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Descriptor Approach to Unknown Input PI Observer Design: application to fault detection

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Abstract—**the descriptor observer approach is improved further to a more suitable observer scheme which is applicable to a more general group of faults and systems. Also a disturbance decoupling scheme is added to the mentioned observer in order to enable the observer to distinguish faults from disturbances.**

Keywords: descriptor systems; observer based fault detection; Descriptor observer approach;PI observer; fault estimation.

I. Introduction

Singular systems represent a more general frame work for linear systems. [1] A singular model is an appropriate model for describing large scale interconnected systems, constrained robots and other differential algebraic systems with linear algebraic constraints [2]. Also singular models can be utilized to model a system when the dependent variable is displacement rather than time [3]. The observer design problem for descriptor systems can be employed in state observation of normal systems with input disturbances. [4, 5] Descriptor observer approach in [4] is a strong method to construct fault signal asymptotically. Compared to other fault estimation schemes descriptor observer approach has two main advantages firstly it leads to a simple design procedure. Secondly it can observe fault signal asymptotically while the other methods like proportional integral observers as in [6, 7, 8] are only able to estimate fault signal by a constant vector or another approximation. Therefore the descriptor approach doesn't need any prior knowledge of fault model and is able to observe any fault signal. However this advantage is gained at the cost of more restrictive existence conditions for the observer. In this paper both methods in [4] and [6] are improved further by adding a disturbance decoupling scheme which enables the observer to distinguish between disturbances and faults. Also a more advanced algorithm is proposed to detect and estimates faults with the descriptor observer approach proposed first in [4]. The proposed method in [4] has two drawbacks which are improved in this paper; first the descriptor approach in [4] uses a descriptor observer which requires the output derivative in order to be implemented easily. Although [8] suggests a mathematic derivation for the output derivative to avoid output differentiation, it will lead to a more complicated pole placement algorithm and the observer will be more sensitive to model inaccuracy because it uses an indirect measurement of output derivative. The second drawback is the assumption that disturbance is only present in the output and doesn't enter the state equation. For the case that disturbance enters the state equation, [4] uses a proportional integral observer. The proposed method in this paper uses a normal observer without need of output derivative and has a simple pole placement algorithm. Another advantage is that disturbance or fault vectors are not assumed to be bounded while [4] made this assumption. It is assumed that fault signals are such that they can be estimated by a step signal with an unknown amplitude and unknown occurrence time. No assumption on disturbance signals is made. Descriptor approach to the observer design has another property which is very essential. It is capable of incorporating optimal estimation schemes via optimal estimation theory for singular systems [5]. Note that ordinary PI observers and unknown input observers cannot use the optimization algorithms directly. The paper is organized as follows: In the next section backgrounds are discussed and Definitions, Lemmas and assumptions are presented. In the third section sufficient conditions for the existence of the proposed observers are derived. A combined version of two algorithms is proposed finally. In the fourth section a comparison between two methods is made by simulation. The paper is concluded in section five.

II. Backgrounds and Preliminaries

A. *Disturbance decoupling and integral estimate*

The problem of disturbance decoupling has been studied in the literature [1, 5, and 9]. Disturbance decoupling was firstly intended for state estimation because proportional observers are not capable of yielding good estimation of states when disturbances perturb the system. However PI observers are more robust and usually require less restrictive existence conditions therefore it is more logical to use a PI observer to have an independent state estimation in the presence of disturbances. However in some applications an unknown input observer is a better choice. For example when the disturbance dynamics are fast an integral variable may not be able to track the unknown input and there will be no choice but to increase the number of integral variables which is equal to using a time polynomial to estimate fault rather than a constant. Another application of decoupling scheme occurs in fault detection. An appropriate fault detection algorithm must be able to distinguish between fault and disturbances. Therefore one can decouple disturbances while estimate faults to detect their occurrence and also to diagnose their type as in [8, 10 and 12]. In [4] it is assumed that disturbance or fault perturbs only the output but here it is assumed that fault disturbs both the state vector and the output. In real situations only the sensor fault can be modeled in this way therefore for detecting actuator or plant faults there is no way but to use integral estimation of fault signal. Utilizing the proposed algorithm in this paper one can estimate actuator faults and/or plant faults asymptotically without any knowledge of their signals (i.e. either the fault is a step, sinusoidal, etc.). The existence conditions for a proportional integral observer are less restrictive than those for descriptor approach observer but it is more logical to use descriptor approach observers when possible because they are

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able to estimate faults exactly while integral observers are only estimating unknown inputs by a few terms of their Taylor expansions. Therefore if one uses a proportional integral observer for a ramp fault, even the fault occurrence may not be detected [10]. The key requirement for the existence of a descriptor approach observer is the presence of fault signal in the output. Particularly all fault signals should appear in the output. As a result a necessary condition for the descriptor approach observer to exist is that the number of outputs should be equal or more than the number of faults. When fault is not present in the output equation or appears partially (i.e. not all fault inputs appear in the output equation) it is suggested to use integral approximation for fault estimation [4, 5, and 6]. Consider a linear time-invariant descriptor system described by the following:

$$E\dot{x}(t) = Ax(t) + Bu(t) + B_d d(t) + B_f f(t)$$

$$y(t) = Cx(t) \qquad (1)$$

Vectors x, u, d, f, y are the system state, input, disturbance, fault and output respectively. In this paper dimension of each vector is described by a letter n subscripted with the signal name for example n_d denotes number of disturbance signals.

Definition1: Having set $E = I$ system (1) becomes a normal state space system and it is called observable if:

$$rank\begin{bmatrix} sI - A \\ C \end{bmatrix} = n \quad \forall s \in C$$

Here C denotes the complex plane.

Remark1: observability condition is necessary for the existence of a disturbance decoupling observer but it is not sufficient.

Remark2: Note that disturbance is supposed not to perturb output directly and system is presumed to be strictly proper from both control input and disturbance input to the output.

Definition2: System (1) is called completely observable if:

$$rank\begin{bmatrix} E \\ C \end{bmatrix} = rank\begin{bmatrix} sE - A \\ C \end{bmatrix} = n \quad \forall s \in C$$

Definition3: A system described by the following equations is called a disturbance decoupling observer or an unknown input observer for system (1) if it is able to decouple disturbances (i.e. give an estimation of the state vector independent of input disturbance in (1)).

$$\dot{z}(t) = \hat{A}z(t) + \hat{B}y(t) + \hat{J}u(t) + \hat{H}w(t)$$

$$\hat{x}(t) = z(t) + \hat{D}y(t)$$

$$\dot{w}(t) = F(y(t) - C\hat{x}(t)) + \Phi w(t) \qquad (2)$$

At the moment suppose $\hat{H} = \Phi = F = 0$. If it is aimed to estimate faults by integral action, then vector $w(t)$ is utilized to estimate faults, F is the fault input vector and Φ is the fault model. (For step faults it is equal to zero and for a ramp fault it is a state matrix of an integrator system and so on.)

Lemma1: Set $f(t) \equiv 0$ then there exists a singular, disturbance decoupling observer which is able to estimate the state independent of input disturbance for system (1) if and only if:

$$rank\begin{bmatrix} sE - A & B_d \\ C & 0 \end{bmatrix} = n + n_d \quad \forall s \in C, \mathrm{Re}(s) \geq 0 \qquad (3)$$

Proof is in [1].

Remark3: If disturbance perturbs the output directly, it is still possible to have a disturbance decoupling observer as shown in [1]. But the existence condition changes a bit.

Remark4: As shown in [1], for estimation of disturbance signal, the decoupling condition of lemma1 combined with the exact knowledge of disturbance dynamic model is sufficient.

Lemma2: For system (1) a normal, disturbance decoupling observer as (2) exists if:

$$rank\, CE^{\#}B_d = rank\, B_d = n_d$$

$$rank\begin{bmatrix} E \\ C \end{bmatrix} = n$$

$$rank\begin{bmatrix} sE - A & B_d \\ C & 0 \end{bmatrix} = n + n_d \quad \forall s \in C, \mathrm{Re}(s) \geq 0 \qquad (4)$$

Matrix $E^{\#}$ is defined such that:

$$\begin{bmatrix} E^{\#} & C^{\#} \end{bmatrix} \times \begin{bmatrix} E \\ C \end{bmatrix} = I$$

It is possible to find the above matrices if the complete observability condition in definition2 is assumed. Proof can be found in [7].

Remark5: In (4) the last condition can be replaced by the observability condition in definition (2).

Corollary1: For constant faults an integral variable is enough to show the amplitude of fault signal. Therefore a Proportional Integral observer can be used in this situation. For polynomial (even unstable as in [11]) faults or any fault signal which can be approximated by a polynomial function of time, it is better to use multiple integral variables.

Lemma3: System (2) is an Unknown Input Proportional Integral observer for system (1) if there exists a matrix U such that:

$$\hat{A}UE + \hat{B}C = UA$$

$$\hat{J} = UB$$

$$UB_d = 0$$

$$UE + \hat{D}C = I$$

$$\hat{H} = UB_f$$

$$\mathrm{Re}\lambda_j\begin{bmatrix} \hat{A} & \hat{H} \\ -FC & \Phi \end{bmatrix} < 0 \qquad (5)$$

Proof is in [10].

Remark6: A simple procedure to design observer (2) which satisfies conditions (5) is proposed in [10].

B. Descriptor approach to the observer design problem

If fault signal appears in the output equation, it is sometimes possible to reconstruct fault signal asymptotically. This kind of observer is advantageous over the integral observes in two aspects. First it doesn't require any knowledge about fault signal and second it can estimate them accurately in an arbitrarily small time. The latter characteristic is very important in fault detection applications. Another advantage of descriptor approach is that it removes the famous trade-off between noise amplification and fast state estimation in ordinary observers as illustrated in [4]. Consider a normal system described by (6):

$$\dot{x}_1(t) = A_1x_1(t) + B_1u(t) + B_{1d}d(t) + B_{1f}f(t)$$
$$y(t) = C_1x_1(t) + C_f f(t) \tag{6}$$

Note that unlike (1) here it is assumed that fault enters the output equation directly. This assumption is essential in the derivation of descriptor approach observer. Also it is assumed that:

$$rank\, C_f = n_f \tag{7}$$

This means all fault signals affect the output and requires that the number of output is greater than the number of faults. Augment fault vector in the system state and add a trivial algebraic constraint like $f - f = 0$ to obtain system description (1) with the following matrices:

$$E = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \; A = \begin{bmatrix} A_1 & B_{f1} \\ 0 & I \end{bmatrix} \; B = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}$$
$$C = \begin{bmatrix} C_1 & C_f \end{bmatrix}, B_d = \begin{bmatrix} B_{1d} \\ 0 \end{bmatrix}, B_f = \begin{bmatrix} 0 \\ -I \end{bmatrix}, B_{da} = \begin{bmatrix} B_{d1} & B_{f1} \end{bmatrix} \tag{8}$$

Now it is sufficient to design an observer for the augmented descriptor system in order to reconstruct faults asymptotically. The following lemma will be useful:

Lemma4: Augmented system is completely observable according to definition 2, if and only if system (6) is observable according to definition 1 and condition (7) is satisfied.

Proof: Observability condition for the augmented descriptor system can be addressed as:

$$rank\begin{bmatrix} E \\ C \end{bmatrix} = rank\begin{bmatrix} sE - A \\ C \end{bmatrix} = n + n_f$$

According to (8) and using assumption (7) it is clear that:

$$rank\begin{bmatrix} E \\ C \end{bmatrix} = rank\begin{bmatrix} I & 0 \\ 0 & 0 \\ C & C_f \end{bmatrix} = n + n_f$$

And:

$$rank\begin{bmatrix} sE - A \\ C \end{bmatrix} = rank\begin{bmatrix} sI - A & -B_{f1} \\ 0 & -I \\ C & C_f \end{bmatrix}$$
$$= n_f + rank\begin{bmatrix} sI - A \\ C \end{bmatrix}$$

These equalities complete the proof.

Note that assumption (7) plays an essential role in the above lemma it depicts that if the fault model is not known as a priori knowledge for designing an observer, it is required that it appears in the output completely. If the fault dynamic model is known a simple augmentation procedure can be applied to estimate fault as shown in [1]. The worst case is when the fault dynamic model is not known and fault doesn't appear in the output equation completely. In this case a proportional (multiple) integral observer can be used to estimate fault signal.

III. THE COMBINED OBSERVER

In [4] an integral observer was used in the presence of input faults but in this case a descriptor approach can still be used. The following theorem suggests a better alternative. In the presence of disturbances a simple PI observer is not effective in state/fault estimation. Therefore disturbances should be decoupled in order to have a good estimation of states and faults.

Theorem1: Suppose system (6) with $rank\, C_f = n_f$ an unknown input observer which is able to decouple any disturbance perfectly and reconstruct faults exactly within an arbitrarily small time, exists if:

$$rank\,[C_f \quad (C - C_f C_f^{-1} C)B_d] = n_d + n_f$$
$$rank\begin{bmatrix} sI - A_1 & B_{1d} \\ C_1 & 0 \end{bmatrix} = n_1 + n_d \;\; \forall s \in C, \mathrm{Re}(s) \geq 0 \tag{9}$$

Proof: Augment fault vector into the system (6) state vector to obtain the augmented descriptor system in (8). According to lemma 2 the following conditions should be satisfied in order for the observer to exist.

$$rank\begin{bmatrix} E \\ C \end{bmatrix} = rank\begin{bmatrix} I & 0 \\ 0 & 0 \\ C_1 & C_f \end{bmatrix} = n_1 + n_f$$

$$rank\; C\;E^{\#}B_{da} = rank\; B_{da} = n_d + n_f$$

$$rank\begin{bmatrix} sE - A & B_{da} \\ C & 0 \end{bmatrix} = n_1 + n_d + n_f$$

The first condition is always satisfied because of the assumption (7) in the theorem. Noting that matrix $E^{\#}$ is of full rank [7], the second condition can be rewritten as:

$$rank\; CE^{\#}B_{da} = rank\begin{bmatrix} C_1 & C_f \end{bmatrix}\begin{bmatrix} I & 0 \\ -C_f^{-1}C_1 & I \end{bmatrix}\begin{bmatrix} B_{1d} & 0 \\ 0 & -I \end{bmatrix}$$
$$= rank\,[C_f \quad (C - C_f C_f^{-1} C)B_d] = n_d + n_f$$

The third condition can be written as:

$$rank\begin{bmatrix} sE - A & B_{da} \\ C & 0 \end{bmatrix} = rank\begin{bmatrix} sI - A_1 & 0 & B_{1d} & B_{1f} \\ 0 & -I & 0 & -I \\ C_1 & C_f & 0 & 0 \end{bmatrix}$$
$$= n_1 + n_d + 2n_f$$

Because of two zero blocks in the latter matrix and considering assumption (7), it's obvious that the last condition can be fulfilled if:

$$rank\begin{bmatrix} sI - A_1 & B_{1d} \\ C_1 & 0 \end{bmatrix} = n_1 + n_d$$

Proof is complete. The above equation is the decoupling condition for disturbance in a normal system described by (6). Therefore one can deduce the following corollary:

Corollary2: The augmented singular system has an unknown input observer for disturbance and fault decoupling if the normal system has an unknown input observer for disturbance decoupling and:

$$rank\, CE^{\#}B_{da} = rank\, B_{da} = n_d + n_f$$

It is worth noting that a more restrictive condition is needed for simultaneous disturbance and fault decoupling since both fault and disturbance are unknown inputs. A necessary condition for the above observer to exist is that the number of outputs should be greater than or equal to the number of unknown inputs (i.e. number of faults plus number of disturbances).

Remark7: Theorem1 shows that by augmenting the fault vector in the state, it is sufficient to decouple all unknown inputs (both fault and disturbance) and make a state estimation of the augmented plant to detect faults. Although the dimensions are increased and existence conditions are more restrictive, an accurate reconstruction of fault is made which is desired in some applications. This requires that unknown inputs enter the plant in an independent manner as well as the decoupling conditions for both of them. (See equation (9)).

Remark8: Note that one choice for $E^{\#}$ is:

$$\begin{bmatrix} E^{\#} & C^{\#} \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ -C_f^{-1}C & I & C_f^{-1} \end{bmatrix}$$

Because of assumption (7) a left inverse for C_f always exists but it is not unique.

Design procedure

For designing the observer one can follow this design procedure:

1. Check the existence conditions as in Lemma1.
2. Find a left inverse for C_f and $\begin{bmatrix} E \\ C \end{bmatrix}$ such that theorem1 conditions are fulfilled.
3. Follow the procedure proposed in [10] to design observer gain matrices.

IV. EXAMPLE

Example: Consider the following matrices for system (1):

$$A_1 = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}, B_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, B_{f1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, B_{d1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$C_f = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$E = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$E^{\#} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & -3 & 1 \end{bmatrix}, C^{\#} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}, G = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$$

The observer gain matrices can be computed as:

$$L = \begin{bmatrix} 1 & -1 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, U = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \hat{D} = L$$

$$\hat{A} = \begin{bmatrix} -1.9664 & 0.8123 & 0.7788 \\ 0.5438 & -1.6576 & -0.2014 \\ 0.6109 & 0.2350 & -2.3759 \end{bmatrix}, \hat{B} = \begin{bmatrix} 0 & -1 \\ 0 & 1 \\ 0 & -1 \end{bmatrix}$$

Following figures show the estimated fault, first and second state variable respectively. The following inputs were simulated:

$$u(t) = \sin t, d(t) = 0.5 \; t > 0, \; f(t) = \begin{cases} 0.5 & t > 5 \\ 0 & t < 5 \end{cases}$$

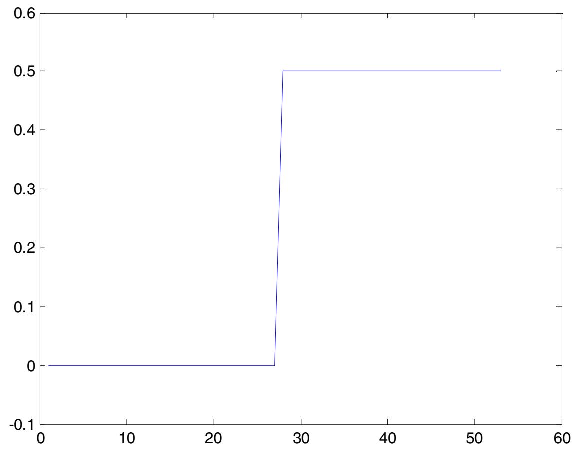

Figure1: estimated fault

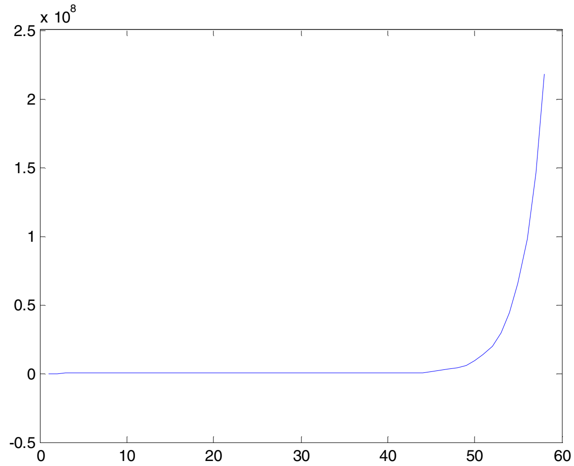

Figure2: estimated x1

The unstable mode in the system is estimated by the observer but the difference between the estimated and the original variable is not recognizable. For the stable pole of the system the estimation error is recognizable though it is fading in time fast.

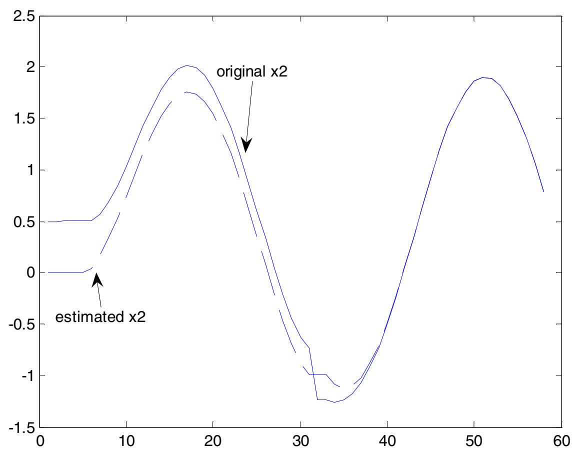

Figure3: estimated x2 and original x2

Note that this example is an unstable system and this is an advantage over the observer proposed in [4] which is only able to estimate output disturbances for stable systems and bounded output disturbances for unstable systems. Consider the following inputs:

$$u(t) = \sin t, f(t) = \begin{pmatrix} 0.5 & t > 5 \\ 0 & t < 5 \end{pmatrix}, \ d(t) = t$$

Figure4 shows the fault estimate which is perfect regardless of the unbounded disturbance.

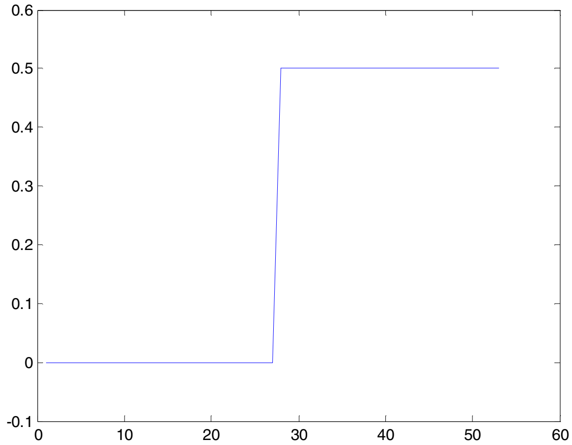

Figure4: estimated fault

If a constant disturbance is added to the system output, state estimates will not be offset free anymore as in figure5.

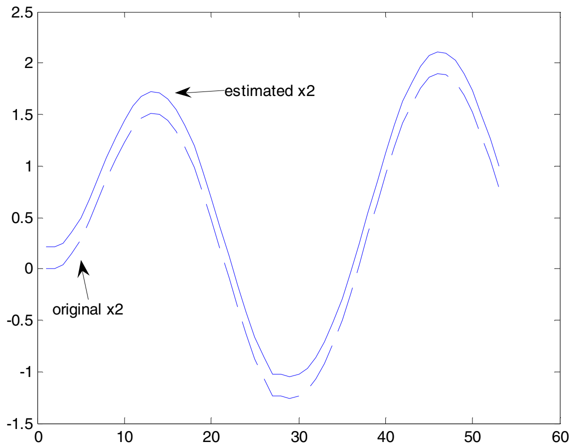

Figure5: estimated x2

However the fault estimation still remains unchanged. This is an interesting characteristic of descriptor approach observer. This advantage is gained at the cost of restrictive existence condition in (7).

V. Conclusion

A new design method is proposed for unknown input observer design. The presented formulation has the advantage of estimating different types of unknown inputs as well as decoupling other unknown inputs. This algorithm is capable of incorporating optimal filtering as discussed. Simulation example showed the effectiveness of the proposed algorithm.

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