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Technical note

Fuzzy error governor: A practical approach to counter actuator saturation on flexible joint robots

S. Ozgoli [a], H.D. Taghirad [b,*]

[a] Tarbiat Modares University, Faculty of Engineering, Department of Electrical Engineering, Tehran, Iran
[b] K.N. Toosi University of Technology, Advanced Robotics and Automated Systems (ARAS), Faculty of Electrical and Computer Engineering, Tehran, Iran

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ABSTRACT

In this paper, a practical method to counter actuator saturation based on a fuzzy error governor is developed and a complete case study is considered. In addition to good performance, the method has two attracting properties: It does not change the structure of the main controller, and therefore, the theoretically proven characteristics of the system are untouched, and it is simply implementable in practice. The proposed controller structure is applied on a flexible joint robot (FJR). The robust stability of the closed loop system for an n-DOF FJR is thoroughly analyzed and the proposed controller is implemented on a laboratory setup to show the ease of implementation and the resulting closed-loop performance. The main controller used for the n-DOF FJR consists of a composite structure, with a PD controller on the fast dynamics and a PID controller on the slow dynamics. The bandwidth of the fast controller is decreased during critical occasions with the fuzzy logic supervisor, which adjusts the loop gain to a proper level. Using Lyapunov direct method, the robust stability of the overall system is analyzed in presence of modeling uncertainties, and it is shown that if the PD and the PID gains are tuned to satisfy certain conditions, the closed loop system becomes UUB stable.

1. Introduction

Although the rigid manipulators are extensively used in industries, the desire for high performance in the structure and mechanical specifications of robot manipulators has inspired engineers to design flexible joint robots (FJR). Several new applications such as space manipulators [1], and multi-fingered articulated hands, [2,3] have naturally adopted this design. On the other hand, traditional controllers implemented on FJRs have failed in performance [4,5], and new control strategies have been developed for FJRs in various linear, nonlinear, robust, adaptive and intelligent regimes [6,7]. Although many topologies have been proposed for the position control of the FJRs, only few of them has been implemented [8,9]. Practical limitations such as actuator saturation have been rarely considered in this area, despite the fact that it is an important practical drawback to achieve good performance [10]. As an example among these few, in [11] a PD control law with on-line gravity compensation has presented, in which global stabilization properties is guaranteed using lower positional gains. Furthermore, by applying the scheme in combination with a point-to-point interpolating trajectory the actuator efforts are prevented from saturation. Another example is [12], in which a practical robust controller with a simple structure is applied to a 6 DOF flexible joint robot. However, in this application the proposed robust controller is applied on an industrial robot, which has strong actuators and stiff enough joints, therefore the actuators does not encounter any saturation limit in the presented experiments.

Actuator saturation has been considered by the control community from early achievements of control engineering. A common classical remedy for this problem is to reduce the bandwidth of the control system such that saturation rarely occurs. This is a trivial weak solution, since even for small reference commands and disturbances the possible performance of the system is significantly degraded. On the other hand, the idea of bandwidth reduction is very practical and *easy*, which motivates some researchers to propose a kind of "adaptive" reduction in bandwidth consistent with the actuation level [13]. The adaptation process is performed by an *error governor* working under supervision of a *supervisory loop*. As proposed in [13], this process can be accomplished through complex computations, using *a priori* information about the reference input, which is not appropriate for practical implementations. In order to remove this drawback, a fuzzy logic supervisory control has been proposed by authors in [14]. In this strategy, only a fuzzy system is added to the existing controller, which can be translated to a time invariant mapping and therefore, can be easily implemented. This idea has been first presented in general form by authors in [14], and then has been modified to incorporate the composite controller for FJRs [15]. It is observed

* Corresponding author.
E-mail addresses: Ozgoli@modares.ac.ir (S. Ozgoli), Taghirad@kntu.ac.ir, hamid@cim.mcgill.ca (H.D. Taghirad).

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in various simulations and experiments that this supervisory loop will preserve the steady state performance of the system. Moreover, the supervisory loop can remove the instability caused by saturation. The stability analysis of the overall system, however, is essential for the closed loop structure in susceptible applications such as space robots, in which the stability is the main concern. This issue is analyzed thoroughly in this paper.

In fact the fuzzy supervisor provided here, is a method to counter actuator saturation in general and is not limited to FJRs. The performance of this method is verified in this paper and some previous papers [14–16]. This method has two important benefits: It is an outer loop method, and therefore it imposes little changes in the principal properties of the closed loop system, and secondly, in contrary to its similar methods [13,17] it is very easy to be implemented. This paper is devoted to show these properties with the focus on these two benefits. First the stability proof for the overall system is given to show that the main property of the pre-designed controller, namely the stability, is preserved by using the fuzzy supervisor. Then the practical implementation results are given to show the effectiveness and simplicity of implementation in practice. Through these examinations a practical method is presented to counter actuator saturation with comparable performance to its similar methods.

The idea of "reduction in bandwidth" for robot control or "reducing the speed of the robot motion" in order to prevent saturation is well understood in the robotic community. A method called "dynamic scaling of trajectory" based on a fundamental property of time scaling of trajectories for rigid robots, has been proposed in [18]. When the traveling time along a geometric path is uniformly scaled by a factor of α, the torque needed for executing the new trajectory is scaled by a factor of α^2 up to the gravitational torque contribution. This fact is used recently to prevent saturation in FJRs [19]. Although a similar idea is used in this paper, a completely different and new controller structure is implemented here. In this structure, a composite controller is proposed, with a PD controller to stabilize the fast dynamics, a PID controller to robustly stabilize the slow dynamics, and a supervisory loop to decrease the bandwidth of the fast controller adaptively during critical occasions. The robust stability of the overall system in presence of the modeling uncertainties is then analyzed in detail, and it is shown that uniformly ultimately bounded (UUB) stability of the overall system is guaranteed, provided that the PD and PID gains are tuned to satisfy certain conditions. Stability analysis for some combinations of P, I and D controllers for FJRs has been presented in some previous papers without considering saturation. Robust stability proof for a control law consisting of a PD action on motor position error and an integral action on the link position error is presented in [20]. Refs. [21,22] have discussed the stability of a composite control law consisting of a PID action on link position error and a PD action on the joint torque in addition to a correcting term. Authors have later removed the correcting term and have presented a new proof in [23]. No one of the mentioned references have considered actuator limitation. In the current paper a robust stability analysis for the same configuration but with a supervisory term proposed to counter saturation is presented. The introduction of such a fuzzy supervisor whose performance is experimentally verified, and the stability analysis of the overall closed loop system including this fuzzy error governor, are the main contributions of this paper. The paper is organized in six sections. In Section 2, the modeling of an FJR and the proposed controller structure are presented. Section 3 is devoted to the detailed description of the proposed supervisor, and Section 4 is allocated for robust stability analysis. Finally, the experimental results are presented in Section 5, and the conclusions are made in Section 6.

2. FJR modeling

In an FJR the actuator positions are not statically related to the link positions, but they are related through the dynamics of the flexile element. Therefore, the overall system state vector consists of the link positions as well as the actuator positions. It is usual to arrange these states in a vector as follows:

$$\vec{Q} = [\theta_1, \theta_2, \ldots, \theta_n, \theta_{n+1} | \theta_{n+2}, \ldots, \theta_{2n}]^T = [\vec{q}_1^T, \vec{q}_2^T]^T \tag{1}$$

where θ_i: $i = 1, 2, \ldots, n$ represents the position of the ith link and the position of the ith actuator is represented by θ_{i+n}: $i = 1, 2, \ldots, n$. With some simplifying assumptions, Spong has proposed a model for FJRs as follows [24]:

$$\begin{cases} \mathbf{M}(\vec{q}_1)\ddot{\vec{q}}_1 + \vec{N}(\vec{q}_1, \dot{\vec{q}}_1) = -\mathbf{K}(\vec{q}_1 - \vec{q}_2) \\ \mathbf{J}\ddot{\vec{q}}_2 - \mathbf{K}(\vec{q}_1 - \vec{q}_2) = \vec{u} \end{cases} \tag{2}$$

where $\mathbf{M}$ is the matrix of the link inertias and $\mathbf{J}$ is that of the motors, $\vec{u}$ is the vector of input torques and $\vec{N}$ is the vector of all gravitational, centrifugal and Coriolis torques as follows:

$$\vec{N}(\vec{q}_1, \dot{\vec{q}}_1) = \mathbf{V_m}(\vec{q}_1, \dot{\vec{q}}_1)\dot{\vec{q}}_1 + \vec{G}(\vec{q}_1) + \mathbf{F_d}\dot{\vec{q}}_1 + \vec{F}_S + (\dot{\vec{q}}_1) + \vec{T}_\mathbf{d} \tag{3}$$

In which, matrix $\mathbf{V_m}(\vec{q}_1, \dot{\vec{q}}_1)$ consists of the Coriolis and centrifugal terms, $\vec{G}(\vec{q}_1)$ is the vector of gravity terms, $\mathbf{F_d}$ is the diagonal matrix of viscous friction, $\vec{F}_S(\vec{q}_1)$ includes the static friction terms, and finally, $\vec{T}_d$ is the vector of disturbances and unmodeled but bounded dynamics. The last term in the model encapsulates the modeling uncertainties. As it is demonstrated in [25], in spite of uncertainty in all parameters, the following quantities are bounded:

$$\underline{m}\mathbf{I} \leqslant \mathbf{M}(\vec{q}_1) \leqslant \bar{m}\mathbf{I} \tag{4}$$

$$\|\vec{N}(\vec{q}_1, \dot{\vec{q}}_1)\| \leqslant \beta_0 + \beta_1\|\dot{\vec{q}}_1\| + \beta_2\|\dot{\vec{q}}_1\|^2 \tag{5}$$

$$\|\mathbf{V_m}(\vec{q}_1, \dot{\vec{q}}_1)\| \leqslant \beta_3 + \beta_4\|\dot{\vec{q}}_1\| \tag{6}$$

where $\underline{m}, \bar{m}$ and β_i's are real positive constants. These uncertainty bounds will be used in robust stability analysis later. It is assumed that all flexible elements are modeled by linear springs and without loss of generality [24], all springs are assumed to have the same spring constant k so the matrix $\mathbf{K}$ is defined as $\mathbf{K} = k\mathbf{I}$.

The inertia matrices are non-singular; therefore, the model can be reformulated into the following singular perturbation form [26]:

$$\begin{cases} \ddot{\vec{q}} = -\mathbf{M}^{-1}(\vec{q})\,\vec{z} - \mathbf{M}^{-1}(\vec{q})\,\vec{N}(\vec{q}, \dot{\vec{q}}) \\ \varepsilon\ddot{\vec{z}} = -(\mathbf{M}^{-1}(\vec{q}) + \mathbf{J}^{-1})\,\vec{z} - \mathbf{M}^{-1}(\vec{q})\,\vec{N}(\vec{q}, \dot{\vec{q}}) - \mathbf{J}^{-1}\,\vec{u} \end{cases} \tag{7}$$

In which, $\vec{q} = \vec{q}_1$, $\vec{z} = \mathbf{K}(\vec{q}_1 - \vec{q}_2)$ and $\varepsilon = 1/k$. Now if ε is chosen to be zero then the slow behavior of $\vec{z}$ could be derived as

$$\begin{aligned} \vec{z}_S &= -(\mathbf{M}^{-1}(\vec{q}) + \mathbf{J}^{-1})^{-1}[\mathbf{M}^{-1}(\vec{q})\,\vec{N}(\vec{q}, \dot{\vec{q}}) + \mathbf{J}^{-1}\vec{u}_S] \\ &= -(\mathbf{I} + \mathbf{M}(\vec{q})\mathbf{J}^{-1})^{-1}\,\vec{N}(\vec{q}, \dot{\vec{q}}) - (\mathbf{J}\,\mathbf{M}(\vec{q})^{-1} + \mathbf{I})^{-1}\vec{u}_S \end{aligned} \tag{8}$$

In which it is assumed that $\vec{u} = \vec{u}_S + \vec{u}_f$ where $\vec{u}_S$ and $\vec{u}_f$ are consequently the slow and the fast terms of the control input. Besides, the fast term is designed to be a function of $\vec{z}_f$ only and to vanish for $\vec{z}_f = 0$. Substitution of $\vec{z} = \vec{z}_S + \vec{z}_f$ into Eq. (7) using matrix inversion lemma [27], which reads $(\mathbf{I} + \mathbf{MJ}^{-1})^{-1} = \mathbf{I} - \mathbf{M}(\mathbf{J} + \mathbf{M})^{-1}$, gives

$$\ddot{\vec{q}}_S = -\mathbf{M}^{-1}(\vec{q})\vec{z}_f - [\mathbf{J} + \mathbf{M}(\vec{q})]^{-1}\,\vec{N}(\vec{q}, \dot{\vec{q}}) + [\mathbf{J} + \mathbf{M}(\vec{q})]^{-1}\vec{u}_S \tag{9}$$

In which $\vec{z}_f$ represents the fast behavior of $\vec{z}$ whose dynamics could be found to be

$$\varepsilon\ddot{\vec{z}}_f = -(\mathbf{M}^{-1}(\vec{q}) + \mathbf{J}^{-1})\vec{z}_f - \mathbf{J}^{-1}\vec{u}_f \tag{10}$$

It should be noted again that $\vec{u}_f$ is chosen to be a function of $\vec{z}_f$ only, so it is plausible to use $\vec{u}_S$ instead of $\vec{u}$ in (9). With $\vec{u}_S$ and $\vec{u}_f$ in hand the last three equations can be solved to find $\vec{q}_S, \vec{z}_f$ and $\vec{z}_S$. Then using Tikhonov theorem [28], one can show that under some stability conditions the overall behavior of the system can be determined from the following equations:

$$\vec{q}(t) = \vec{q}_S(t) + O(\varepsilon) \quad t \in [0, T]$$
$$\vec{z}(t) = \vec{z}_S(t) + \vec{z}_f(t) \quad t \in [0, T]$$
$$\exists t_1 \; \vec{z}(t) = \vec{z}_S(t) + O(\varepsilon) \quad t \in [t_1, T]$$

In which, $O(\varepsilon)$ determines the terms with order of ε. The stability conditions can be satisfied by proper selection of $\vec{u}_f$. Considering the dynamics of $\vec{z}_f$ (Eq. (10)), a proper selection of $\vec{u}_f$ is a simple PD controller [21]:

$$\vec{u}_f = [\mathbf{K_{Pf}}\vec{z}_f + \mathbf{K_{Df}}\dot{\vec{z}}_f] \tag{11}$$

For the slow subsystem different control strategies can be used. However, using a well designed PID controller for the $\vec{u}_S$, provides the following benefits: (1) no need for rate measurements, (2) no need for offline computations, particularly derivations of the reference input, and (3) guaranteed robust stability under the conditions detailed in [21]. These attributes make this structure attractive for practical implementation; hence, it is proposed to use such structure:

$$\vec{u}_S = \mathbf{K_P}\,\vec{e} + \mathbf{K_D}\,\dot{\vec{e}} + \mathbf{K_I}\int_0^t \vec{e}(\tau)\,dt \tag{12}$$

in which the error vector is defined as:

$$\vec{e} = \vec{q}_d - \vec{q} \tag{13}$$

The overall control system is shown in Fig. 1 by which the desirable performance can be achieved permitting large control effort, which may result in actuator saturation. This drawback is remedied by a supervisory loop added to the control structure which will be detailed in the proceeding section.

3. The supervisory loop

In this section, the fuzzy anti-saturation scheme as it was first proposed by the authors in [14] is described and then the modifications needed to use this idea for the FJR are provided. Without loss of generality one can assume that each element $u_i(t)$ of the control vector has a saturation limit of 1. The proposed method is a two step design procedure: first the compensator is designed without considering any saturation limit, then a time varying scalar gain $0 < \lambda(t) \leqslant 1$ is added which modifies the error (so called *error governor*) and is adjusted via a supervisory loop in order to cope with saturation (see Fig. 2).

Intuitively one can state the logic of adjustment as follows:

- If the system is close to experiencing saturation make λ smaller.
- Otherwise increase λ up to one.

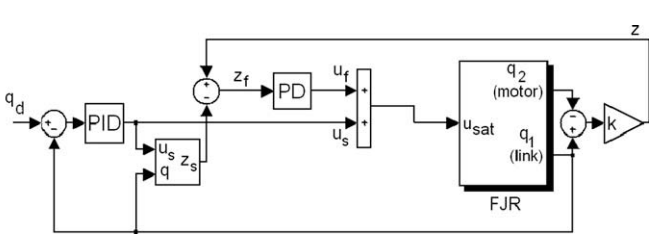

Fig. 1. The FJR control system.

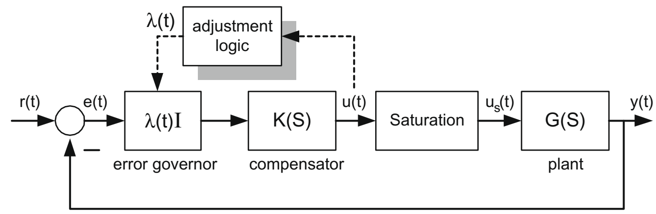

Fig. 2. The closed loop system with error governor.

This logic decreases the bandwidth when the system is close to experiencing saturation but in normal conditions the error governor is turned off by making $\lambda = 1$. This configuration reduces the amplitude of the control effort as is done by saturation itself but there are some important differences to that: (1) this is a dynamic compensator and not a hard nonlinearity as is the case with saturation. (2) This approach limits the control effort by affecting the controller states while saturation will limit the control effort independent of the controller states. In other words, it acts in a closed loop fashion rather than an open loop structure which is the case with a saturation block. Hence, the dynamic behavior of this approach can be used to preserve stability.

A rigorous mathematical model has been proposed in [13] to calculate $\lambda(t)$ which is very difficult to implement. However, fuzzy logic can be easily employed here because there is a good linguistic description of what is required. In order to have a good measure to quantify the closeness to saturation limit, the absolute value of the amplitude of the control effort $|u(t)|$ can be used. In addition, to add a kind of prediction to the logic and so to make the decision fast enough, $\dot{u}(t)$ is also taken into account. Thus, the above logic can be interpreted with fuzzy notation as follows (see Table 1):

- If $|u(t)|$ is *NEAR* to one and $\dot{u}(t)$ is *POSITIVE* make λ *LESS* than one.
- When $|u(t)|$ is *OVER* one, make λ *SMALL* if $\dot{u}(t)$ is negative and make λ *VERY SMALL* if $\dot{u}(t)$ is not negative.
- Otherwise, make it *ONE*.

To implement this logic, fuzzy sets are defined as shown in Fig. 3. Since the logic is based on a model free routine, the proposed method can be implemented not only on FJRs but also on a variety of systems which have limiting actuators. Effectiveness of this structure has been verified in different applications [14–16] and it is observed that it can preserve the stability which may be lost due to saturation. In addition, the steady state behavior of the closed loop system remains unchanged. Theoretical reasons for this important characteristic of the proposed structure will be elaborated in the next section.

In order to use this strategy for the FJR, some adjustments to the general structure has been made, which is briefed as follows:

(1) The supervisor is applied to the fast subsystem, which mainly causes the instability when limited by saturation.
(2) The saturation limit is not 1 in the FJR configuration, so the control effort $u(t)$ must be scaled by this factor before feeding to the supervisor.

Table 1
Fuzzy rules.

$\dot{u}$	$\lvert u\rvert$		
	Small	Near	Over
Neg	One	One	Small
Zero	One	One	Very small
Pos	One	Less	Very small

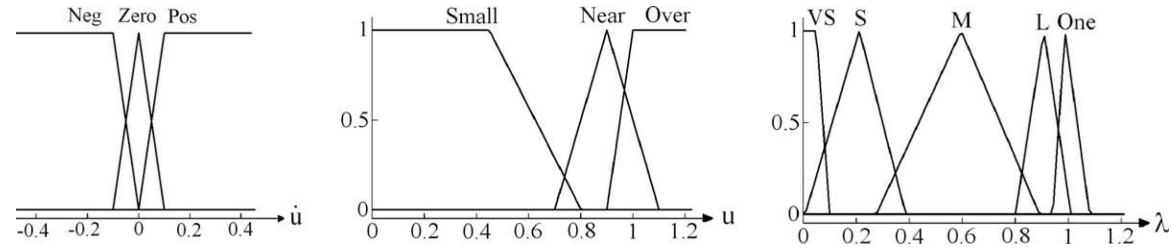

Fig. 3. Fuzzy sets.

The modified supervisory loop for the FJR is shown in Fig. 4. A filter is used to estimate $\dot{u}(t)$ from $u(t)$ so that the only required measurement would be $u(t)$. As mentioned in the previous section the composite controller composed of a fast PD controller and a slow PID controller has been shown to be robustly stable in absence of actuator limitation [21,23]. The robust stability of the system after adding a fuzzy supervisor and under the effect of the term $\lambda(t)$, will be studied in this paper at the following section.

4. Robust stability analysis

Recall the system equations:

$$\ddot{\vec{q}}_S = -\mathbf{M}^{-1}(\vec{q})\vec{z}_f - \mathbf{M}_t(\vec{q})^{-1}\vec{N}(\vec{q},\dot{\vec{q}}) + \mathbf{M}_t(\vec{q})^{-1}\vec{u}_S \tag{14}$$

$$\varepsilon\ddot{\vec{z}}_f = -(\mathbf{M}^{-1}(\vec{q}) + \mathbf{J}^{-1})\vec{z}_f - \mathbf{J}^{-1}\vec{u}_f \tag{15}$$

in which, $\mathbf{M}_t(\vec{q}) = \mathbf{J} + \mathbf{M}(\vec{q})$. It should be noted that utilizing the concept of integral manifold, a reduced-order model is derived for the system which is of the same dimension as the rigid system but which incorporates the effects of the flexibility [26]. Based on this reduced flexible model, a corrective control strategy is devised to compensate for the flexibility in the system. The overall control strategy $\vec{u}$ consists of a "rigid control" $\vec{u}_S$ designed for the rigid system and a "corrective control" $\vec{u}_f$ to compensate for deviations of the flexible system response from the response of the rigid system, therefore, the overall control input is denoted as $\vec{u} = \vec{u}_S + \vec{u}_f$. The control terms in presence of supervisory loop have been chosen to be

$$\vec{u}_S = \mathbf{K_P}\vec{e} + \mathbf{K_D}\dot{\vec{e}} + \mathbf{K_I}\int_0^t \vec{e}(\tau)d\tau \tag{16}$$

$$\vec{u}_f = \lambda(t)\cdot[\mathbf{K_{Pf}}\vec{z}_f + \mathbf{K_{Df}}\dot{\vec{z}}_f] \tag{17}$$

Thus the error dynamics can be written as

$$\ddot{\vec{e}} = \ddot{\vec{q}}_d + \mathbf{M}^{-1}(\vec{q})\vec{z}_f + \mathbf{M_t}^{-1}(\vec{q})\mathbf{N}(\vec{q},\dot{\vec{q}}) - \mathbf{M_t}^{-1}(\vec{q})(\mathbf{K_P}\vec{e} + \mathbf{K_D}\dot{\vec{e}} + \mathbf{K_I}\int_0^t \vec{e}(\tau)d\tau) \tag{18}$$

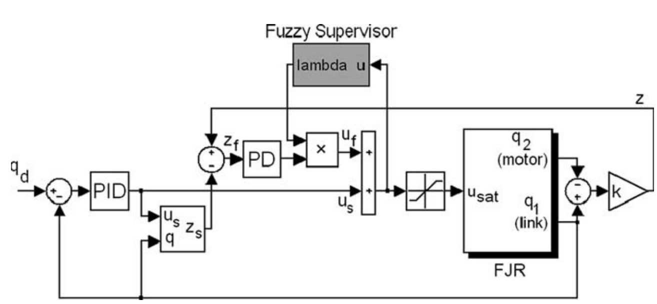

Fig. 4. The complete control system for the FJR with fuzzy supervisor.

Moreover, the fast dynamics is governed by

$$\varepsilon\ddot{\vec{z}}_f = -(\mathbf{M}^{-1}(\vec{q}) + \mathbf{J}^{-1})\vec{z}_f - \mathbf{J}^{-1}\lambda(t)\cdot[\mathbf{K_{Pf}}\vec{z}_f + \mathbf{K_{Df}}\dot{\vec{z}}_f] \tag{19}$$

We can rewrite this dynamics as

$$\ddot{\vec{z}}_f = -\mathbf{K_1}\dot{\vec{z}}_f - \mathbf{K_2}\vec{z}_f \tag{20}$$

where

$$\mathbf{K_1} = \frac{\lambda(t)}{\varepsilon}\mathbf{J}^{-1}\mathbf{K_{Df}}, \quad \mathbf{K_2} = (1/\varepsilon)[\mathbf{M}^{-1}(\vec{q}) + \mathbf{J}^{-1}(\mathbf{I} + \lambda(t)\mathbf{K_{Pf}})] \tag{21}$$

These equations can be rearranged into state space structure as

$$\dot{\vec{x}} = \mathbf{A}\vec{x} + \mathbf{B}\Delta\mathbf{A} + \mathbf{C}[\mathbf{I} \quad \mathbf{0}]\vec{y} \tag{22}$$

$$\dot{\vec{y}} = \mathbf{A_f}\vec{y} \tag{23}$$

In which,

$$\vec{x} = \begin{bmatrix} \int_0^t \vec{e}(\tau)d\tau \\ \vec{e} \\ \dot{\vec{e}} \end{bmatrix}, \quad \vec{y} = \begin{bmatrix} \vec{z}_f \\ \dot{\vec{z}}_f \end{bmatrix} \tag{24}$$

and,

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \\ -\mathbf{M_t}^{-1}(\vec{q})\mathbf{K_I} & -\mathbf{M_t}^{-1}(\vec{q})\mathbf{K_P} & -\mathbf{M_t}^{-1}(\vec{q})\mathbf{K_D} \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{M_t}^{-1}(\vec{q}) \end{bmatrix}, \quad \Delta\mathbf{A} = (\mathbf{N}(\vec{q},\dot{\vec{q}}) + \mathbf{M_t}(\vec{q})\ddot{\vec{q}}_d), \tag{25}$$

$$\mathbf{C} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{M}^{-1}(\vec{q}) \end{bmatrix}$$

$$\mathbf{A_f} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{K_2} & -\mathbf{K_1} \end{bmatrix} \tag{26}$$

In the next subsection the effect of $\lambda(t)$ on the stability of the fast subsystem is provided.

4.1. Stability of the fast subsystem

In this section the stability of the fast subsystem is first studied. This analysis is not directly used in the stability analysis of the overall system. However, it is essential to prove that the fast subsystem is stable, in order to apply the Tikhonov theorem in our case. For this study consider the following Lyapunov function candidate:

$$V_f(\vec{y}) = \vec{y}^T\mathbf{S}\vec{y} \tag{27}$$

In which $\mathbf{S}$ is defined as

$$\mathbf{S} = 1/2\begin{bmatrix} 2\mathbf{I} & \mathbf{K_1}^{-1} \\ \mathbf{K_1}^{-1} & \mathbf{K_2}^{-1} \end{bmatrix} \tag{28}$$

Lemma 1. *The matrix **S** is positive definite.*

The proof is based on Shur complement [29] and is the same as what can be found in [22] for the case $\lambda(t) = 1$. Therefore, the function V_f is positive.

Theorem 1. *The fast subsystem, (23), with the matrix A_f introduced in (26) is stable provided that the lower bounds on $\lambda, \dot{\lambda}$ and conditions on $\mathbf{K_{PF}}$ and $\mathbf{K_{DF}}$ stated in Appendix I are met.*

This theorem reveals that there are some bounds on the parameters $\mathbf{K_{PF}}$, $\mathbf{K_{DF}}$, $\lambda(t)$ used in control term (17) that if satisfied, the dynamics (15) becomes stable in a closed loop configuration.

Proof. To prove stability using Lyapunov direct method, consider the time derivative of V_f along trajectory (23):

$$\dot{V}_f(\vec{y}) = \dot{\vec{y}}^T\mathbf{S}\vec{y} + \vec{y}^T\mathbf{S}\dot{\vec{y}} + \vec{y}^T\dot{\mathbf{S}}\vec{y} = \vec{y}^T[\mathbf{A_f}^T\mathbf{S} + \mathbf{SA_f}]\vec{y} + \vec{y}^T\dot{\mathbf{S}}\vec{y} \tag{29}$$

We define

$$\mathbf{A_f}^T\mathbf{S} + \mathbf{SA_f} + \dot{\mathbf{S}} = -\mathbf{W} \tag{30}$$

It is shown in Appendix I for a 1DOF case that the matrix $\mathbf{W}$ is positive definite provided that some bounds are satisfied, and for higher order systems similar analysis can be studied. Therefore, from the above equations $\dot{V}_f$ is negative. Thus V_f is a Lyapunov function and the stability is guaranteed. □

In the later subsection the stability of the complete system will be studied. Before that, let us review some preliminary lemmas.

4.2. Preliminary lemmas for stability analysis

To prove the robust stability of the closed loop system in presence of modeling uncertainties, the Lyapunov direct method is used. Let V be the Lyapunov function candidate:

$$V(\vec{x},\vec{y}) = \vec{x}^T\mathbf{P}\vec{x} + \vec{y}^T\mathbf{S}\vec{y} \tag{31}$$

In which $\mathbf{S}$ is defined as before (Eq. (28)) and $\mathbf{P}$ is chosen to be

$$\mathbf{P} = 1/2\begin{bmatrix} \alpha_2\mathbf{K_P} + \alpha_1\mathbf{K_I} + \alpha_2^2\mathbf{M} & \alpha_2\mathbf{K_D} + \mathbf{K_I} + \alpha_1\alpha_2\mathbf{M} & \alpha_2\mathbf{M} \\ \alpha_2\mathbf{K_D} + \mathbf{K_I} + \alpha_1\alpha_2\mathbf{M} & \alpha_1\mathbf{K_D} + \mathbf{K_P} + \alpha_1^2\mathbf{M} & \alpha_1\mathbf{M} \\ \alpha_2\mathbf{M} & \alpha_1\mathbf{M} & \mathbf{M} \end{bmatrix} \tag{32}$$

In which, α_i s are real positive constants. The above function has a quadratic form and it would be positive definite provided that $\mathbf{P}$ and $\mathbf{S}$ are positive definite. Positive definiteness of $\mathbf{S}$ has been already shown in Lemma 1 and the following lemma guarantees that $\mathbf{P}$ is also positive definite in presence of modeling uncertainties (Eq. (4)).

Lemma 2. *The matrix **P** is positive definite if*

$$\alpha_1 > 0, \quad \alpha_2 > 0, \quad \alpha_1 + \alpha_2 < 1 \tag{33}$$

$$\alpha_2(k_P - k_D) - (1-\alpha_1)k_I - \alpha_2(1+\alpha_1-\alpha_2)\bar{m} > 0 \tag{34}$$

$$k_P + (\alpha_1 - \alpha_2)k_D - k_I - \alpha_1(1+\alpha_2-\alpha_1)\bar{m} > 0 \tag{35}$$

in which,

$$\mathbf{K_P} = k_P\mathbf{I}, \quad \mathbf{K_I} = k_I\mathbf{I}, \quad \mathbf{K_D} = k_D\mathbf{I} \tag{36}$$

Proof is given in [30].

Now for the stability analysis, differentiate V along trajectories (22) and (23). This yields to

$$\begin{aligned}\dot{V}(\vec{x},\vec{y}) &= \dot{\vec{x}}^T\mathbf{P}\vec{x} + \vec{x}^T\mathbf{P}\dot{\vec{x}} + \vec{x}^T\dot{\mathbf{P}}\vec{x} + \dot{\vec{y}}^T\mathbf{S}\vec{y} + \vec{y}^T\mathbf{S}\dot{\vec{y}} + \vec{y}^T\dot{\mathbf{S}}\vec{y} \\ &= \vec{x}^T[\mathbf{PA} + \mathbf{A}^T\mathbf{P}]\vec{x} + \vec{x}^T\mathbf{B}\Delta\mathbf{A} + \vec{x}^T\dot{\mathbf{P}}\vec{x} + 2\vec{x}^T\mathbf{PC}[\mathbf{I} \quad \mathbf{0}]\vec{y} \\ &\quad + \vec{y}^T[\mathbf{A_f}^T\mathbf{S} + \mathbf{SA_f}]\vec{y} + \vec{y}^T\dot{\mathbf{S}}\vec{y}\end{aligned} \tag{37}$$

The following lemmas will be used to prove that V is a Lyapunov function.

Lemma 3. *For the matrices **P**, **A**, **B** and $\Delta\mathbf{A}$ defined previously, the following inequality holds*

$$\vec{x}^T[\mathbf{PA} + \mathbf{A}^T\mathbf{P}]\vec{x} + \vec{x}^T\mathbf{B}\Delta\mathbf{A} + \vec{x}^T\dot{\mathbf{P}}\vec{x} \leqslant \|\vec{x}\|(\varepsilon_0 - \varepsilon_1\|\vec{x}\| + \varepsilon_2\|\vec{x}\|^2) \tag{38}$$

Here ε_0, ε_1 and ε_2 are some real positive constants that depend only on α_1, α_2 and the uncertainty bounds (see Eqs. (4)–(6)) as follows:

$$\varepsilon_0 = \gamma_1(\beta_0 + \bar{m}\gamma_3) \tag{39}$$

$$\varepsilon_1 = \gamma_4 - \gamma_1\beta_3 - \bar{m}\gamma_2 - \gamma_1\beta_1 \tag{40}$$

$$\varepsilon_2 = \gamma_1\beta_4 + \gamma_1\beta_2 \tag{41}$$

where

$$\begin{aligned}\gamma_1 &= \bar{\lambda}(\mathbf{R}_1), \quad \gamma_2 = \bar{\lambda}(\mathbf{R}_2) \\ \gamma_3 &= \|\ddot{q}_d(t)\|_\infty, \quad \gamma_4 = \underline{\lambda}(\mathbf{Q})\end{aligned} \tag{42}$$

In which,

$$\mathbf{Q} = \begin{bmatrix} \alpha_2 k_I\mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & (\alpha_1 k_P - \alpha_2 k_D - k_I)\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & k_D\mathbf{I} \end{bmatrix} \tag{43}$$

$$\mathbf{R}_1 = \begin{bmatrix} \alpha_2^2\mathbf{I} & \alpha_1\alpha_2\mathbf{I} & \alpha_2\mathbf{I} \\ \alpha_1\alpha_2\mathbf{I} & \alpha_1^2\mathbf{I} & \alpha_1\mathbf{I} \\ \alpha_2\mathbf{I} & \alpha_1\mathbf{I} & \mathbf{I} \end{bmatrix} \tag{44}$$

$$\mathbf{R}_2 = \begin{bmatrix} \mathbf{0} & \alpha_2^2\mathbf{I} & \alpha_1\alpha_2\mathbf{I} \\ \alpha_2\mathbf{I} & 2\alpha_1\alpha_2\mathbf{I} & (\alpha_1^2+\alpha_2)\mathbf{I} \\ \alpha_1\alpha_2\mathbf{I} & (\alpha_1^2+\alpha_2)\mathbf{I} & \alpha_1\mathbf{I} \end{bmatrix} \tag{45}$$

Proof. Consider the first two terms of the left hand side of (38), one can write

$$\begin{aligned}&\vec{x}^T[\mathbf{PA} + \mathbf{A}^T\mathbf{P}]\vec{x} + \vec{x}^T\mathbf{PB}\Delta\mathbf{A} \\ &= \vec{x}^T\left[-\mathbf{Q} + \mathbf{R}_2\begin{bmatrix} \mathbf{M} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{M}\end{bmatrix}\right]\vec{x} + \vec{x}^T\begin{bmatrix}\alpha_2\mathbf{I} \\ \alpha_1\mathbf{I} \\ \mathbf{I}\end{bmatrix}\Delta\mathbf{A}\end{aligned} \tag{46}$$

In which, matrices $\mathbf{R}_2$ and $\mathbf{Q}$ have been defined in Eqs. (45) and (43). Thus

$$\vec{x}^T[\mathbf{PA} + \mathbf{A}^T\mathbf{P}]\vec{x} + \vec{x}^T\mathbf{PB}\Delta\mathbf{A} \leqslant (-\gamma_4 + \gamma_2\bar{m})\|\vec{x}\|^2 + \gamma_1\|\vec{x}\|\|\Delta\mathbf{A}\| \tag{47}$$

where γ_1, γ_2 and γ_4 have been defined in (42). Now consider the last term in (38)

$$\vec{x}^T\dot{\mathbf{P}}\vec{x} = \frac{1}{2}\vec{x}^T\begin{bmatrix}\alpha_2\mathbf{I} \\ \alpha_1\mathbf{I} \\ \mathbf{I}\end{bmatrix}\dot{\mathbf{M}}[\alpha_2\mathbf{I} \quad \alpha_1\mathbf{I} \quad \mathbf{I}]\vec{x} \tag{48}$$

Taking into account the fact that for robot manipulators $\vec{v}^T\dot{\mathbf{M}}(\vec{q})\vec{v} = 2\vec{v}^T\mathbf{V_m}(\vec{q},\dot{\vec{q}})\vec{v}$ holds for any vector $\vec{v}$ [31], Eq. (48) can be changed to

$$\vec{x}^T\dot{\mathbf{P}}\vec{x} = \vec{x}^T\begin{bmatrix}\alpha_2\mathbf{I} \\ \alpha_1\mathbf{I} \\ \mathbf{I}\end{bmatrix}\mathbf{V_m}[\alpha_2\mathbf{I} \quad \alpha_1\mathbf{I} \quad \mathbf{I}]\vec{x} \tag{49}$$

which yields to

$$\vec{x}^T\dot{\mathbf{P}}\vec{x} \leqslant \gamma_1\|\vec{x}\|^2\|\mathbf{V_m}\| \tag{50}$$

Adding Eqs. (47) and (50) and considering the uncertainty bounds (4) and (5) concludes the proof. □

Lemma 4. *For the matrix* **C** *defined previously the following inequality holds*

$$2\vec{x}^T\mathbf{PC}[\mathbf{I}\quad \mathbf{0}]\vec{y} \leqslant 2\|\vec{x}\|\bar{\lambda}(\mathbf{P})\bar{\lambda}(\mathbf{M}^{-1})\|\vec{y}\| \tag{51}$$

Proof. Proof is straightforward considering that

$$\mathbf{C}[\mathbf{I}\quad \mathbf{0}] = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{M}^{-1}(\vec{q}) & \mathbf{0} \end{bmatrix} \qquad \square \tag{52}$$

Lemma 5. *Suppose that the Lyapunov function of a dynamic system has the following properties*

$$\dot{V}(X) \leqslant \|X\|(\phi_0 - \phi_1\|X\| + \phi_2\|X\|^2) \tag{53}$$

and

$$\underline{\gamma}\|X\|^2 \leqslant V(X) \leqslant \bar{\gamma}\|X\|^2 \tag{54}$$

where $\underline{\gamma}, \bar{\gamma}$ *and* ϕ_i*s are some constants. Given that*

$$d_1 = \frac{2\phi_0}{\phi_1 + \sqrt{\phi_1^2 - 4\phi_0\phi_2}} \times \sqrt{\frac{\bar{\gamma}}{\underline{\gamma}}} \tag{55}$$

then the system is UUB stable with respect to B(0, d1), provided that

$$\phi_1 > 2\sqrt{\phi_0\phi_2} \tag{56}$$

$$\phi_1[\phi_1 + \sqrt{\phi_1^2 - 4\phi_0\phi_2}] > 2\phi_0\phi_2\left(1 + \sqrt{\frac{\bar{\gamma}}{\underline{\gamma}}}\right) \tag{57}$$

$$\phi_1 + \sqrt{\phi_1^2 - 4\phi_0\phi_2} > 2\phi_2\|X_0\|\sqrt{\frac{\bar{\gamma}}{\underline{\gamma}}} \tag{58}$$

where $\|X_0\|$ *denotes the two norm of the initial condition.*

Proof can be found in [25] under lemma 3.5.

4.3. Stability of the complete system

In this subsection, the main result is presented. To be compact it is referred to the equations simply by their numbers.

Theorem 2. *Consider the flexible joint manipulator of Eqs. (14) and (15) with the composite controller structure of Eqs. (16) and (17), under supervisory loop. The overall closed loop system with governing equations of motion (22) and (23) is UUB stable and the state variables converge to the origin under conditions of Theorem 1, conditions of Lemma 2, and some new certain limits imposed on the fast (PD) and slow (PID) controller gains, which will be given by Eqs. (69)–(71).*

Proof. The Lyapunov function candidate V introduced in (31) has been shown to be positive definite. This imposes conditions of *Lemma* 2 to be satisfied. Now in order to study the negative definiteness of $\dot{V}(\vec{x},\vec{y})$, consider (37), (38), (51), and (30) which yields to

$$\begin{aligned}\dot{V}(\vec{x},\vec{y}) &\leqslant \|\vec{x}\|(\varepsilon_0 - \varepsilon_1\|\vec{x}\| + \varepsilon_2\|\vec{x}\|^2) \\ &\quad + 2\|\vec{x}\|\bar{\lambda}(\mathbf{P})\bar{\lambda}(\mathbf{M}^{-1})\|\vec{y}\| - \bar{\lambda}(\mathbf{W})\|\vec{y}\|^2 \\ &= \begin{bmatrix}\|\vec{x}\| & \|\vec{y}\|\end{bmatrix}\begin{bmatrix} -\varepsilon_1 & \bar{\lambda}(\mathbf{P})\bar{\lambda}(\mathbf{M}^{-1}) \\ \bar{\lambda}(\mathbf{P})\bar{\lambda}(\mathbf{M}^{-1}) & -\bar{\lambda}(\mathbf{W})\end{bmatrix}\begin{bmatrix}\|\vec{x}\| \\ \|\vec{y}\|\end{bmatrix} \\ &\quad + \varepsilon_0\|\vec{x}\| + \varepsilon_2\|\vec{y}\|^3\end{aligned} \tag{59}$$

or

$$\dot{V}(\vec{x},\vec{y}) \leqslant -\vec{z}_1^T\mathbf{R}\vec{z}_1 + \varepsilon_0\|\vec{x}\| + \varepsilon_2\|\vec{x}\|^3 \tag{60}$$

where

$$\vec{z}_1 = \begin{bmatrix}\|\vec{x}\| \\ \|\vec{y}\|\end{bmatrix}, \quad \mathbf{R} = -\begin{bmatrix} -\varepsilon_1 & \bar{\lambda}(\mathbf{P})\bar{\lambda}(\mathbf{M}^{-1}) \\ \bar{\lambda}(\mathbf{P})\bar{\lambda}(\mathbf{M}^{-1}) & -\bar{\lambda}(\mathbf{W})\end{bmatrix} \tag{61}$$

In here it is seen that the matrix **W** must be positive definite thus conditions of *Theorem* 1 must be satisfied, as well. Now if it is defined

$$\vec{z} = \begin{bmatrix}\vec{x} \\ \vec{y}\end{bmatrix} \tag{62}$$

then $\|\vec{z}\| = \|\vec{z}_1\|, \|\vec{z}\| \geq \|\vec{x}\|$ thus we have

$$\dot{V}(\vec{z}) \leqslant \|\vec{z}\|(\varepsilon_0 - \bar{\lambda}(\mathbf{R})\|\vec{z}\| + \varepsilon_2\|\vec{z}\|^2) \tag{63}$$

Applying the Railey Ritz inequality for **P** and **S** which reads

$$\begin{aligned}\underline{\lambda}(\mathbf{P})\|\vec{x}\|^2 \leqslant \vec{x}^T\mathbf{P}\vec{x} \leqslant \bar{\lambda}(\mathbf{P})\|\vec{x}\|^2 \\ \underline{\lambda}(\mathbf{S})\|\vec{y}\|^2 \leqslant \vec{y}^T\mathbf{S}\vec{y} \leqslant \bar{\lambda}(\mathbf{S})\|\vec{y}\|^2\end{aligned} \tag{64}$$

and adding these two equations yields to

$$\vec{z}_1^T\begin{bmatrix}\lambda(\mathbf{P}) & 00 \\ 0 & \lambda(\mathbf{S})\end{bmatrix}\vec{z}_1 \leqslant V(\vec{z}) \leqslant \vec{z}_1^T\begin{bmatrix}\bar{\lambda}(\mathbf{P}) & 0 \\ 0 & \bar{\lambda}(\mathbf{S})\end{bmatrix}\vec{z}_1 \tag{65}$$

In other words

$$\underline{\gamma}\|\vec{z}\|^2 \leqslant V(\vec{z}) \leqslant \bar{\gamma}\|\vec{z}\|^2 \tag{66}$$

In which

$$\begin{aligned}\bar{\gamma} &= \max\{\bar{\lambda}(\mathbf{P}), \bar{\lambda}(\mathbf{S})\} \\ \underline{\gamma} &= \min\{\underline{\lambda}(\mathbf{P}), \underline{\lambda}(\mathbf{S})\}\end{aligned} \tag{67}$$

Now from (63) and (66) and by *Lemma* 5 it can be stated: given that

$$d = \frac{2\varepsilon_0}{\bar{\lambda}(\mathbf{R}) + \sqrt{\bar{\lambda}(\mathbf{R})^2 - 4\varepsilon_0\varepsilon_2}} \times \sqrt{\frac{\bar{\gamma}}{\underline{\gamma}}} \tag{68}$$

the system is UUB stable with respect to $\mathbf{B}(0, d)$, provided the following stability conditions are satisfied

$$\bar{\lambda}(\mathbf{R}) > 2\sqrt{\varepsilon_0\varepsilon_2} \tag{69}$$

$$\bar{\lambda}(\mathbf{R})[\bar{\lambda}(\mathbf{R}) + \sqrt{\bar{\lambda}(\mathbf{R})^2 - 4\varepsilon_0\varepsilon_2}] > 2\varepsilon_0\varepsilon_2\left(1 + \sqrt{\frac{\bar{\gamma}}{\underline{\gamma}}}\right) \tag{70}$$

$$\bar{\lambda}(\mathbf{R}) + \sqrt{\bar{\lambda}(\mathbf{R})^2 - 4\varepsilon_0\varepsilon_2} > 2\varepsilon_2\|z_0\|\sqrt{\frac{\bar{\gamma}}{\underline{\gamma}}} \tag{71}$$

where $\|z_0\|$ denotes the two norm of the initial condition. $\square$

This proof reveals an important aspect of the supervisory loop dynamics included in the proposed controller law. Since the supervisory loop adjusts only the controller gain by an error multiplier $\lambda(t)$ and does not change the main control structure, it will not perturb the stability of the original system. This general idea which was observed through various simulations of implemented supervisory loop for different systems has been analytically proven here for the FJR. Another aspect that can be concluded from this analysis is the robustness of stability in presence of modeling uncertainties. Since the unmodeled but bounded dynamics of the system is systematically encapsulated in the system model (as stated in Eqs. (4)–(6)), the only influence this will impose on the stability is the respective bounds on the controller gains depicted in conditions of *Theorem* 2. At the next section some experimental results are given to verify the effectiveness of the supervisory loop in practice.

5. Experimental results

The laboratory set up which has been considered for experimental study is shown in Fig. 5. It is a 2 DOF flexible joint manipulator. In the first joint a harmonic drive is used for power transmission. Its spring constant is empirically derived to be 6340 N m/rad [32].

The flexible element used in power transmission system of the second joint is shown in Fig. 6. It has been made from Polyurethane and is designed to possess high flexibility. Its equivalent spring constant is as low as 8.5 N m/rad which makes the control problem more challenging. In order to show the effectiveness of the proposed algorithm in presence of actuator saturation, the experimental results on the second joint with lower stiffness are presented here. Specifications of the second motor are given in Table 2 [10].

In order to control the system by means of a PC, a PCL-818 I/O card and a PCL-833 encoder data acquisition card of the Advantech Company are used for hardware interfacing. The "Real Time Workshop" facilities of the MATLAB SIMULINK are used for user interface. A block diagram of the system is shown in Fig. 7.

Experimental results are shown in Figs. 8–12. In the first experiment a sine wave with frequency 0.5 rad/sec is given as the reference signal to the system (Fig. 8). At time 30.2 s the supervisor is turned on, i.e. the value of $\lambda(t)$ is set to be adapted by the fuzzy supervisor. In this experiment, the reference frequency is intentionally selected to be low and the composite PD and PID gains

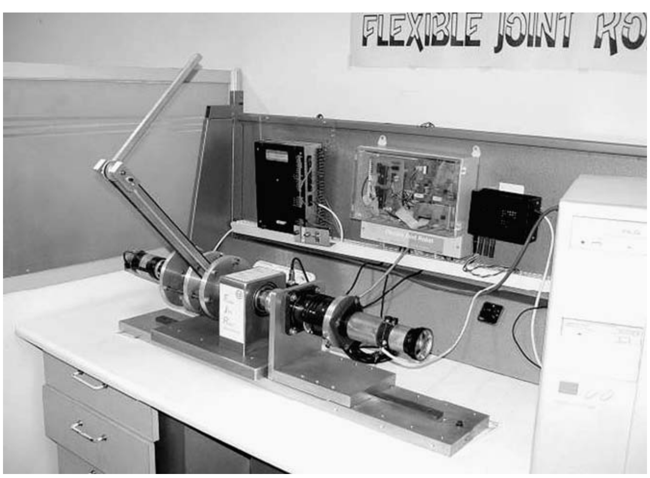

Fig. 5. Experimental setup.

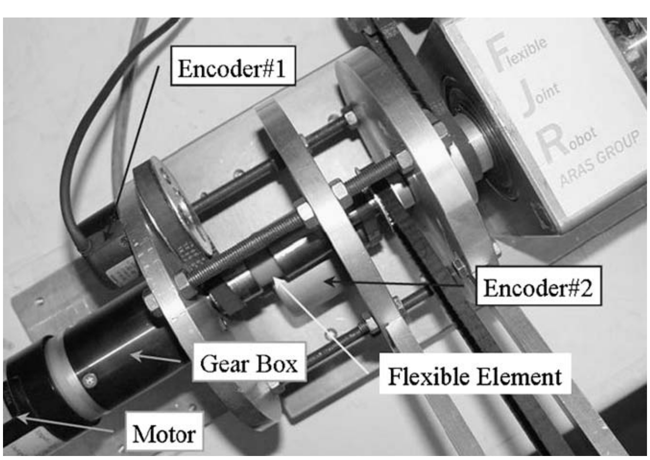

Fig. 6. The flexible element.

Table 2
Motor specifications for the experimental setup.

Continuous torque (N m)	13
Maximum rated input (V DC)	12
Maximum continuous power (W)	62
Rated speed (rpm)	26

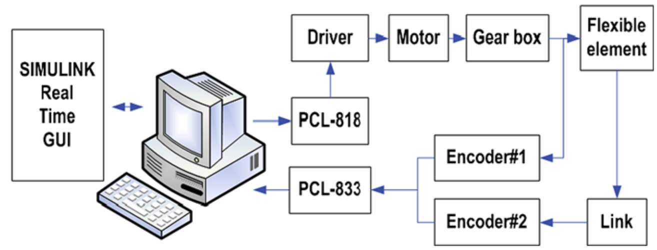

Fig. 7. Block diagram of the system.

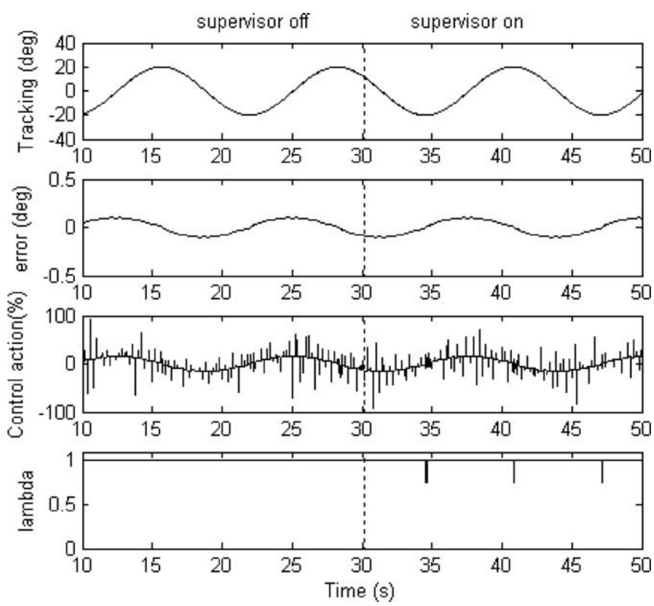

Fig. 8. Experimental results for sine input.

are tuned such that the tracking error is suitable and the closed loop tracking of the system is smooth. In such case, since the required actuation power is low, it is less suffered from the saturation limits and therefore there is no oscillation seen in the output tracking. This experiment verifies that in cases where the demanded actuator amplitude is well within the saturation limits, the resulting performance is good without any oscillations despite low stiffness joints used in the structure of the FJR. However, since the supervisor is less triggered in this experiment, its crucial effect can be less obviously seen in the result. In order to examine the effect of the supervisor in a more stringent case, another experiment is designed, in which by increasing the internal PD and PID controller gains, the control input will suffer from the saturation limits in a wider range. Fig. 9 shows the result for tracking the same sine wave with these controller gains, in which, during the interval 20.3–30.4 s the supervisor is turned off. In this experiment due to higher controller gains, a much better tracking performance can be achieved, if there is no actuator saturation. The actuator saturation limits, though, introduce oscillations in the tracking performance; however, as it is seen in the result the supervisory loop can effectively reduce the level of oscillation in such cases. The same

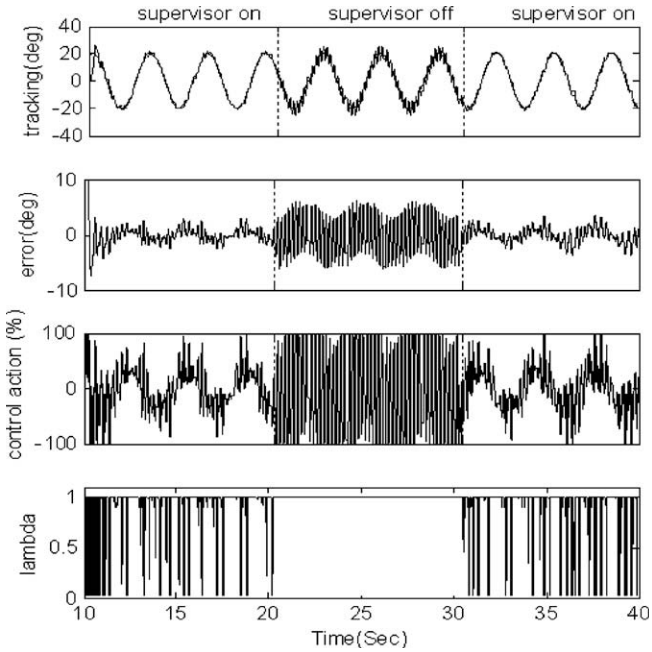

Fig. 9. Experimental results for sine input with a new high gain controller.

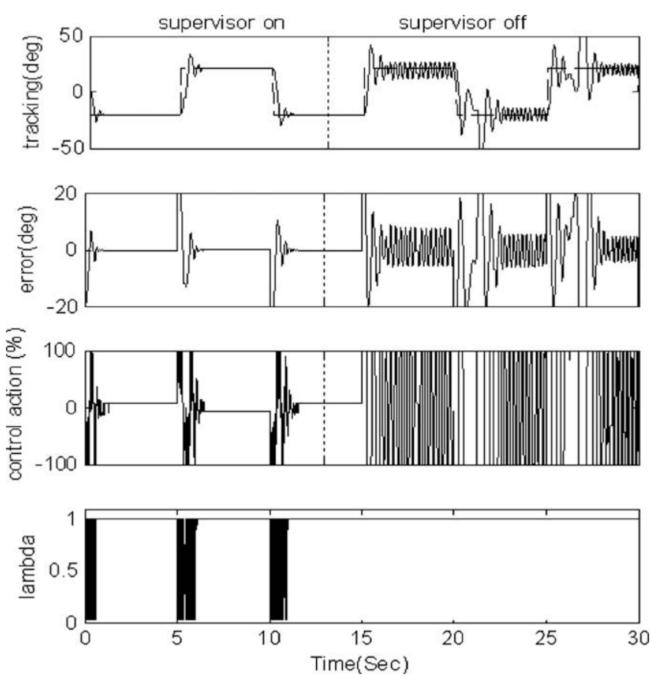

Fig. 10. Experimental results for pulse input.

result is observed for a pulse reference input in Fig. 10 in which the supervisor is turned off at time $t = 13$ s. Due to the same reason high resonance after this moment are observed.

In order to verify that one of the main sources of oscillations in the response is the effect of actuator saturation, a new experiment is performed. One may think that the oscillations are solely due to the low stiffness of the joint. However to verify that the root of oscillations in previous figures stems out from the actuator limitations, a new experiment is performed in which the polyurethane flexible element in the joint is substituted with a stiffer one, result-

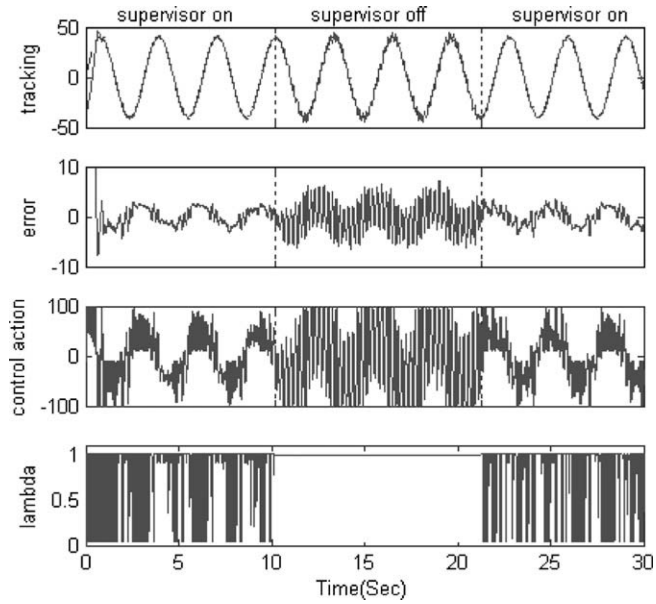

Fig. 11. Experimental results for a stiffer joint.

ing in an order of magnitude stiffer joint. Fig. 11 shows the result for a sine wave reference applied to this set up. Comparing the behavior of the system to that of Fig. 9, it is observed that similar oscillation occurs when the PD–PID gains are adjusted to be high enough. These set of experiments verify the main claim of this paper that the supervisory controller is able to significantly reduce the amount of oscillation caused by the saturation limit.

Furthermore, in order to show that the supervisory loop is superior to the idea of reducing the bandwidth, a special experiment is designed for the system whose results are given in Fig. 12. In this experiment λ is switched on and off alternately between the $\lambda(t)$ determined by the supervisory loop and a constant value less than one. As it is expected and experimentally verified in this figure, using a constant value for λ results in relatively higher tracking errors. These experimental observations confirm intuitively that the supervisory loop is capable to reduce the high oscillations caused by actuator saturation. Moreover, it confirms the availability of some feasible mapping regions where the controller gains and the supervisory parameters can be selected to meet the sufficient conditions stated in the *Theorems* 1 *and* 2.

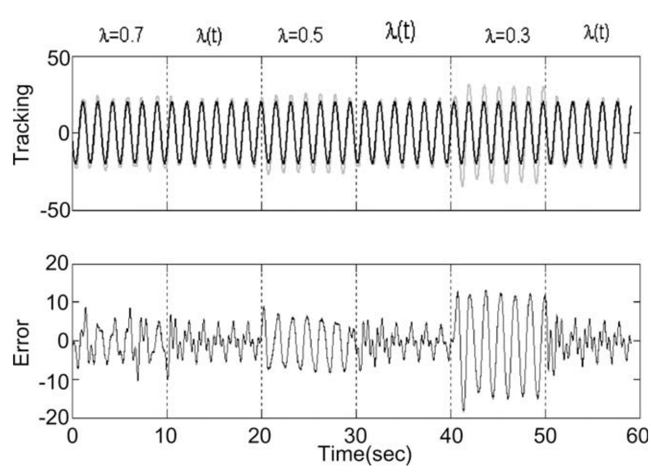

Fig. 12. Experimental results for switching λ to amounts less than one.

6. Conclusions

In this paper, the problem of controller synthesis for flexible joint robots in presence of actuator saturation is investigated in detail. The singularly perturbed model of the system is first introduced briefly, and a composite controller is proposed for the system. The composite controller consists of a robust PID term for slow (rigid) dynamics and a PD controller for fast (flexible) dynamics. In order to reduce the limitations caused by actuator bounds, a supervisory loop is added to this structure, and it is shown that a model free fuzzy supervisory loop makes it plausible to preserve stability, without great loss in performance. It is shown through a Lyapunov based stability analysis that due to the structure of the supervisory loop the overall variation of the system energy is dissipative, provided some conditions on the controller gains and supervisory loop parameters are met. These considerations have enabled us to propose a controller, whose stability is thoroughly analyzed and can be easily implemented. Such requirements are essential for susceptible applications such as space robotics. Then some experimental results are presented to confirm the effectiveness of the fuzzy supervisor in practice to reduce the high oscillations caused by actuator saturation. Finally, it should be noted that the focus in this paper is on FJR but the proposed method is applicable on several other systems [14,16].

Appendix A. Positive definiteness of W

In this section, the conditions for positive definiteness of matrix **W**, for a first order FJR is derived where the same progress could be pursued for an n-DOF case. These conditions are used for the stability analysis of the fast subsystem as stated in *Theorem* 1. Governing equations of motion for a 1-DOF FJR is [24]:

$$\begin{cases} I\ddot{q}_1 + mgL\sin(q_1) = -k(q_1 - q_2) \\ J\ddot{q}_2 - k(q_1 - q_2) = u \end{cases} \tag{I.1}$$

In which I is the link moment of inertia, J is that of the motor, $2L$ is the length of the link, k is the elasticity of the flexible element and u is the input torque. Now from (21) we can write

$$\mathbf{K}_1 = \frac{\lambda}{\varepsilon J}K_{Df}, \quad \mathbf{K}_2 = \frac{I + J + I\lambda K_{Pf}}{\varepsilon IJ} \tag{I.2}$$

Therefore,

$$\dot{\mathbf{S}} = -\frac{1}{2}\begin{bmatrix} 0 & \frac{\varepsilon J\dot{\lambda}}{\lambda^2 K_{Df}} \\ \frac{\varepsilon J\dot{\lambda}}{\lambda^2 K_{Df}} & \frac{\varepsilon I^2 JK_{Pf}\dot{\lambda}}{(I+J+I\lambda K_{Pf})^2} \end{bmatrix} \tag{I.3}$$

and thus from (30)

$$\mathbf{W} = \begin{bmatrix} \frac{(I+J+I\lambda K_{Pf})}{I\lambda K_{Df}} & \frac{\varepsilon J\dot{\lambda}}{2K_{Df}\lambda^2} \\ \frac{\varepsilon J\dot{\lambda}}{2K_{Df}\lambda^2} & \frac{IK_{Df}\lambda}{(I+J+I\lambda K_{Pf})} - \frac{\varepsilon J}{K_{Df}\lambda} + \frac{\varepsilon I^2 JK_{Pf}\dot{\lambda}}{2(I+J+I\lambda K_{Pf})^2} \end{bmatrix} \tag{I.4}$$

Note that the sufficient condition for the matrix **W** to be positive definite can be derived from the following conditions on its elements

$$\begin{aligned} &W_{11} + W_{22} > 0 \\ &W_{11}W_{22} - W_{12}^2 > 0 \end{aligned} \tag{I.5}$$

First consider $W_{11} + W_{22}$,

$$\begin{aligned} W_{11} + W_{22} &= \frac{(I+J+I\lambda K_{Pf})}{I\lambda K_{Df}} - \frac{\varepsilon J}{K_{Df}\lambda} + \frac{IK_{Df}\lambda}{(I+J+I\lambda K_{Pf})} + \frac{\varepsilon I^2 JK_{Pf}\dot{\lambda}}{2(I+J+I\lambda K_{Pf})^2} \\ &= \frac{(I+J+I\lambda K_{Pf}) - \varepsilon IJ}{I\lambda K_{Df}} + \frac{2IK_{Df}\lambda(I+J+I\lambda K_{Pf}) + \varepsilon I^2 JK_{Pf}\dot{\lambda}}{2(I+J+I\lambda K_{Pf})^2} \end{aligned} \tag{I.6}$$

As all parameters except than $\dot{\lambda}$ are positive, a sufficient condition for $W_{11} + W_{22}$ to be positive is

$$\begin{aligned} &(I+J+I\lambda K_{Pf}) - \varepsilon IJ > 0 \\ &2K_{Df}\lambda(I+J+I\lambda K_{Pf}) + \varepsilon IJK_{Pf}\dot{\lambda} > 0 \end{aligned} \tag{I.7}$$

which imposes a lower bound on λ and one on $\dot{\lambda}$ as follows:

$$\lambda > \frac{\varepsilon IJ - I - J}{IK_{Pf}}, \quad \dot{\lambda} > -\frac{2K_{Df}(I+J+IK_{Pf})}{\varepsilon IJK_{Pf}} \tag{I.8}$$

Note that ε is a very small parameter, thus for typical values of I and J the lower bound of λ would be negative which is not a limitation at all. Also the lower bound of $\dot{\lambda}$ would be very large negative number which is not really limiting. Now consider $W_{11}W_{22} - W_{12}^2$

$$\begin{aligned} W_{11}W_{22} - W_{12}^2 &= 1 - \frac{\varepsilon J(I+J+I\lambda K_{Pf})}{IK_{Df}^2\lambda^2} + \frac{\varepsilon IJK_{Pf}\dot{\lambda}}{2\lambda K_{Df}(I+J+I\lambda K_{Pf})} - \left(\frac{\varepsilon J\dot{\lambda}}{2K_{Df}\lambda^2}\right)^2 \\ &= \frac{c_1 + b_1\dot{\lambda} + a_1\dot{\lambda}^2}{4\lambda^4 K_{Df}^2 I(I+J+I\lambda K_{Pf})} \end{aligned} \tag{I.9}$$

where

$$\begin{aligned} c_1 &= 4\lambda^2(I+J+I\lambda K_{Pf})[\lambda^2 K_{Df}^2 I - \varepsilon JI\lambda K_{Pf} - \varepsilon J(I+J)] \\ b_1 &= 2\varepsilon I^2 JK_{Pf}K_{Df}\lambda^3 \\ a_1 &= -\varepsilon^2 J^2 I(I+J+I\lambda K_{Pf}) \end{aligned} \tag{I.10}$$

The numerator of Eq. (I.9) is a polynomial of degree 2 in $\dot{\lambda}$ with $a_1 < 0$. Thus for $W_{11}W_{22} - W_{12}^2$ to be positive, $\dot{\lambda}$ should remain between the two roots:

$$\frac{-b_1 - \sqrt{b_1^2 - 4a_1c_1}}{2a_1} < \dot{\lambda} < \frac{-b_1 + \sqrt{b_1^2 - 4a_1c_1}}{2a_1} \tag{I.11}$$

And for the roots to lie in both sides of zero, c_1 must be positive. c_1 includes a polynomial of degree 2 in λ, for which Δ and a are positive, thus the following bounds should be satisfied

$$\begin{aligned} &\frac{\varepsilon IJK_{Pf} - \sqrt{\Delta}}{2K_{Df}^2 I} < \lambda < \frac{\varepsilon IJK_{Pf} + \sqrt{\Delta}}{2K_{Df}^2 I} \\ &\Delta = (\varepsilon IJK_{Pf})^2 + 4K_{Df}^2\varepsilon IJ(I+J) \end{aligned} \tag{I.12}$$

Considering the fact that $0 < \lambda < 1$, the following bound is a sufficient condition for (I.12)

$$IK_{Df}^2 < \varepsilon J(I+J) + \varepsilon IJK_{Pf} \tag{I.13}$$

In other word, for the matrix **W** to be positive definite, the bounds (I.8), (I.11), and (I.13) must be met on λ, $\dot{\lambda}$, K_{Pf} and K_{Df}.

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