreover, Merlet reviewed most of the manipulability and dexterity measures for parallel manipulators. His latest comprehensive review leads to doubt on dexterity indices as an open problem. Additionally, in his opinion, the most appropriate global accuracy indices are the determination of the maximal positioning errors, their average values, and their variance. On the other hand, for serial spatial manipulators, Gosselin [27] proposes to assume three virtual points as vertexes of a triangle on the end-effector and to build a Jacobian matrix for their translational velocities. Then, the method is used for a conventional parallel mechanism like Stewart-Gough platform by Kim and Ryu [28]. Although the method is useful for the Stewart-Gough platform, it is not a suitable method for spatial shapes of the attachment points, since, the method cannot cover replacement of the attachment points if they are moved on any normal direction to the trianglular plane. In other words, for a manipulator with spatial shaped end-effector, at least four virtual points are required to modify the Gosselin method and determine effects of the attachment point replacement while it is hard to avoid rank deficiency during the mapping. Therefore, it is proposed to measure the dexterity of the KNTU CDRPM by Jacobian condition number [29]:

$$Cn = \frac{\sigma_{max}}{\sigma_{min}} \tag{11}$$

where, σ_{max} and σ_{min} are the maximum and minimum singular values of the Jacobian matrix, respectively. Therefore, it is suitable to have $Cn = 1$ which describes an isotropic Jacobian matrix in the given position and an infinity or a big value of the condition number explains singular condition of the manipulator at the given position. Repeating this test for each grid point of the workspace, determines singular points within the workspace. As shown in figure 7, singularity exists within the workspace of the robot. Analysis of the Jacobian condition in the singular positions shows that a major problem exists in the Neuron design of the KNTU CDRPM; rotation about z axis of the end-effectors which strongly depends on the translation along z axis especially in the central locations of the workspace. In other words, linear independency of 4th and 6th columns of the Jacobian fails in the $z = 0$ plane and the Jacobian rank deficiency disturbs motion control of the robot at the position. Therefore, rearrangement of the attachment points is necessary to decouple the role of the cables in translations and rotations of the end-effector. Since the shape of the end-effector is suitable to avoid cable to body collision [12], arrangement of the fixed attachment points should be studied. To achieve a better condition number on the $z = 0$ plane, we had to disturb symmetric arrangement of the fixed attachment points, A_is. The change contains width to length ratio of the fixed frame and rotation of the top and bottom plates about z axis as shown in figure 6. Therefore, symmetric and role dependence of the cables can be remedied by changing $f_a, f_b, \Delta\theta$. Effects of changing these parameters on the condition number are illustrated in figure 11. Note that the global condition number used in the latest analysis is:

$$GC = \frac{\max(\sigma_{max_i})}{\min(\sigma_{min_i})} \tag{12}$$

in which, $\max(\sigma_{max_i})$ and $\min(\sigma_{min_i})$ are the global maximum and minimum singular values of the Jacobian matrix through the entire workspace, respectively. Therefore, if the *Global Condition Number*(GC) approaches to 1, the workspace is more dexterous and homogeneous. Analysis of the results shows that:

- Increment of $\Delta\theta$ has significant role in the singularity avoidance within the workspace.
- Same values of f_a and f_b, and near zero values of $\Delta\theta$ lead to singularity of the KNTU CDRPM.
- There exists a region in the results where the manipulator workspace is singular free.

However, the values of design parameters $\Delta\theta$, f_a and f_b cannot set to the ideal values because another problem may significantly decrease the workspace of the robot. Cable to cable and cable to end-effector collisions can bound the end-effector motion. Implementation of the collision detection methods [12] leads to figure 10. This figure shows the ratio of the accessible collision free workspace to the volume of the fixed frame ratio while the design parameters are changing. These results show that *the best global condition number region located in the worse collision free workspace* and vice versa. Consequently, to detect performance of a point in the workspace, a program proposed as shown in flowchart of figure 8. For a given $\boldsymbol{x}$ vector,inverse kinematics calculates $\boldsymbol{L}$ and proposed FK returns it again in Cartesian space to examine kinematics solutions. Moreover, collision detection algorithms proposed in [12] is used to determine collision avoidance. Then, proposed singularity analysis and the force feasibility algorithm [13] is applied to the KNTU CDRPM to verify its controllability. Finally, the result of this examination shows that the manipulator workspace is 85% without rotation of the end-effector and 72% when the end-effector rotates 20° about its x axis. This workspace is much larger compared to that of a similar eight actuator CDRPM [10].

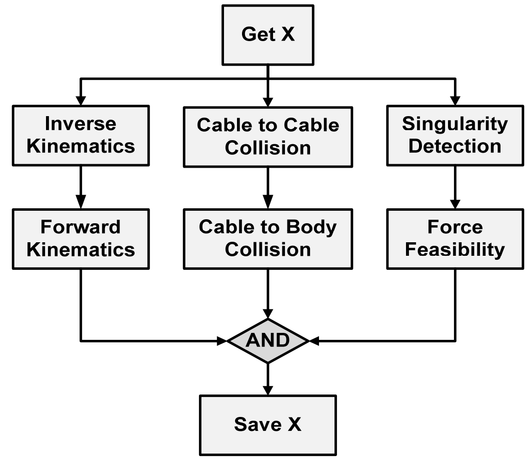

Fig. 8. Flowchart of iterative routine used to determine workspace of a cable driven manipulator

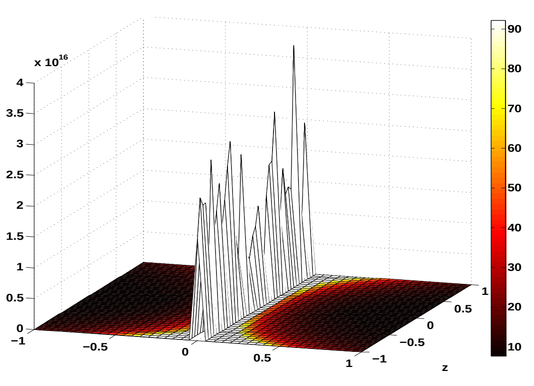

Fig. 7. Condition number on $x = 0$ plane

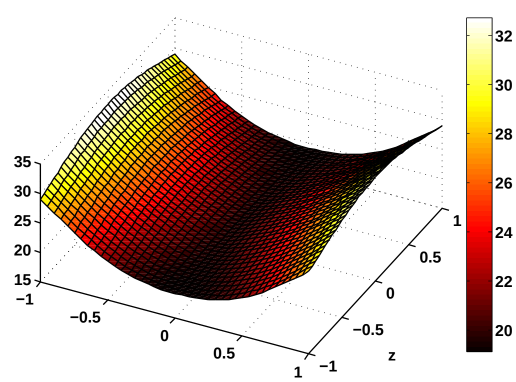

Fig. 9. The condition number values on the $x = 0$ after the change in design parameters

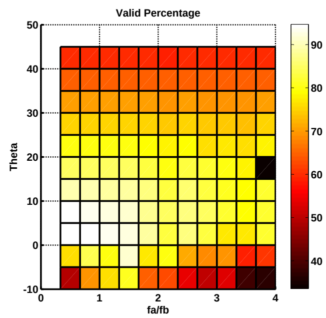

Fig. 10. Collision free workspace percentage

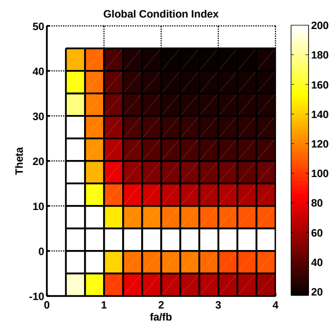

Fig. 11. Dependency of global condition number on the design parameters

IV. Conclusion

In this paper, inverse kinematics and forward kinematics are derived. Then, the 8×6 Jacobian matrix of the manipulator is extracted. Furthermore, singularity of the Jacobian is analyzed to determine condition of the manipulator. Next, effects of design parameters on the condition are examined. The results show that a singular configuration exists right at the middle of the workspace. A thorough analysis in the Jacobian explains dependency of the actuator force and torque on the two motion directions. Therefore, the role of three design parameters of the manipulator is examined by the analysis of the global condition number. Through the resulting maps of dexterity, design of the KNTU CDRPM is significantly improved. This revision leads to a singular free workspace for the KNTU CDRPM. On the other hand, a method is proposed to examine whole kinematic properties of a manipulator through its entire workspace, which is necessary for an optimal design problem. The method contains the analysis of inverse and forward kinematics, collision detection, dexterity index, and tension force feasibility. The proposed method is useful in workspace analysis, trajectory planning, and optimal design of CDRPMs.

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