

Adaptive Cascade Control of the KNTU CDRPM: A Cable Driven Redundant Parallel Manipulator

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Abstract—The challenging control problem of the cable driven redundant manipulators is due to the complexity of its dynamic and the required stringent performance for the its promising applications. This paper presents an approach to the control of the KNTU CDRPM using an adaptive cascade control scheme. The goal in this approach is achieving accurate trajectory tracking while assuring positive tension in the cables. The cascade control topology uses two loops, namely the internal and external loops. The inherent nonlinear behavior of the cable manipulator is controlled by the internal loop, while the external loop can effectively reduce the target tracking errors of the end-effector in the presence of disturbance force/torques. The cascade strategy reduces the tracking error by 80% compared to that of a single loop controller in the KNTU CDRPM. Moreover, adaptation of the cascade controller gains can significantly improve the overall tracking performance. The closed-loop performance of various control topologies are analyzed by a simulation study that is performed on the KNTU CDRPM. Since, the dynamic equations of this parallel manipulator is implicit in its general form, special integration routines are used for integration. The simulation study verifies that the proposed controller is not only promising to be implemented on the KNTU CDRPM, but also being suitable for other cable driven manipulators.

I. INTRODUCTION

Increasing performance requirements necessitates design of new types of manipulators working in a larger dexterous workspace with higher accelerations. Parallel manipulators can generally perform better than serial manipulators in terms of stringent stiffness and acceleration requirements [1]. However, limited workspace and existence of many singular regions inside the workspace of a typical parallel manipulator, limits the use of parallel manipulators in various applications. In the case of *cable driven redundant parallel manipulators* (CDRPM), the conventional linear actuators of parallel manipulators are replaced with electrical powered cable drivers. This novel engineering design idea leads immediately to a wider workspace, and higher accelerations of the moving platform due to the fact of using lighter moving parts [2]. However, forward kinematics of parallel manipulators such as CDRPM's are very complicated and difficult to solve [3]. Cables are sagged under compression forces [4], and therefore, to achieve tension forces in the cables throughout the whole *dexterous workspace*, the moving platform must be designed over-constrained [5]. In this case $m = n + 2$ cables are proposed to be used in

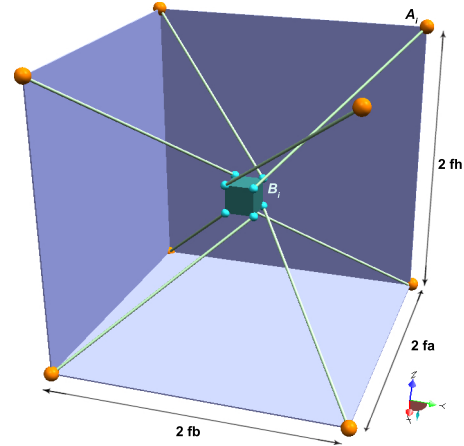


Fig. 1. The KNTU CDRPM, a perspective view

order to *dextrously* move the redundant actuated end-effector in an n -dimensional space [6]. Redundancy resolution is needed to assure tension force along each cable, however, this is usually computationally expensive [7]. The KNTU CDRPM uses a novel design to achieve high stiffness, accurate positioning for high-speed maneuvers. This paper presents an approach to the control of the CDRPM's using an adaptive cascade control scheme to achieve a stringent tracking performance while all the cables are in tension for such maneuvers.

Over the last decades, several control methods have been proposed for parallel manipulators. Among them the control of redundantly actuated parallel manipulators has attracted the attention of fewer researchers. However, only a few of the proposed topologies can be implemented in cable driven redundant parallel manipulators. Most of the proposed control schemes are based on dynamic model of the robot. Representatives of such inverse dynamic control schemes can be viewed in [8],[9] and [10]. Moreover, Fang et al. have proposed a motion control scheme on cable length coordinates [4], De Luca et al. have presented a proportional and derivative (PD) controller with on-line gravity compensation for robots with elastic joints [11], Ryeok and Agrawal developed a method for control based on feedback linearization [12], Ryeok et al. have designed a two level controller for a helicopter carrying a payload using a cable suspended robot [13] and Duchaine et al. have proposed an approach to the control of manipulators using a computationally efficient-model-based predictive control scheme [14].

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This paper presents a different control topology examined for possible implementation on KNTU CDRPM using an adaptive cascade control scheme. The proposed controller structure guarantees fully tension forces on the cables, in a more trusted fashion, and is capable to fulfill the stringent positioning requirements for these type of manipulators. This paper is organized as follows. In section II-A the inverse kinematics is derived first. Section II-B recalls dynamic modeling of the KNTU CDRPM. The cascade control with on-line gravity compensation is introduced in section III-A accompanying with the simulation analysis. In section III-B the adaptation control law for the cascade controller gains is elaborated, and the simulation results are presented. The proposed redundancy resolution scheme is examined section III-C, and finally, the concluding remarks and contributions of this work are enlightened in the last section.

II. KINEMATICS AND DYNAMICS

A. Kinematics

The KNTU Cable Driven Redundant Parallel Manipulator is illustrated in figure 1. This figure shows a spatial six degrees of freedom manipulator with two degrees of redundancy. This robot has eight identical cable limbs. The cable driven limbs are modeled as spherical-prismatic-spherical (SPS) joints, for cables can only bear tension force and not radial or bending force. Two cartesian coordinate systems $A(x, y, z)$ and $B(u, v, w)$ are attached to the fixed base and the moving platform. Points A_1, A_2, \dots, A_8 lie on the fixed cubic frame and B_1, B_2, \dots, B_8 lie on the moving platform. The origin O of the fixed coordinate system is located at the centroid of the cubic frame. Similarly, the origin G of the moving coordinate system is located at centroid of the cubic moving platform. The transformation from the moving platform to the fixed base can be described by a position vector $\vec{g} = \overrightarrow{OG}$ and a 3×3 rotation matrix ${}^A R_B$. Consider that \vec{a}_i and ${}^B \vec{b}_i$ denote the position vectors of points A_i and B_i in the coordinate system A and B , respectively. Although in the analysis of the KNTU CDRPM, all the attachment points, are considered to be arbitrary, the geometric and inertial parameters given in table I are used in the simulations. Similar to other parallel manipulator, CDRPM has a complicated forward kinematic solution [3]. However, the inverse kinematic analysis is sufficient for dynamic modeling. As illustrated in figure 1, the B_i points lie at the vertexes of the cube. For inverse kinematic analysis of the cable driven parallel manipulator, it is assumed that the position and orientation of the moving platform $x = [x_G, y_G, z_G]^T$, ${}^A R_B$ are given and the problem is to find the joint variable of the CDRPM, $\mathbf{L} = [L_1, L_2, \dots, L_8]^T$. From the geometry of the manipulator as illustrated in figure 2 the following vector loops can be derived:

$${}^A \overrightarrow{A_i B_i} + {}^A \vec{a}_i = {}^A \vec{g} + \vec{E}_i \quad (1)$$

in which, the vectors \vec{g} , \vec{E}_i , and \vec{a}_i are illustrated in figure 2. The length of the i 'th limb is obtained through taking the dot product of the vector $\overrightarrow{A_i B_i}$ with itself. Therefore, for

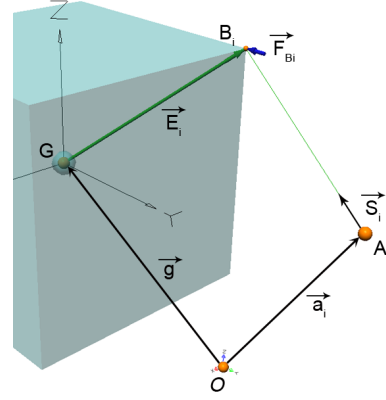


Fig. 2. i th Attachment point on the moving platform and related vectors

$$i = 1, 2, \dots, 8:$$

$$L_i = \{[\vec{g} + \vec{E}_i - \vec{a}_i]^T [\vec{g} + \vec{E}_i - \vec{a}_i]\}^{\frac{1}{2}}. \quad (2)$$

B. Dynamics

Newton-Euler method is used for dynamic modeling of CDRPM. According to acceleration of rotating velocity vector [15], the Newton-Euler equations for varying mass cable results into:

$$\begin{aligned} \mathbf{F}_{Bi} = & -\frac{1}{2}\rho L_i^2 [\dot{L}_i \boldsymbol{\omega}_i \times \hat{\mathbf{S}}_i + \dot{\boldsymbol{\omega}}_i \times \hat{\mathbf{S}}_i + \boldsymbol{\omega}_i \times (\boldsymbol{\omega}_i \times \hat{\mathbf{S}}_i)] \\ & - \frac{\rho}{2}(\dot{L}_i^2 + L_i \ddot{L}_i) \hat{\mathbf{S}}_i + \mathbf{F}_{Ai} \end{aligned} \quad (3)$$

Where \mathbf{F}_{Bi} , \mathbf{F}_{Ai} , \dot{L}_i , $\hat{\mathbf{S}}_i$, $\boldsymbol{\omega}_i$ and $\dot{\boldsymbol{\omega}}_i$ are resultant acting force on the each moving attachment point, acting forces on the A_i fixed joint, cable linear velocity along its straight, the unit vector on i th cable straight as shown in figure 2, the i th cable angular velocity about the fixed attachment point and the i th cable angular acceleration about the fixed attachment point, respectively. By using light weight cables such as the ones used in this manipulator, the gravity force effects on the cables can be ignored compared to the dynamic induced forces [16]. The cable tension force applied by cable driver unit, \mathbf{F}_{Ai}^S , can be represented by:

$$\mathbf{F}_{Ai}^S = -\boldsymbol{\tau} \quad (4)$$

TABLE I
GEOMETRIC AND INERTIAL PARAMETERS OF THE KNTU CDRPM

Description	Quantity
f_a : Fixed cube half length	1 m
f_b : Fixed cube half width	2 m
f_h : Fixed cube half height	0.5 m
C : The moving platform cube half dimension	0.1 m
M : The moving platform mass	5 Kg
I : The moving platform moment of inertia	0.033 Kg · m ²
ρ : The limb density per length	0.007 Kg/m

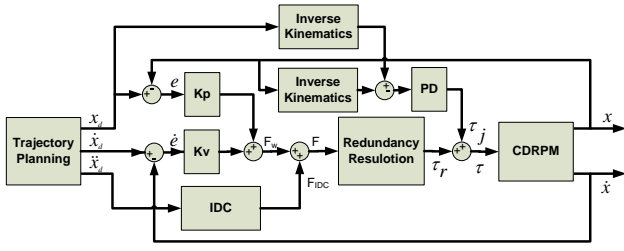


Fig. 3. The cascade control scheme

Relations between actuator forces and the end-effector affected forces had been studied in cable-affected forces. Writing the Newton-Euler equations for the moving platform describes the relation between forces, torques and acceleration of the moving platform as following:

$$M\ddot{x} = F_D + G + \sum_{i=1}^m F_{Bi} \quad (5)$$

$$I_G\ddot{\theta} = \tau_D - \sum_{i=1}^m E_i \times F_{Bi} \quad (6)$$

In which, M and I_G are moving platform mass and moment of inertia and m is number of the cables. G is effect of gravity force on the end-effector, F_D and τ_D are disturbance forces and torques effects on the moving platform with respect to the fixed frame coordinate. Also, F_{Bi} is calculated by the equation 3. Angular and linear acceleration of each cable $\ddot{\omega}_i$ and \ddot{L}_i in equation 3, depend on the end-effector acceleration. The dependency makes the motion equations implicit. Therefore, equations 5 and 6 can be viewed in a implicit 6×1 vector differential equations of the form:

$$f_f(x, \dot{x}, \ddot{x}, \mathcal{F}_D, \tau) = 0 \quad (7)$$

Where, \mathcal{F}_D is the vector of disturbance forces and moments. The governing motion equations of the manipulator can be implemented for dynamic simulation of the system. For dynamic simulation, it is assumed that the actuator forces $\tau(t)$, are given and the manipulator motion trajectory $x(t)$, is needed to be determined. Owing to implicit nature of the dynamic equation of the parallel manipulators, usual numerical integration routines such as Runge-Kutta methods [17], cannot be used to solve the problem, and a special implicit numerical solution is used to derive dynamical behavior of the CDRPM [18]. Therefore, all the dynamic components including the controller, inverse and forward dynamics and redundancy resolution routines, have to be solved simultaneously by an implicit solver as ODE15i in Matlab software [19].

III. CONTROL

In a thorough study of the dynamic behavior of the system it has been shown that due to high stiffness of the robot, there are inherent oscillations observed around the equilibrium points [15]. Therefore a controller is needed to damp the oscillations and improve trajectory tracking while robot nonlinear behavior is controlled. The details and advantages

of several joint space and task space controller topologies are addressed in [8]. However, a task space controller tries to reduce the end-effector positioning error while ignores the corresponding cables length errors. On the other hand, joint space controller cannot achieve a good tracking performance because the end-effector position is not measured. Therefore, to achieve a better tracking performance, both the joint space and workspace controllers are needed simultaneously. Thus, a cascade control scheme is proposed in this paper. In the following subsections first the topology proposed for the cascade controller is elaborated, and then, the adaptation law for the cascade controller is presented.

A. The Cascade Control

The block diagram of the cascade control is shown in figure 3. In this control scheme, two control loops are used, namely, the internal loop, which is based on decentralized PD controller in the joint space and the external loop, which is based on a decentralized PD controller in the workspace. Inherent nonlinear behavior of the cable manipulator is controlled by the internal loop, while external loop can effectively reduce the target tracking errors of the end-effector in the presence of disturbance force/torques. The gains of each controller are tuned such that the required tracking performance is achieved. Note that in this topology the redundancy resolution block is elaborated in section III-C.

Assume that the desired path of the manipulator in 3D is cylindrical as shown in figure 4. As illustrated in figure 3 the vector force, \mathcal{F} , in the external loop is determined by:

$$\mathcal{F} = \mathcal{F}_w + \mathcal{F}_{IDC} \quad (8)$$

Where \mathcal{F}_w is the created vector force by PD controller $\mathcal{F}_w = K_{pw}e(t) + K_{vw}\dot{e}(t)$. Where $e(t) = x_d(t) - x(t)$ is the trajectory tracking error and K_{pw}, K_{vw} are appropriate position and velocity gain matrices, whose values in the simulations are $10^4 \times I_{6 \times 6}$ and $10^3 \times I_{6 \times 6}$ respectively. \mathcal{F}_{IDC} is the generated vector force by the IDC [20]. Inverse dynamics generated force, preserves the end-effector current state of acceleration and obtains required force in the external loop in the form of a feedback linearization:

$$\mathcal{F}_{IDC} = \hat{M}\ddot{x}_d + \hat{G} \quad (9)$$

Where, \hat{M}, \hat{G} are inertia and gravity computed forces of the end-effector represented in the task coordinate.

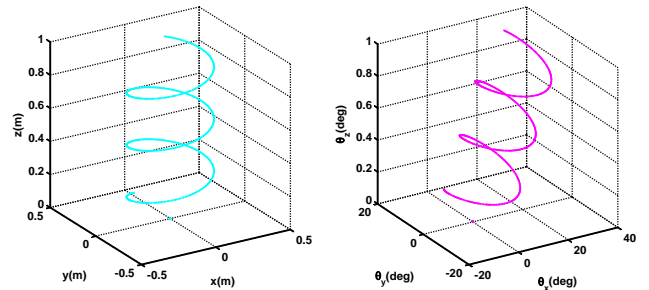


Fig. 4. Desired path in the workspace

In the internal loop, the cables length L are measured and its time derivative \dot{L} are either measured or estimated. Let L_d, \dot{L}_d denote the desired cable length and its velocity which can be easily obtained through computing the inverse kinematics. In this part, the control efforts are directly applied through the cable driver units. In this loop, the control law is as follows:

$$\tau = \tau_j + \tau_r \quad (10)$$

In which, τ is an 8×1 tension force vector along each cable, τ_r is redundancy resolution distributed forces vector and τ_j is part of the tension force in the joint space coordinate that is provided by PD controller by $\tau_j = K_{pj}e + K_{vj}\dot{e}$. The values of K_{pj} and K_{vj} which is used in the simulations are $2 \times 10^5 \times \mathbf{I}_{8 \times 8}$ and $2 \times 10^4 \times \mathbf{I}_{8 \times 8}$, respectively. These gains are tuned such that the required tracking performance is achieved. Let $e(t) = L_d - L$ and $\dot{e}(t) = \dot{L}_d - \dot{L}$ denote the error of actual cable length to that of the desired one and its derivative.

The tracking performance of the CDRPM using the proposed cascade controller is illustrated in figure 5. As seen in this figure, the proposed control topology is capable of reducing the tracking errors less than $4\mu m$ in position and less than $4 \times 10^{-5}^\circ$ in orientation. In order to compare the tracking performance of this control topology to that of a single loop controller, consider the two and infinity norms of the tracking performance as shown in figure 10, and notice the logarithmic scale that is used to represent the errors. As it is seen from this chart this proposed topology can significantly improve the tracking error norms in all the translational and rotational degrees of freedom. This significant improvement is due to the fact that the internal loop has a linearizing effect on the system, while the external loop ensures better tracking of the robot manipulator.

B. The Adaptive Cascade Control

In this section, the adaptive cascade control topology is presented. An important characteristic of this topology is the ability to adapt rapidly to any changes in system. Due to weighted role of some cable in a specific motion which is depending on the end-effector position, fixed coefficients of internal controller gain may not satisfy the necessary tracking performance. Therefore, an adaptive PD controller

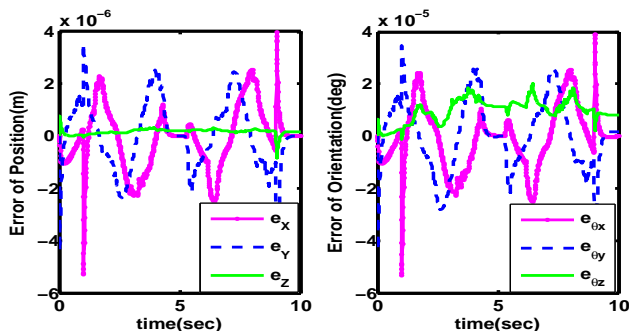


Fig. 5. The tracking error in the cascade controller

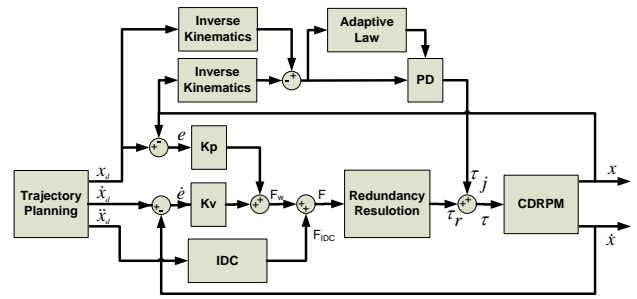


Fig. 6. Adaptive cascade control scheme

is used in the internal loop of the cascade controller. This control strategy not only preserves the advantages of previous topology, but also decreases the tracking error and increases the robot bandwidth, while guarantees fully tension forces on the cables, in a more trusted fashion. The topology of an adaptive cascade control is shown in figure 6. The difference between this strategy and previous one is that the internal gains changing according to an adaptive law [21], as following:

$$K_{pj}(t) = K_{pj}(0) + \beta_p \left(\gamma_1 e(t) + \gamma_2 \dot{e}(t) \right) e(t) + \alpha_p \int_0^t \left[\left(\gamma_1 e(t) + \gamma_2 \dot{e}(t) \right) e(t) \right] dt \quad (11)$$

$$K_{vj}(t) = K_{vj}(0) + \beta_v \left(\gamma_1 e(t) + \gamma_2 \dot{e}(t) \right) \dot{e}(t) + \alpha_v \int_0^t \left[\left(\gamma_1 e(t) + \gamma_2 \dot{e}(t) \right) \dot{e}(t) \right] dt \quad (12)$$

Where α_p, α_v are positive scalar integral adaptation gains, β_p, β_v are proportional adaptation gains, and γ_1, γ_2 are scalar weighting factors which reflect the significance of the position and velocity errors $e(t)$ and $\dot{e}(t)$ in the adaptation law. These parameters are tuned such that the required tracking performance is achieved. The parameter values are presented in table II and the adaptation of the gains are shown in figure 8.

The positioning tracking errors are given in figure 7 in a similar scale as in the normal cascade control in figure 5. Comparing these figures clearly shows the effectiveness of the adaptive cascade controller in terms of achieving

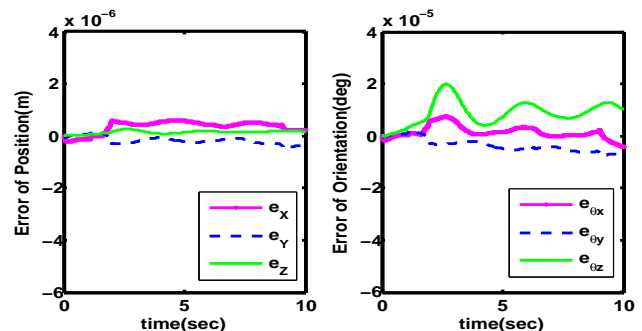


Fig. 7. The tracking error in the adaptive cascade controller

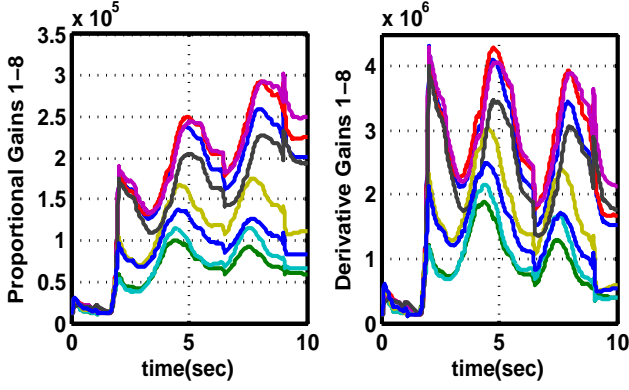


Fig. 8. The adaptive proportional and derivative gains

better performance. The tracking errors are decreased to less than $0.6\mu m$ in position and less than $2 \times 10^{-5}^\circ$ in orientation. In order to compare the tracking performance of this control topology to that of a cascade controller, and a single loop controller, consider the two and infinity norms of the tracking performance as shown in figure 10, and notice the logarithmic scale that is used to represent the errors. As it is seen from this chart this proposed adaptive cascade controller can significantly improve the tracking error norms in all the translational and rotational degrees of freedom except that in θ_z . Although the positioning error in this direction is still acceptable, it seems that the arrangement of the attachment point considered in the structure of the simulated CDRPM desensitizes the effectiveness of the internal control gains to reduce the error. This important observation leads us to re-examine a better arrangement of the attachment points for the KNTU CDRPM in the forthcoming research.

C. Redundancy Resolution

Actuator redundancy of CDRPMs is an inherent requirement in order to move the end-effector by tension forces of the cables. Redundancy resolution is an essential tool to optimally project a desired wrench in the cartesian space on the cable forces into the joint space. The KNTU CDRPM uses 8 actuators in a 6 dimensional motion. Therefore, there are infinitely many solutions for the eight actuators forces to solve the six dynamic equations. Let us denote the resulting cartesian force/torque applied to the manipulator moving platforms \mathcal{F} . In this definition \mathcal{F} is calculated from the summation of all inertial, and external forces *excluding the*

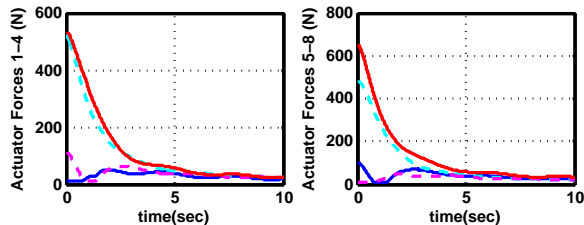


Fig. 9. The actuator forces in the cascade controller

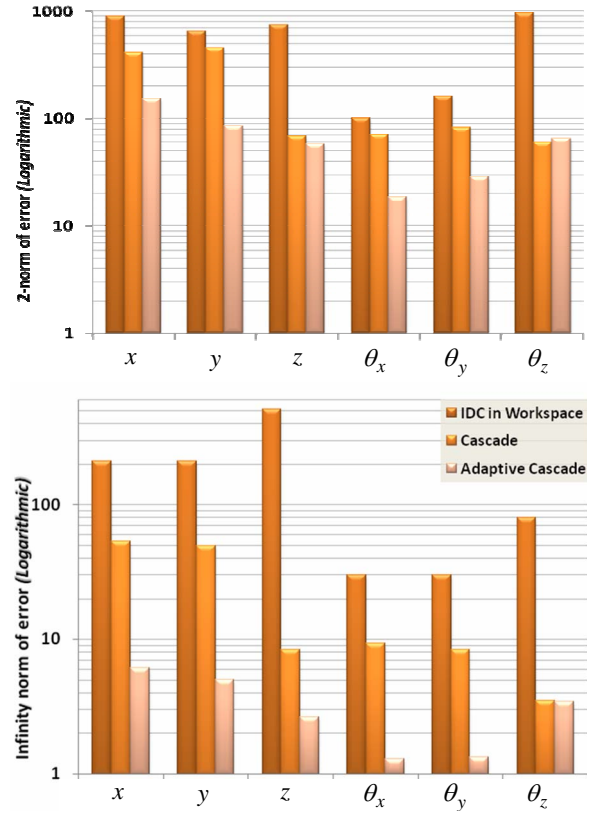


Fig. 10. Comparison of norms between three control topologies

actuator torques τ in the dynamic equations 7. Due to the projection property of the Jacobian matrix [22], $\mathcal{F} = \mathbf{J}^T \tau$ is the projection of the actuator forces onto the moving platform, and can be uniquely determined from the dynamic equations by excluding the actuator forces from them. If the manipulator has no redundancy in actuation, the Jacobian matrix, \mathbf{J} , would be squared and the actuator forces could be uniquely determined by $\tau = \mathbf{J}^{-T} \mathcal{F}$, provided that \mathbf{J} is nonsingular. For redundant manipulators, however, there are infinity many solution for τ to be projected into \mathcal{F} . The simplest solution is a minimum norm solution, which can be determined by the pseudo-inverse of \mathbf{J}^T , through:

$$\mathbf{J}^{T\dagger} = \mathbf{J}^T (\mathbf{J} \mathbf{J}^T)^{-1} \quad (13)$$

By this means, $\tau_0 = \mathbf{J}^{T\dagger} \mathcal{F}$ determines the minimum required force of each cable to generate the corresponding force, \mathcal{F} . However, this solution can result into positive or negative tensions of the cables. Since the cable forces must be kept in tension in all maneuvers, a constrained optimization technique is proposed in here to resolve the redundancy. Note that all the solutions of the projection can be determined using the null space of the Jacobian matrix by:

$$f_\tau(\gamma) = \tau_0 + (\mathbf{I}_{m \times m} - \mathbf{J}^{T\dagger} \cdot \mathbf{J}^T) \gamma \quad (14)$$

in which, \mathbf{I} is the identity matrix and γ is an m dimensional vector in the joint space. $f_\tau(\gamma)$ determines an affine hyperplane as an intersection of n subspaces of

TABLE II
ADAPTIVE CONTROLLER PARAMETERS

Parameters	Quantity	Parameters	Quantity
α_p	2×10^{13}	α_v	10^{16}
β_p	10^{14}	β_v	2×10^{12}
γ_1	4×10^3	γ_2	10^3
$K_{pj}(0)$	10^5	$K_{vj}(0)$	10^4
K_{pw}	$10^4 \times \mathbf{I}_{6 \times 6}$	K_{vw}	$10^3 \times \mathbf{I}_{6 \times 6}$

linear equality constraint defined by Jacobian transpose. To achieve a solution for the actuator forces of the CDRPM, the constrained optimization is numerically solved in order to find an optimum value for $f_\tau(\gamma)$ by finding an appropriate value for γ vector in the equation 14. In this optimization the norm of actuator efforts are minimized subject to:

$$\tau = f_\tau(\gamma) \Rightarrow (\forall i, i \in \{1, 2, \dots, m\} \Rightarrow \tau_i > \tau_{min}) \quad (15)$$

where, τ_{min} is the lower bound of the actuator forces. Other optimization techniques can be used to find the actuator forces projected from, \mathcal{F} which can minimize another user defined cost function [7].

The simulation result for the cascade controller using the proposed redundancy resolution scheme is shown in figure 9. As it is seen the proposed redundancy resolution scheme is capable to keep the actuator forces of the CDRPM always positive. Furthermore, comparing the required forces in these figures, no significant increase in the amount of required forces is seen in the adaptive scheme.

IV. CONCLUSION

A cascade control strategy is proposed to improve the overall tracking performance of a cable driven redundant parallel manipulator while system nonlinear behavior is remedied and sensitivity of the external loop to nonlinearity of the cables dynamics is decreased. The main idea in this controller algorithm is to use of two control loops simultaneously, namely the internal loop, which is based on decentralized PD controller in the joint space and the external loop, which is based on a decentralized PD controller in the workspace. Inherent nonlinear behavior of the cable manipulator is significantly reduced by internal loop, while the external loop can effectively reduce the tracking errors of the end-effector in the presence of disturbance force/torques. The work presented here represents an effective attempt to use two control loop for performance improvement in trajectory-following tasks of this type of robot manipulators. On the other hand, studying Jacobian matrix shows significant variation in the role of each cable in motion along the same axis depending on the end-effector position which can affect the loop-gain of the controller especially in the internal loop. Hence, fixed coefficients of internal controller cannot satisfy necessary tracking performance. Consequently, an adaptation method is used to achieve the required tracking performance. The simulation analysis presented on the KNTU CDRPM verifies the expected theoretical claims and demonstrated that the proposed algorithm can significantly improve the overall tracking performance

while keeping the cables under positive tension. As shown in chart 10 the cascade strategy can overall decrease the tracking error by 80% with respect to the previously advised inverse dynamic control. Moreover, adaptation of the cascade controller improves the overall tracking performance seven times than that in the cascade controller. The investigated control topologies can be carefully implemented for the other cable parallel redundant manipulators in real-time applications.

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