Nonlinear Sensorless Speed Control of PM Synchronous Motor via an SDRE Observer-Controller Combination

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***Abstract*—In this paper, a nonlinear control approach for permanent magnet synchronous motor (PMSM) drives, without mechanical sensors, is presented. The proposed method is based on a combination of the well-known nonlinear regulation technique, state-dependent Riccati equation (SDRE), and its filtering counterpart (SDREF) which is derived by constructing the dual of this control method. The SDREF is designed for online estimation of the rotor speed, position and also the load torque by only measuring the motor voltages and currents. The estimated values are applied in the controller structure as state variables. We demonstrate the performance and applicability of this new nonlinear control for PMSM derives by an illustrative simulation in presence of variable load torque. The sensitivity and robustness of the proposed method are also investigated for the motor parameters variations.**

***Index Terms*—Nonlinear sensorless drive, permanent magnet synchronous motor (PMSM), state-dependent Riccati equation (SDRE) technique, SDRE filter.**

I. INTRODUCTION

PERMANENT magnet synchronous motors (PMSM) are widely used in industry because of their high power density, high efficiency, large torque to inertia ratio, rugged construction, easy maintenance and ease of control [1], [2]. Due to emergent applications using these machines a lot of researchers try to optimize the control of such drives and mainly to suppress mechanical sensors.

PMSMs are nonlinear multivariable systems thus, achieving high performance speed or position control with conventional control strategies is difficult. Therefore it is more reliable to use a nonlinear control technique. Nonlinear speed control techniques such as feedback linearization control and sliding mode control have been implemented for the PMSM derives [3]-[5].

In recent years, state-dependent Riccati equation (SDRE) techniques are increasingly being used in a wide variety of nonlinear control and nonlinear filtering applications [6]. The SDRE techniques are defined by their linear-like structures which have state-dependent coefficient (SDC) matrices. The SDRE nonlinear regulator has the same structure as infinite-horizon linear quadratic regulator (LQR) which requires solving an algebraic Riccati equation (ARE) to find a nonlinear control law [7]-[9]. With this method one can trade off between the control accuracy and control effort by minimizing a quadratic cost function.

Although, there exist a number of other schemes for this purpose, SDRE is one of the few successful approaches that have important properties such as applicability to large class of nonlinear systems, and its systematic and relatively easy formulation [10]. Furthermore, it can be applied to the PMSM derives because we can easily parameterized the motor equations into the SDC form.

The implementation of this control method requires an accurate knowledge of the rotor shaft speed and also load torque. This implies the need for speed or position sensors such as an absolute encoder, a magnetic resolver or a Hall-Effect sensor attached to the shaft of the motor. However, in most applications, the use of such sensors will increase the complexity, weight and cost of the system and reduce the overall reliability of the controlled drive system [1]. As a result, several sensorless control methods for PMSMs are provided (see [11]-[13] and the references cited therein).

Stochastic filtering techniques seem to be one of the most suitable strategies for sensorless drives [14]. The basic idea is to estimate rotor position and/or speed through measured stator terminal quantities. The extended Kalman filter (EKF) has found wide application for this purpose [13], [15]. The EKF is based on linearization about the current estimate of the state. If the state estimate is fairly accurate, this method has been shown to be effective. If, however, the estimates are poor, linearization can cause errors in the filter, which can lead to degraded performance or possible divergence of the state estimate.

A recently proposed nonlinear estimation technique, called the state-dependent Riccati equation filter (SDREF) [16], which originates from the SDRE control method discussed above, allows the incorporation of nonlinear dynamics and/or measurements in the filter deign without the need to linearize the equations. This filtering technique has the same structure as the steady state linear Kalman filter and has additional degrees of freedom that can be used to either enhance filter performance, avoid singularities or avoid loss of observability.

In presence of load torque disturbance conventional sensorless schemes have steady state- and/or transient-state errors which can be suppressed or reduced by injection of a load compensation signal [10], [17] and [18]. Here, this problem is addressed by developing the SDREF for simultaneous state and parameter estimation [19], i.e. the load torque variations are also estimated. In this work we combine the SDRE control technique with this filtering method, to design a new sensorless control system for PMSM drives. Consequently in this way, a uniform nonlinear control structure is obtained, which possesses the beneficial features of the SDRE control technique as well as the SDRE filter, simultaneously.

The outline of the paper is as follows. In Section II, the mathematical model of the PMSM drives is presented. In Section III, the SDRE nonlinear regulator is introduced and an SDRE-based tracking control strategy for PMSM is developed. Section IV reviews the SDREF estimation method and explains how it can be employed for our purpose. Then, we use MATLAB/Simulink to implement the proposed sensorless control strategy and we assess its robustness property against motor parameters variations in Section V. Finally, we conclude and summarize in Section VI.

II. PMSM MODEL

For the purpose of control and estimation we need the mathematical model of the PMSM. The dynamic model of the PMSM is similar to that of the wound rotor synchronous motor. It can be described by a set equations in phase variables, but a simpler representation for the machine is obtained when either $\alpha\beta$ or dq reference frames are applied. As it is well known, the $\alpha\beta$ and dq variables are derived from the a-b-c variables thorough the 3/2 and S/R (station to rotation) transformations respectively, which are defined as

$$\begin{bmatrix} f_\alpha \\ f_\beta \end{bmatrix} = 2/3 \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} \tag{2.1}$$

$$\begin{bmatrix} f_d \\ f_q \end{bmatrix} = \begin{bmatrix} \cos\theta_r & \sin\theta_r \\ -\sin\theta_r & \cos\theta_r \end{bmatrix} \begin{bmatrix} f_\alpha \\ f_\beta \end{bmatrix} \tag{2.2}$$

where f can represent voltage, current or flux and θ_r is the rotor electrical position, i.e., the angle between the d-axis and the α-axis.

Choosing the rotor fixed currents i_d, i_q, and the rotor mechanical angular velocity ω_r as state variables x, and the fundamental voltages, v_d, v_q, as the input vector u, in the dq reference frame, a three-phase non-salient PMSM can be described by the following equations [10], [13]:

$$\begin{aligned} \dot{x}_1 &= -\frac{r_s}{L_d} x_1 + N_p \frac{L_q}{L_d} x_2 x_3 + \frac{1}{L_d} u_1 \\ \dot{x}_2 &= -N_p \frac{L_d}{L_q} x_1 x_3 - \frac{r_s}{L_q} x_2 - N_p \frac{\lambda_r}{L_q} x_3 + \frac{1}{L_q} u_2 \\ \dot{x}_3 &= \frac{3}{2} N_p \frac{\lambda_r}{J} x_2 - \frac{D}{J} x_3 - \frac{1}{J} T_L \end{aligned} \tag{2.3}$$

where $x = [x_1 \quad x_2 \quad x_3]^T = [i_d \quad i_q \quad \omega_r]^T$ is the state vector, $u = [u_1 \quad u_2]^T = [v_d \quad v_q]^T$ is the control input, and T_L is the load torque which can be treated such as an unknown exogenous input. r_s is the stator resistance, L_d is the inductance in the direct axis, L_q is the inductance in the quadrature axis, N_p is the number of the pair poles, λ_r is the rotor magnetic flux, J is the rotor moment of inertia, and D is the viscous friction coefficient.

The equilibrium points of (2.3) are given by

$$\begin{aligned} x_2 &= \frac{2}{3N_p\lambda_r}(Dx_3 + T_L) \\ u_1 &= r_s x_1 - N_p L_q x_2 x_3 \\ u_2 &= N_p L_d x_1 x_3 + r_s x_2 + N_p \lambda_r x_3 \end{aligned} \tag{2.4}$$

For the fixed desired speed, x_{3d}, and assume that field orientation principle is satisfied we can set $x_{1d} = 0$. Then the desired value for x_2 would be

$$x_{2d} = \frac{2}{3N_p\lambda_r}\left(Dx_{3d} + T_L\right) \tag{2.5}$$

The d- and q-axis currents are chosen as the measurement vector, $y = [y_1 \quad y_2]^T$, then the output equations are as follows

$$\begin{aligned} y_1 &= i_d \\ y_2 &= i_q \end{aligned} \tag{2.6}$$

The nonlinear optimal control design for the system given by (2.3) is presented in the following section.

III. THE SDRE CONTROL TECHNIQUE

In this section we provide a brief overview of the suboptimal nonlinear regulator, SDRE method. Moreover, we develop a tracking SDRE-based control system for the PMSM drives which is similar to linear state feedback with integral action [20].

The SDRE technique can be illustrated by considering the following autonomous infinite-horizon nonlinear regulator problem [7]:

Minimize the performance index

$$I(x,u) = \frac{1}{2}\int_0^\infty (x^T Q x + u^T R u) dt \tag{3.1}$$

with respect to the state x and control u associated with the nonlinear differential constraint

$$\dot{x} = f(x) + g(x)u \tag{3.2}$$

where $x \in R^n$, $u \in R^m$, $Q \in R^{n \times n}$ is symmetric positive semi-definite (SPSD) and $R \in R^{m \times m}$ is symmetric positive definite (SPD). Moreover, $x^T Q x$ is a measure of control accuracy and $u^T R u$ is a measure of control effort. Here it is assumed that $f(0) = 0$ and $f(x)$ is a continuously differentiable function of x, i.e., $f(x) \in C^l$.

The SDRE approach for obtaining a suboptimal locally asymptotically stabilizing solution of problem (3.1)-(3.2) is:

1) Use direct parameterization to bring the nonlinear dynamics to the state-dependent coefficient (SDC) form

$$\dot{x} = A(x)x + B(x)u \tag{3.3}$$

where

$$f(x) = A(x)x \qquad g(x) = B(x) \tag{3.4}$$

2) Find the unique symmetric positive definite solution $P(x)$ of the state-dependent Riccati equation (SDRE)

$$\begin{aligned} &A^T(x)P(x) + P(x)A(x) \\ &\qquad - P(x)B(x)R^{-1}B^T(x)P(x) + Q = 0 \end{aligned} \tag{3.5}$$

3) Construct the nonlinear feedback controller via

$$u(x) = -K(x)x = -R^{-1}B^T(x)P(x)x \tag{3.6}$$

In [8], it is shown that under given mild conditions of stabilizability and detecability, this method has desirable properties of local stability and suboptimality.

Remarks:

1) Here, we formulated the SDRE control technique with constant weighting matrices, Q and R. The theory for state-dependent weighting matrices, $Q(x)$ and $R(x)$, is developed in [6], [8] and [9].

2) It is shown that (see, e.g., [8]), in the multivariable case like our problem, if $f(x) \in C^l$ then there exists an infinite number of ways to factor $f(x)$ into $A(x)x$. In order to obtain a valid solution of the SDRE, the pair $\{A(x), B(x)\}$ has to be pointwise stabilizable in the linear sense for all x in the domain of interest.

The PMSM model given by the state equations (2.3) can be put into the SDC form (3.3) by

$$A(x) = \begin{bmatrix} -\dfrac{r_s}{L_d} & N_p \dfrac{L_q}{L_d} x_3 & 0 \\ -N_p \dfrac{L_d}{L_q} x_3 & -\dfrac{r_s}{L_q} & -N_p \dfrac{\lambda_r}{L_q} \\ 0 & \dfrac{3}{2} N_p \dfrac{\lambda_r}{J} & -\dfrac{D}{J} \end{bmatrix} \tag{3.7}$$

$$B(x) = \begin{bmatrix} 1/L_d & 0 & 0 \\ 0 & 1/L_q & 0 \end{bmatrix}^T$$

Note that, here the control matrix $B(x)$ is a constant matrix.

In order to perform command following, the above technique should be modified appropriately. In [6], the SDRE controller is implemented as an integral servomechanism. This is accomplished by decomposition of the state x into tracking and regulating components and then augmentation of the state vector with the integral states of the tracking component.

Here, based on the theory of linear tracking systems with integral control, we propose an alternate SDRE nonlinear tracking scheme for PMSM. The output equation (2.4) can be written as

$$y(t) = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}_{C} x(t) \tag{3.8}$$

It is desired that the rotor speed (ω_r) tracks some reference commands, but the output vector, y , does not include the corresponding state variable (x_3). Thus, y is augmented with ω_r as

$$\tilde{y}(t) = \begin{bmatrix} y(t) \\ \omega_r(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\tilde{C}} x(t) \tag{3.9}$$

It is assumed that the pair $\{A(x), B(x)\}$ is controllable in linear sense. We define the integral state $q(t)$ as follows

$$\dot{q}(t) = r(t) - \tilde{y}(t) = r(t) - \tilde{C}x(t) \tag{3.10}$$

where $r(t)$ is the reference input. Considering the integral state, the augmented system becomes (parameter t is omitted)

$$\begin{aligned} &\dot{\bar{x}} = \bar{A}(\bar{x})\bar{x} + \bar{B}(\bar{x})u + \begin{bmatrix} 0 \\ I \end{bmatrix} r \\ &\tilde{y}(t) = \bar{C}(\bar{x})\bar{x} \end{aligned} \tag{3.11}$$

with $\bar{x} = \begin{bmatrix} x \\ q \end{bmatrix}$, $\bar{A}(\bar{x}) = \begin{bmatrix} A(x) & 0 \\ -\tilde{C} & 0 \end{bmatrix}$, $\bar{B}(\bar{x}) = \begin{bmatrix} B \\ 0 \end{bmatrix}$ and $\bar{C}(\bar{x}) = \begin{bmatrix} \tilde{C} & 0 \end{bmatrix}$. The matrices $A(x)$ and B are given by (3.7) and the matrix $\tilde{C}$ is defined by (3.9).

It is straightforward to show that, the augmented system is controllable if the matrix

$$M(x) = \begin{bmatrix} B & A(x) \\ 0 & -\tilde{C} \end{bmatrix} \tag{3.12}$$

has full rank for all x in the domain of interest. This condition follows directly from [20, chapter 19, pp. 651-653].

Applying the SDRE technique, equations (3.3)-(3.6), to the augmented system (3.11) yields a state feedback control law of the form

$$u(\bar{x}) = -R^{-1}\bar{B}^T(\bar{x})\bar{P}(\bar{x})\bar{x} \tag{3.13}$$

where $\bar{P}(\bar{x})$ satisfies the Riccati equation

$$\begin{aligned} &\bar{A}^T(\bar{x})\bar{P}(\bar{x}) + \bar{P}(\bar{x})\bar{A}(\bar{x}) \\ &\qquad - \bar{P}(\bar{x})\bar{B}(\bar{x})R^{-1}\bar{B}^T(\bar{x})\bar{P}(\bar{x}) + \bar{Q} = 0 \end{aligned} \tag{3.14}$$

Note that $\overline{Q}$ has appropriate dimensions proportional to the augmented state, $\overline{x}$.

This leads to a locally asymptotically stable closed loop system. Therefore, the steady state values of $x(t)$ and $q(t)$ will be constant and we have

$$\lim_{t\to\infty}\dot{q}(t) = \lim_{t\to\infty}(r(t) - y(t)) = 0 \tag{3.15}$$

Hence

$$\lim_{t\to\infty} y(t) = r(t) \tag{3.16}$$

which implies that the output of the closed loop system, here means the rotor speed, tracks the desired reference input and we have an SDRE-based tracking system.

The control strategy found by the above method is a full-state feedback and can be implemented for the PMSM drives with the knowledge of the system states (current components and speed) and also the load torque T_L. However, according to (2.4), the only measurable states are currents components (i_d, i_q). Moreover, there are always noise sources such as process noise, due to inaccuracy of the model, and measurement noise. Hence, we need to estimate the rotor speed and load torque from a noise compelled equations of PM synchronous motor.

The SDRE filter is used to estimate the unknown states and uncertain load torque of PMSM, which is explained in the next section.

IV. The SDRE Filter

In this section, the continuous-time SDRE filter (SDREF) design is briefly reviewed and is developed to simultaneously estimate the states and load torque of the motor. The filtering counterpart of the SDRE control algorithm is obtained by taking the dual of steady state linear regulator and then allowing the coefficient matrices of the dual to be state-dependent [16].

Consider the stochastic nonlinear system

$$\begin{aligned}\dot{x} &= f(x) + g(x)u + w\\ y &= h(x) + v\end{aligned} \tag{4.1}$$

where v and w are uncorrelated Gaussian zero-mean white noise processes effecting on the system.

After bringing the system to the SDC form

$$\begin{aligned}\dot{x} &= F(x)x + G(x)u + w\\ y &= H(x)x + v\end{aligned} \tag{4.2}$$

The SDREF is given by the steady state linear continuous Kalman filter equations as

$$\dot{\hat{x}} = F(\hat{x})\hat{x} + G(\hat{x})u + K_f(\hat{x})\left[y(x) - H(\hat{x})\hat{x}\right] \tag{4.3}$$

where

$$K_f(\hat{x}) = \Gamma H^T(\hat{x})V^{-1} \tag{4.4}$$

and Γ is a symmetric positive definite matrix, which satisfies the following state-dependent Riccati equation

$$F(\hat{x})\Gamma + \Gamma F^T(\hat{x}) - \Gamma H^T(\hat{x})V^{-1}H(\hat{x})\Gamma + W = 0 \tag{4.5}$$

W and V are assumed to be, respectively, PSD and PD matrices with appropriate dimensions.

Remark 3) A usual choice for the matrices W and V are the covariances for the corrupting noise terms in (4.1), i.e.,

$$E\left[w(t)w^T(t+\tau)\right] = W(t)\delta(\tau) \tag{4.6}$$

$$E\left[v(t)v^T(t+\tau)\right] = V(t)\delta(\tau) \tag{4.7}$$

However, this is not the only possibility. Any other PD and PSD matrices may be chosen as well.

Consider the PMSM model given by (2.3)-(2.4) in a noisy environment like (4.1). Then, the SDRE filter can be applied to estimate the rotor speed and position using stator voltage and current measurements only. Although it is possible to choose a different SDC parameterization for the PMSM model in order to implement the SDRE filter, we use the same state-dependent matrices as it was used for the proposed control strategy in the previous section, i.e.

$$F(x) = A(x) \qquad G(x) = B \qquad H(x) = C \tag{4.8}$$

where $A(x)$ and B are given by (3.7) and C is defined by (3.8).

Now we use the common approach of state augmentation to estimate the unknown load torque simultaneously. By adjoining T_L to x we obtain a new state vector

$$z^T = \begin{bmatrix} z_1 & z_2 & z_3 & z_4\end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 & T_L\end{bmatrix} \tag{4.9}$$

Assuming that z_4 is evolved in accordance with $\dot{z}_4 = \beta$, where β is a Gaussian zero mean white noise, then the dynamic model of the motor can be written as

$$\begin{aligned}\dot{z} &= \overline{F}(z)z + \overline{G}(z)u + \overline{w}\\ y &= \overline{H}(z)z + v\end{aligned} \tag{4.10}$$

where

$$\overline{F}(z) = \begin{bmatrix} -\dfrac{r_s}{L_d} & N_p\dfrac{L_q}{L_d}z_3 & 0 & 0\\ -N_p\dfrac{L_d}{L_q}z_3 & -\dfrac{r_s}{L_q} & -N_p\dfrac{\lambda_r}{L_q} & 0\\ 0 & \dfrac{3}{2}N_p\dfrac{\lambda_r}{J} & -\dfrac{D}{J} & -\dfrac{1}{J}\\ 0 & 0 & 0 & 0\end{bmatrix} \tag{4.11}$$

$$\overline{G}(z) = \begin{bmatrix} G(x)\\ 0_{1\times 2}\end{bmatrix}$$

$$\overline{H}(z) = \begin{bmatrix} H(x) & 0_{2\times 1}\end{bmatrix} \qquad \overline{w} = \begin{bmatrix} w\\ \beta\end{bmatrix} \tag{4.12}$$

Application of the SDREF method to the augmented system (4.10) results in

$$\dot{\hat{z}} = \overline{F}(\hat{z})\hat{z} + \overline{G}(\hat{z})u + \overline{K}_f(\overline{z})(y - \overline{H}(\hat{z})\hat{z}) \tag{4.13}$$

where the filter gain matrix is obtained by equations similar to (4.4) and (4.5).

Using the estimated states, $\hat{z}(t)$, in the control law which is obtained by (3.13)-(3.14), the motor speed can be controlled readily.

V. SIMULATION RESUILTS

In this section, the performance of the proposed nonlinear sensorless control for the PMSM drives has been verified by simulations using MATLAB/Simulink software.

First we use the actual parameters of the PMSM model which are given in Appendix [see 1], i.e. the drive parameters are considered exactly known. The current limit is set at 3 A. Table I presents the design parameters for the SDRE controller and SDRE filter. Note that in general, once the dynamics of system become complicated, it is difficult to solve the state-dependent Riccati equations like (3.5) and (4.5) analytically. Here, the numerical methodology which is called the Taylor series method and is presented in [7], has been used for this purpose.

TABLE I. CONTROLLER AND OBSERVER PARAMETERS

Design matrices	Values
$\bar{Q}$	I_6
R	diag(1, 10)
W	diag(10^{-3}, 10^{-3}, 10^{-4}, 10^{-6})
V	$50I_2$

The motor is assumed to be at rest and unloaded at the time $t = 0^-$. Then a speed command step of 50 rad/s and a load torque equal to 3 Nm are applied. The angular speed reference has a step change of 100 rad/s per sec. Besides, a variable load torque is considered, which increases to 5 Nm at $t = 0.5$sec and then decreases to 1 Nm at $t = 1.5$sec. Under these assumptions, the simulation results which are shown in Fig. 1-Fig. 4, obtained.

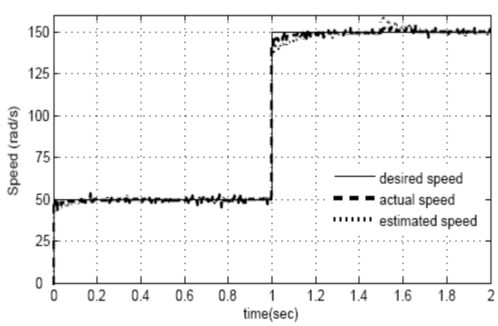

Figure 1. Speed reference, estimated and actual speed responses

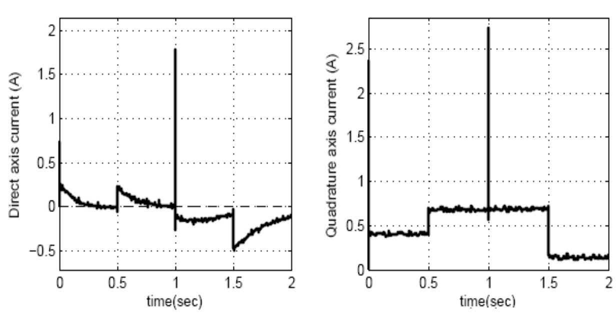

Figure 2. Actual direct and quadrature axis currents

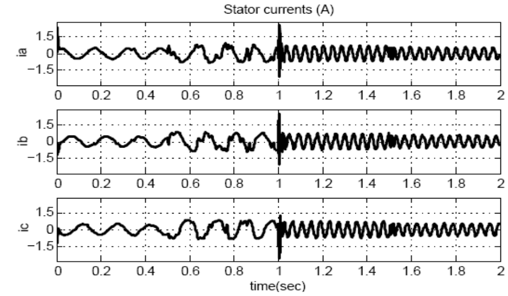

Figure 3. Three phase PMSM currents

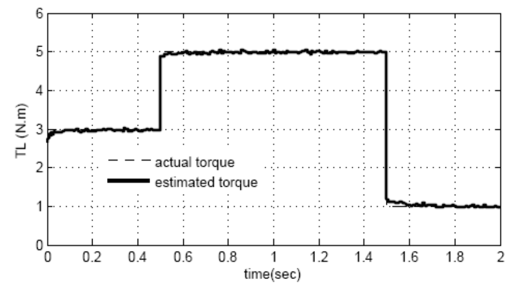

Figure 4. Actual and estimated load torque

Fig. 1 shows the actual and estimated rotor speed. It is obvious that the motor speed can track its desired value in less than 0.2 sec, even during the transient due to load torque variations. The proposed control method rejects the load torque disturbances and the motor speed follows the reference trajectory successfully. Moreover, the figure point out that the filter converges to the actual speed. The corresponding direct axis and quadrature axis currents are shown in Fig. 2. Also, Fig. 3 shows the three phase PMSM currents. Fig. 4 provides the online identification of load torque which is done by SDREF. One can note that the state/parameter filter estimates the actual values successfully.

Now to verify the robustness of the recent sensorless scheme, two distinct parametric uncertainties are considered. The first case refers to the situation in which half of the actual stator winding resistance, r_s, is applied in the SDRE controller and the SDREF. In the second case, the *dq*-inductances are assumed to have 30% inaccuracy. Fig. 5 and Fig. 6 compare the speed response of the nominal model to that of the uncertain model in these two cases.

Clearly, by tuning the matrix parameters of the SDREF, we may reduce the sensitivity of the proposed sensorless scheme to model uncertainties.

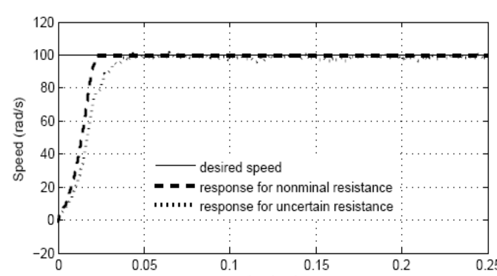

Figure 5. Estimated speed for 50% uncertainty in stator resistance

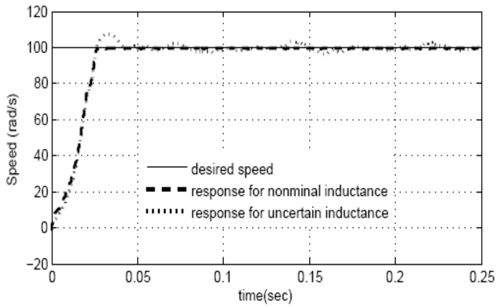

Figure 6. Estimated speed for 70% uncertainty in stator inductance

SDRE robustness analyses have been done individually for example problems rather than explicitly investigated as a separate topic. Although not quantified, it has been reported from experience that due to its LQR-like nature, SDRE exhibits robustness to disturbances and unmodeled dynamics. In addition, SDRE nonlinear H_∞ control has been proposed in [8] which may be applied for speed control of PMSMs in a future work.

VI. CONCLUSIONS

A new continuous-time nonlinear optimal control approach based on the SDRE control technique is developed to control PMSM drives, without requiring speed sensors, to increase reliability of PMSM drives. The SDREF technique is used to develop a state/parameter estimator for simultaneously estimating the state and load torque variations. The estimated values are then utilized in the nonlinear optimal control scheme. This new approach provides relatively fast control ability, a wide operating range and a good dynamic performance despite the parametric uncertainty in the motor parameters.

Promising simulation results suggest that the proposed nonlinear control is capable for effective real-time implementation on the sensorless control of PMSM drives. Issues such as the close loop stability analysis of the proposed SDRE-based control scheme and investigation of its robustness properties in an analytical framework is under current research.

APPENDIX

The technical data for the PMSM model used in simulations are as follows

TABLE II. MOTOR PARAMETERS

Parameter	Symbol	Value
Stator resistance	r_s	1.4 Ω
d-axis inductance	L_d	5.47 mH
q-axis inductance	L_q	7.58 mH
Number of pole pairs	N_p	4
Rotor magnetic flux	λ_r	0.167 Wb
Moment of inertia	J	2.9×10^{-3} kgm^2
Viscous friction coefficient	D	8.6×10^{-4} Nm/rad/s

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