Unknown Input-Proportional Integral Observer for Singular Systems: Application to fault detection

M. M. Share Pasand*, M. Aliyari Sh. **, and **H. D. Taghirad***

*Department of Systems and Control,
Faculty of Electrical and Computer Engineering,
**Department of Mechatronics
K.N. Toosi University of Technology,
P.O. Box 16315-1355, Tehran, Iran
E-mail: m.m.sharepasand@ee.kntu.ac.ir, aliyari@eetd.kntu.ac.ir, taghirad@kntu.ac.ir

Abstract—**a new approach to the observer design for descriptor continuous time systems is proposed and its application in the fault diagnosis problem is illustrated. In this observer, two features of disturbance decoupling and fault estimation are combined. Also a more general frame for fault estimation is used. Some numerical examples and simulation results are shown to justify the effectiveness of the algorithm.**

Keywords-descriptor systems; observer based fault detection; Proportional integral observer; unknown input observers

I. Introduction

Singular systems have been attracting the interest of many researches since they were firstly introduced, since they represent a more general description for linear systems [1]. In addition these systems arise naturally in modeling of economic, computer network and chemical systems [1]. The observer design problem for descriptor systems can be used to estimate the sates of a normal system with disturbances [3]. Most of the existing methods for fault detection in descriptor systems are based on the design of an appropriate observer for the system. Among these methods we can address the parameterization approach in [4], eigen-structure assignment methods in [5], and algebraic methods used in [6]. Compared with proportional observers, Proportional Integral observers provide more robust estimation against model uncertainties as shown in [9] and better disturbance attenuating as shown in [3]. Other methods like co prime factorization approach of [10] and LMI approaches of [9] made the observer more robust. However they don't provide ideal disturbance decoupling and only bound the effect of disturbance. A derivative term is added to the observer in [11, 12]. Since the derivative term leads to noise amplification, it is not considered here. In [12] only disturbances were considered to perturb the states and an observer was proposed to estimate disturbances, therefore it can't decouple any type of disturbances but the proposed observer can decouple any kind of disturbances.

Although many papers have dealt with the problem of observer design for descriptor systems, a few works have been made in simultaneous disturbance rejection and fault detection which is one of the most significant features of a fault detection algorithm. When a system is affected by probable unknown fault and disturbances, an effective fault detection algorithm should be able to decouple disturbances from the estimated fault. In this paper a method is proposed to accomplish this task, in which, the two strategies are combined, namely, the unknown input observer strategy like the one proposed in [6] for disturbance decoupling, and the integral observer design introduced in [3] for fault detection (estimation). The proposed method preserves each method advantages while avoiding their drawbacks. Compared with [6], the new method is able to estimate time varying and even unstable faults while the method introduced in [6] can only detect step faults. Compared with Integral observers of [3, 7], the proposed method has the advantage of distinguishing between fault and disturbances. Therefore, this method can detect faults, even in the presence of unstable disturbances. In addition this algorithm has the capability of incorporating some priori knowledge about the fault model in the observer design. Estimating disturbances by their Taylor series will lead to an inaccurate disturbance decoupling and this is a common drawback of integral observers when used for fault detection. On the other hand unknown input observer schemes don't have the capability of estimating faults. The latter is a common drawback of unknown input observers. From another point of view an Unknown input observer can't tolerate model inaccuracy and its behavior is unpredictable in the presence of model mismatch; therefore, an unknown input observer compromises the robustness of the observer in order to decouple any disturbances. On the other hand a proportional integral observer can tolerate model uncertainties to some extent at the cost of limiting its performance to a specific group of fault/disturbance signals, i.e. step or ramp disturbances. In [6] the general structure observer is suggested for time varying disturbances but the proposed method in this paper is much simpler than a general structure observer. The proposed method composed the above strategies in order to gain better performance. The algorithm proposed in this paper has two advantages over the one proposed in [6, 9]. First it works in the presence of a larger group of faults, and second it has a much simpler design method which can be done by conventional pole placement methods. Even the optimal estimation scheme of Kalman can be applied to this method in order to optimize the observer. This paper is organized as follows. In the second section the preliminary information and assumptions are illustrated, in the third section the main results and theorems are proposed and existence conditions are derived. In the fourth section some simulation results and comparisons are made, and the conclusion and remarks are drawn in the last section.

Proceedings of ICEE 2010, May 11-13, 2010
978-1-4244-6760-0/10/$26.00 ©2010 IEEE

II. BACK GROUANDS AND PRELIMINARIES

A. Model based Fault detection

Consider a descriptor system described by:

$$E\dot{x}(t) = Ax(t) + Bu(t) + B_f f(t) + B_d d(t) \\ y(t) = Cx(t) \tag{1}$$

Vectors $x(t), u(t), d(t), f(t), y(t)$ are the system state, input vector, disturbance vector, fault vector and the output respectively. Matrices A, B, B_d, B_f, C are real valued, constant matrices of appropriate dimensions.

Assumption 1: If $f(t) \in R^{n_f}, d(t) \in R^{n_d}$, we assume:

$$rank\, B_d = n_d$$

$$rank\, B_f = n_f$$

The above assumptions are not restrictive as if they weren't satisfied; we can redefine fault and disturbance vectors. Define:

$$B_d = T\overline{B}_d$$

$$B_f = Q\overline{B}_f$$

Such that T and Q are of ranks n_d and n_f respectively. Then we can set:

$$\overline{d}(t) = \overline{B}_d d(t)$$

$$\overline{f}(t) = \overline{B}_f f(t)$$

Therefore, assumption 1 will be satisfied.

Definition 1: A signal type is its highest non-zero time derivative.

From the above definition, a step signal is of type 0 and a ramp signal has of type 1. Also it is worth noting that a signal type is defined for piecewise continuously differentiable signals.

Corollary 1: a signal of type *n* can be modeled as the impulse response of a system of degree *n+1*.

Corollary 2: Any signal of type *n* can be exactly described by *n+1* terms of its Taylor series. And can be estimated by *n+1* terms of its Taylor series if its *n+1* and higher derivatives are bounded.

Definition 2: A singular system is called impulse free if it doesn't exhibit impulsive behavior in its state response.

Lemma 1: A singular system described by (1) is impulse free if and only if:

$$\deg\ (\det(sE - A)) = rankE$$

Proof: The proof is given in [1].

Definition 3: A singular system is called regular if:

$$\exists s \in C \ni \det(sE - A) \neq 0$$

Since the above polynomial has finite number of roots, the regularity condition can be addressed as:

$$\det(sE - A) \neq 0 \quad \text{For almost all } s \in C$$

Definition 4: A singular system is observable if and only if:

$$rank\begin{bmatrix} E \\ C \end{bmatrix} = rank\begin{bmatrix} sE-A \\ C \end{bmatrix} = n \quad \forall s \in C, \mathrm{Re}(s) \geq 0$$

B. Fault model

The signal $f(t)$ can be modeled as the output of linear time invariant system.

$$\dot{f}(t) = \Phi f(t) \\ f(0) = \begin{bmatrix} f_0^T & \ldots & f_n^T \end{bmatrix}^T \tag{2}$$

There is no need to assume a singular model for fault signal, because the fault effect on the output can be impulsive even if we assume a normal model as described in (2). There are several fault signals which can be modeled by (2) like step, ramp, sinusoids and exponentials. We may add a white noise to the above equation in order to model the uncertainties in the aforementioned model.

III. UNKNOWN INPUT PI OBSERVER FOR FAULT DETECTION

In this section we extend the results in [6] which are only valid for constant faults, into a more general group of faults. Consider the system described by (1), we wish to design a normal observer described by (4) such that:

$$\lim_{t \to \infty} (x - \hat{x}) = 0 \\ \lim_{t \to \infty} (f - w) = 0 \tag{3}$$

The proposed observer has the form:

$$\dot{z}(t) = \hat{A}z(t) + \hat{B}y(t) + \hat{J}u(t) + \hat{H}w(t) \quad (4.a)$$

$$\hat{x}(t) = z(t) + \hat{D}y(t) \quad (4.b)$$

$$\dot{w}(t) = F(y(t) - C\hat{x}(t)) + \hat{\Phi}w(t) \quad (4.c)$$

This is essential to use output to construct the estimated state of the system in (4.b). As shown in [1] it is impossible to design a normal observer like (4) without including the output in equation (4.b). It should be noted that defining matrix F in the above equations is necessary and if it is omitted, as in [6], we must set number of fault signals equal to number of outputs which is meaningless in general situations. Equations (4) suggest a general description for PI observers. In the rest of this paper we shall assume a special case of the above equations because fault model is not known and we construct it by integrating the estimation error. The fault signal can be modeled as:

$$f(t) = f_0 + f_1 t + \ldots. f_n t^n \tag{5}$$

We can model unstable faults with the above equation as well as stable exponentials and polynomials. Matrices $f_0, f_1 \ldots f_n$ are unknown coefficient of fault vector which are aimed to be approximated by an observer. Define:

$$\dot{\omega}(t) = \Phi^* \omega(t)$$

$$w(t) = C_w^* \omega(t)$$

$$\Phi^* = diag(\Phi_1, \ldots. \Phi_{n_f})$$

$$C_w^* = \begin{bmatrix} C_{w1} & \ldots & C_{wn_f} \end{bmatrix}$$

$$\Phi_j = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \cdot & \cdot & 0 & \ldots & . \\ . & . & & & \\ 0 & 0 & \ldots & 0 & 1 \\ 0 & 0 & \ldots & \ldots & 0 \end{bmatrix}_{n_j} \quad (6)$$

$$C_{wj} = [1 \quad 0 \quad \ldots 0]$$

$$\dot{\omega}_j(t) = \Phi_j(t)\omega_j(t)$$

$$w_j(t) = C_{wj}\omega_j(t)$$

The above model for fault is designated to estimate the fault signal by the first n_Φ terms of its Taylor series. It should be noted that depending of the fault signal type, n_Φ can be equal to or greater than n_f . If we set Φ to zero, the proposed observer will be the same as in [6]. Then n_Φ and n_f will be equal and the observer will be efficient for step faults only. It then approximates fault by a constant signal namely $w(t)$. Estimated fault vector is the first n_f integral variables. In the rest of paper we assume $n_f = 1$ (i.e. scalar fault) for simplicity.

Lemma 3: An unknown input observer described by (4) exists for system (1) if there is a matrix U∈ $R^{n\times n}$ such that:

$$\hat{A}UE + \hat{B}C = UA$$
$$\hat{J} = UB$$
$$UB_d = 0$$
$$\hat{C}UE + \hat{D}C = I$$
$$\hat{H} = UB_f$$

And:

$$\mathrm{Re}\lambda_i \begin{bmatrix} \hat{A} & \hat{H} \\ -FC & \Phi \end{bmatrix} < 0 \quad i = 1,..n + n_\Phi \quad (7)$$

Proof: Define:

$$\varepsilon(t) = z(t) - UEx(t)$$
$$\zeta(t) = w(t) - f(t)$$

Then by using (6) and assuming that $\hat{\Phi}$=Φ the estimation error dynamics become:

$$\begin{bmatrix} \dot{\varepsilon}(t) \\ \dot{\zeta}(t) \end{bmatrix} = \begin{bmatrix} \hat{A} & \hat{H} \\ -FC & \hat{\Phi} \end{bmatrix} \begin{bmatrix} \varepsilon \\ \zeta \end{bmatrix} \quad (8)$$

Therefore, the condition (7) is derived easily from this equation. ■

Lemma 4: The system described by (1) without fault input, has an unknown input PI observer described by (4) which can estimate the state vector independent of disturbances, if:

$$1)\, rank \;\; CE^{\#}B_d = rank B_d = n_d$$
$$2)\, rank \begin{bmatrix} A - sE & B_d \\ C & 0 \end{bmatrix} = n + n_d \quad (9)$$

The proof can be found in [6].

In the above lemma, $E^{\#}$ is defined such that:

$$\begin{bmatrix} E^{\#} & C^{\#} \end{bmatrix} \times \begin{bmatrix} E \\ C \end{bmatrix} = I_n \quad (10)$$

From (9) and (10) it is clear that observability of the system (1) is a necessary condition for existence of an Unknown input PI observer.

Lemma 5: One solution to the equations (6) can be addressed as:

$$L = E^{\#}B_d(CE^{\#}B_d)^{+} + G(I_{n_y} - CE^{\#}B_d(CE^{\#}B_d)^{+})$$
$$U = (I_n - GC)(I_n - E^{\#}B_d(CE^{\#}B_d)^{+}C)E^{\#} \quad (11)$$
$$\hat{D} = C^{\#} + L(I_{n_y} - CC^{\#})$$
$$\hat{J} = UB$$
$$\hat{H} = UB_f$$

In (11) G is an arbitrary matrix of appropriate dimensions. Furthermore, if we find $\hat{A}$ such that (7) is satisfied, we then have:

$$\hat{B} = \hat{A}\hat{D} + K \quad (12)$$

Where K satisfies:

$$\hat{A} = UA - KC$$

The design procedure for K when $\Phi = 0$ (i.e. for step faults) is stated is [9], however setting K to a value which can satisfy (7), we can easily design the observer by using equations (11) and (12). The design method for matrix K is a bit complicated so we take another approach which is stated in the following.

Consider the system (1), if we augment fault signal to the system state vector, the augmented system will have the following dynamics:

$$\underbrace{\begin{bmatrix} E & 0 \\ 0 & I_{n_f} \end{bmatrix}}_{E_a} \dot{x}_a(t) = \underbrace{\begin{bmatrix} A & B_f \\ 0 & \Phi \end{bmatrix}}_{A_a} x_a(t) + \underbrace{\begin{bmatrix} B_d \\ 0 \end{bmatrix}}_{B_{da}} d(t) + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{B_a} u(t) \quad (13)$$

$$y(t) = \begin{bmatrix} C & 0 \end{bmatrix} x_a(t)$$

Therefore one can obtain the following equalities:

$$rank \begin{bmatrix} E_a \\ C_a \end{bmatrix} = rank \begin{bmatrix} E & 0 \\ 0 & I \\ C & 0 \end{bmatrix} = rank \begin{bmatrix} E \\ C \end{bmatrix} + n_\Phi \quad (14.a)$$

$$rank \begin{bmatrix} A_a - sE_a \\ C_a \end{bmatrix} = rank \begin{bmatrix} A - sE & B_f \\ 0 & \Phi - sI \\ C & 0 \end{bmatrix} \quad (14)$$

$$\forall s \in C \; \mathrm{Re}(s) \geq 0 \quad (14.b)$$

Theorem 1: The augmented system described by (13) is observable if the system (1) is observable and:

$$rank\begin{bmatrix} B_f \\ sI-\Phi \end{bmatrix} = n_f \quad \forall s \in C, \mathrm{Re}(s) \geq 0 \qquad (15)$$

Proof: The observability matrix for the augmented system is stated in (14). Because of the structure defined for Φ in (6), for $s \neq 0$ in (14.b), from the n^{th} row to the $n+n_\Phi$ th row, there are n_Φ linearly independent rows. Therefore, condition (14.b) is equal to:

$$rank \begin{bmatrix} sE-A & B_f \\ C & 0 \end{bmatrix} = n \;\; \forall s \in C, \mathrm{Re}(s) \geq 0$$

A sufficient condition for the above equation is:

$$rank \begin{bmatrix} sE - A \\ C \end{bmatrix} = n$$

For $s = 0$, this condition should be satisfied:

$$rank \begin{bmatrix} A & B_f \\ 0 & \Phi \\ C & 0 \end{bmatrix} = n + n_\Phi$$

Noting that Φ provides $(n_\Phi - 1)$ linearly independent columns, and that the system in (1) is observable, by using assumption 1, the above matrix is proved to be of full column rank. This completes the proof.

Remark 1: Theorem 1 is only valid for scalar faults and for vector faults we shall apply lemma 4 to equations (14).

Theorem 2: The unknown input PI observer (4) exists for the augmented system (13) if an unknown input PI observer exists for system (1).

Proof: Note that:

$$E_a^{\#} = \begin{bmatrix} E^{\#} & 0 \\ 0 & I \end{bmatrix}, C_a = \begin{bmatrix} C & 0 \end{bmatrix}, B_{da} = \begin{bmatrix} B_d \\ 0 \end{bmatrix}$$

As a result, existence conditions (9) from lemma 4, for augmented system can be obtained as:

$$1)\, rank\;\; C_a E_a^{\#} B_{da} = rank\; CE^{\#} B_d = n_d$$

$$2)\, rank \begin{bmatrix} A_a - sE_a & \begin{matrix} B_d \\ 0 \end{matrix} \\ C_a & 0 \end{bmatrix} = n + n_\Phi + n_d \qquad (17)$$

The latter equality is obtained using assumption 1 and theorem 1 and the observability assumption. This completes the proof of theorem 2.

Corollary 3: Designing an unknown input PI observer (4) for system (1) for fault detection and state estimation can be replaced by designing an unknown input observer for system (13) for state estimation.

Define the estimation error for augmented system as:

$$\xi_a(t) = z(t) - U_a E_a x_a(t)$$

Then the error dynamics obey:

$$\begin{aligned} \dot{\xi}_a(t) &= (\hat{A}_a U_a E_a + \hat{B}_a C_a - U_a A_a)\; x_a(t) \\ &\quad + (\hat{J}_a - U_a B_a)\; u(t) - U_a B_{da}\, d(t) + \hat{A}_a \xi_a(t) \qquad (18) \\ \hat{x}_a(t) &= z(t) + \hat{D}_a y(t) = \xi_a(t) + (U_a E_a + \hat{D}_a C_a)\; x_a(t) \end{aligned}$$

Subscript a shows that the subscripted matrices are to be designed for augmented system. We assume $\hat{H}_a = 0$ because there is no further need to include integral action in augmented system. To have an unknown input observer which estimates the state asymptotically we must set:

$$\begin{aligned} &\hat{A}_a U_a E_a + \hat{B}_a C_a = U_a A_a \\ &\hat{J}_a = U_a B_a \\ &U_a B_{da} = 0 \\ &U_a E_a + \hat{D}_a C_a = I \end{aligned} \qquad (19)$$

Then the error dynamics become:

$$\dot{\xi}(t) = \hat{A}_a \xi_a(t)$$

Lemma 6: If the conditions of theorem 2 are satisfied then the pair $(C_a, U_a A_a)$ is observable, i.e.

$$rank \begin{bmatrix} sI - U_a A_a \\ C_a \end{bmatrix} = n + n_\Phi \;\; \forall s \in C$$

The proof is stated in [6].

Now we can propose the following design procedure:

Design procedure 1:

1) Construct the augmented system in equation (13) by defining an appropriate model for fault in (6).

2) Compute matrices $E^{\#}, C^{\#}$ such that:

$$E^{\#}E + C^{\#}C = I$$

3) Define:

$$E_a^{\#} = \begin{bmatrix} E^{\#} & 0 \\ 0 & I \end{bmatrix}, C_a^{\#} = \begin{bmatrix} C^{\#} \\ 0 \end{bmatrix}$$

4) Select matrix G such that $I - GC$ is non-singular.

5) Compute matrices U_a, L_a according to (11) but for the augmented system.

6) Calculate $\hat{D}_a$ as:

$$\hat{D}_a = C_a^{\#} + L(I - CC^{\#})$$

7) Design K_a such that $\hat{A}_a$ is stable according to the following:

$$\hat{A}_a = U_a A_a - K_a C_a$$

This can be done by a simple pole placement algorithm. Observability of the pair $(U_a A_a, C_a)$ is guaranteed by lemma 6.

8) Calculate $\hat{B}_a$ and $\hat{J}_a$ as:

$$\hat{B}_a = \hat{A}_a \hat{D}_a + K_a$$

$$\hat{J}_a = U_a B_a$$

This completes the design procedure. ■

This procedure has two advantages over the one proposed in [6]. Firstly, it is able to estimate different fault types while the proposed method in [6] is only able to estimate step faults. Secondly, it has a simple and familiar design method of pole placement in step 7, while method in [6] requires a complicated procedure proposed in [9] which demands finding $\hat{A}$ such that (8) is stabilized. The proposed method also provides more design parameters by presenting F in equations (4) and the idea of state augmentation, which leads to a simple design procedure for observer gain. However, it is shown in the following theorem that the proposed method is a more general framework compared to that in [6] and other similar algorithms. Consider an observer in the form of (4) is obtained for a system (1) by using the procedure presented in [6]. Then, suppose we want to design an observer for augmented system of (13) in order to detect more general group of faults. Then the following theorem is essential.

Theorem 3: If (4) is an unknown input PI observer for (1) with step faults, then one solution to the observer design problem for the augmented system can be found as:

$$\dot{z}_a(t) = \hat{A}_a z_a(t) + \hat{B}_a y(t) + \hat{J}_a u(t)$$

$$\hat{x}_a(t) = z(t) + \hat{D}_a y(t)$$

$$\begin{bmatrix} \hat{x} \\ \hat{f} \end{bmatrix} = \begin{bmatrix} I_{n+n_f} & 0 \end{bmatrix} \hat{x}_a(t)$$

$$U_a = \begin{bmatrix} U & 0 \\ 0 & I \end{bmatrix}, L_a = \begin{bmatrix} L \\ 0 \end{bmatrix}, K_a = \begin{bmatrix} K \\ F \end{bmatrix} \tag{20}$$

$$\hat{D}_a = \begin{bmatrix} \hat{D} \\ 0 \end{bmatrix}, \hat{A}_a = \begin{bmatrix} \hat{A} & \hat{H} \\ -FC & \Phi \end{bmatrix} \tag{21}$$

$$\hat{B}_a = \begin{bmatrix} \hat{B} \\ F(I - C\hat{D}) \end{bmatrix}, \hat{J}_a = \begin{bmatrix} \hat{J} \\ 0 \end{bmatrix} \tag{22}$$

Proof: The proof is constructive; consider the design procedure 1, depicted in this section. First note that from equations (13) and the proof of theorem 2, it is clear that:

$$CE^{\#}B_d = C_a E_a^{\#} B_{da},\ E_a^{\#} B_{da} = \begin{bmatrix} E^{\#} B_d \\ 0 \end{bmatrix}$$

Then note that if G is selected such that $I - GC$ is non-singular in the 4th step of design procedure 1, using the following equation:

$$I_{n+n_\Phi} - \begin{bmatrix} G \\ 0 \end{bmatrix} C_a = \begin{bmatrix} I - GC & 0 \\ 0 & I \end{bmatrix}$$

It is obtained that:

$$\det\left(I_{n+n_\Phi} - \begin{bmatrix} G \\ 0 \end{bmatrix} C_a\right) \neq 0$$

Furthermore using equations (11), matrices U_a, L_a are obtained as in (20). If one choose matrices $\hat{A}$ and K as in design procedure 1, then from equations (12), equations (21) and (22) can be easily obtained, and this concludes the proof ■

Remark 2: Note that matrix F is calculated by partitioning the matrix K_a as:

$$K_a = \begin{bmatrix} K \\ F \end{bmatrix} \tag{23}$$

Remark 3: Although we present a method of obtaining the augmented observer from the original observer presented in [6], because of the simplicity and design degree of freedom in the proposed method it is recommended that one follows the design procedure 1, until step 7 for the original system (1) and then switch to the augmented system for the pole placement procedure. Therefore, we present the following design procedure. The following procedure has an advantage of simple pole placement compared with procedure in [6] combined with its low computational cost in comparison with design procedure 1.

Design procedure 2:

1) Consider the system described by (1), and satisfy the existence conditions of lemma 4 and 5, find $E^{\#}, C^{\#}$ as in (10) and select G such that $I - GC$ is not singular.

2) Compute matrices U and L according to (11).

3) Construct A_a, U_a, L_a, C_a according to (13) and (20).

4) Find K_a such that the eigen values of matrix $\hat{A}_a = U_a A_a - K_a C_a$ are located in appropriate places.

5) Partition K_a as in (23) and compute $\hat{D}$ and $\hat{B}$ according to (11) and (12).

6) Construct the observer (4) with $\hat{H} = UB_f$ and Φ as defined in (6) and F as in (23).

IV. Numerical Example

Consider system (1) with following matrices:

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad A = \begin{bmatrix} -5 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & -3 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, B_d = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, B_f = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

We assume ramp fault and ramp disturbance and a step input. It was assumed that although the occurrence of fault and its magnitude is not known, it is known that it has the form of:

$f(t) = at + b$

For disturbance signal there is no assumption and it is considered as a ramp signal added by a sinusoidal. Also a sinusoidal fault signal is simulated in order to examine the efficiency of proposed algorithm in presence of unpredicted fault signal types.

Following the procedure presented in [6] we obtain:

$$E^{\#} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, C^{\#} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Assume:

$$G = \begin{bmatrix} 0 & 1 \\ 1 & -1 \\ 0 & 0 \end{bmatrix}$$

Then we have the following:

$$L = \begin{bmatrix} 1 & 0 \\ -0.5 & 0.5 \\ 0 & 0 \end{bmatrix}, U = \begin{bmatrix} 0 & 0 & 0 \\ 0.5 & 0 & 0.5 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\hat{D} = \begin{bmatrix} 1 & 0 \\ -0.5 & 1 \\ 0 & 0 \end{bmatrix}, \hat{A} = \begin{bmatrix} -20.023 & 0.354 & 0 \\ 0.939 & -39.977 & -0.5 \\ -24.452 & 599.736 & 0 \end{bmatrix}$$

$$\hat{H}=\begin{bmatrix}0\\1\\1\end{bmatrix},\ \hat{B}=\begin{bmatrix}-0.177 & 0\\17.488 & -1.5\\-299.868 & -2\end{bmatrix},\ \hat{J}=\begin{bmatrix}0\\0\\1\end{bmatrix}$$

Figure 1 shows the fault signal and the estimated fault (i.e. fault residue). Fault input is a ramp signal occurred at t=5 sec the algorithm shows fault occurrence and estimates it after 2 sec effectively. Compared with figure 4 it is clear that using the proposed observer is vital in the presence of ramp faults. Figure 2 shows an estimated and original state for the presented observer. It shows that disturbance decoupling was done successfully. Figure 4 shows the estimated fault in case of a sinusoidal fault which shows fault occurrence. Although fault input was not estimated correctly, residue is a fine indicative of fault occurrence. Figure 5 shows the original fault and its estimation by the algorithm proposed in [6]. In the following figures dashed lines are indicative of estimated variables while normal lines show original signals.

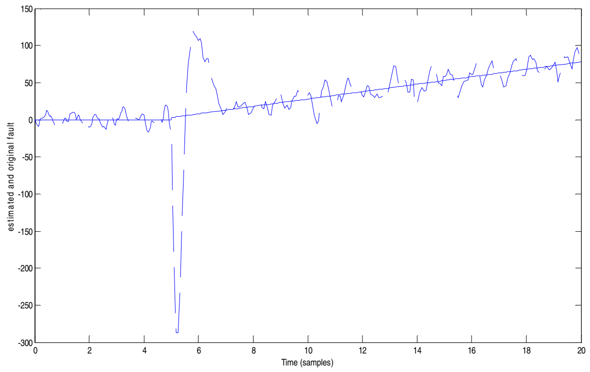

Figure 1: estimated fault signal

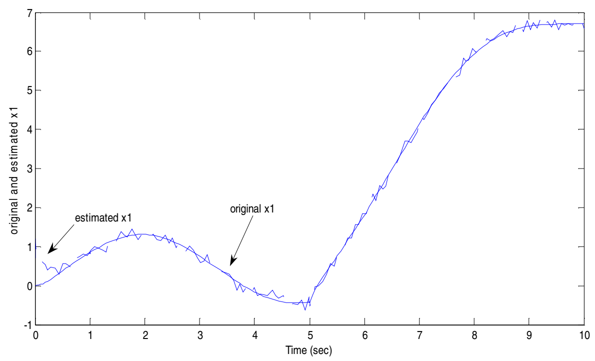

Figure 2: estimated state in presence of fault and disturbance

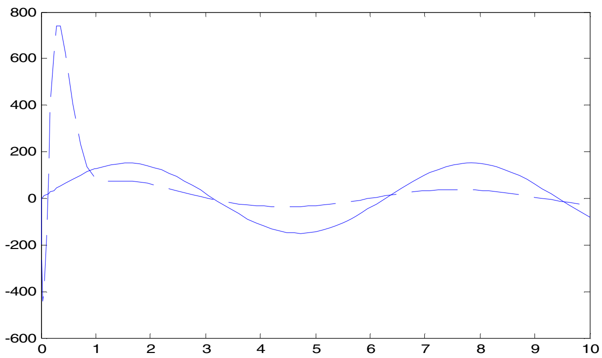

Figure 3: estimated sinusoidal fault

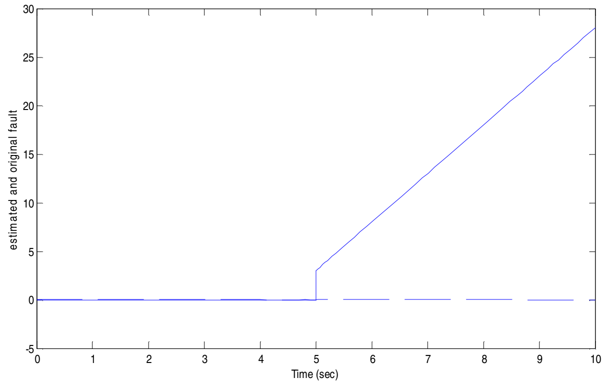

Figure 4: estimated ramp fault using method given in [6]

V. Conclusion

A new method for state observation, disturbance decoupling and fault detection was proposed. The effectiveness of this algorithm is to estimate fault signals asymptotically which is useful in fault diagnosis. The presented method can decouple any type of disturbances and estimate polynomial faults effectively. Also it can detect other types of faults.

References

[1] L. Dai “singular control systems” Springer-Verlag, 1989

[2] R. Nikoukhah, B.C. Levy and A.S. Willsky, “Kalman filtering and ricatii equations foe descriptor systems”, IEEE Trans. Autom. Contr., No. 9, 1992, pp. 1325-1342.

[3] Z. Gao, H. Wang “Descriptor observer approaches for multivariable systems with measurement noises and application in fault detection and diagnosis”, Systems and control letters, No. 55, 2006, pp 304-313.

[4] A.G. Wu, G.R. Duan “Generalized PID Observer Design for Descriptor Linear Systems”, IEEE Trans. On Systems, Man and Cybernetics-Part B, vol. 37, No. 5, october 2007.

[5] A.G. Wu, G.R. Duan “Design of PI Observers for Continuous –Time Descriptor Linear Systems” IEEE Trans. On Systems, Man and Cybernetics-Part B, vol. 36, No. 6, december 2006.

[6] H.S. Kim, T.K Yeu and S. Kawaji “Fault Detection in Linear Descriptor Systems via Unknown Input PI Observer” Trans. Control, Autom. Syst. Eng., vol.3, No.2, June 2001, pp 77-82.

[7] Z. Gao and D.W.C. Ho “ Proportional Multiple Integral Observer Design for descriptor linear systems with measurement output disturbances”proc. Inst. Elect. Eng., Control Theory appl., vol. 151,No. 3, pp. 279-288, May 2004.

[8] Darouach, M. Boutayeb and M. Zasadzinski “Kalman filtering for continuous time descriptor systems”, Proc. American Contr. Conferenece. June 1997.

[9] D. Koenig and S. Mammar “Design of proportional-Integral observer for unknown input descriptor systems” IEEE Trans. Autom. Contr. Vol. 47, No. 12, December 2002.

[10] B. Marx, D. Koenig and D. Georges “ Robust fault diagnosis for descriptor systems via coprime factorization and LMI” proc. Of the IFACSAFEPROCESS’ 03, 2003.

[11] A.G. Wu and G.R Duan “design of PD observers in descriptor linear systems” Int. Journal of Contr., Automation and sys., vol. 5, No. 1, pp 93-98, February 2007.

[12] Z. Gao and S. Ding “ PMID observer design for unknown input generalized dynamical systems” proc. Of 16th IFAC conference, 2005.