International Conference on Control, Automation and Systems 2010
Oct. 27-30, 2010 in KINTEX, Gyeonggi-do, Korea

Disturbance Retrieving Unknown Input Proportional Integral Observer for Generalized Linear Systems

Application to fault diagnosis

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Abstract— **An unknown input retrieving observer scheme is proposed which not only decouples unknown inputs in estimation but also gives an estimation of the decoupled input. The provided method benefits from its low computational cost as well as its less restrictive existence conditions compared to existing ones.**

Keywords-descriptor systems; fault detection;Perfect disturbance decoupling ;PI observer.

I. INTRODUCTION

The problem of unknown input estimation has received considerable amount of interest during last decades [1-4]. Unknown inputs may be used to model disturbances, faults or model uncertainties in a general system representation. In the presence of an unknown input, simple observers fail to yield to a good estimation of the states; therefore, it is required to provide an observer with an error dynamics which is independent of unknown inputs. This problem leads to the unknown input observer schemes developed in the literature [5, 6 and 7]. The capability of decoupling unknown inputs in the state estimation is the main advantage of these observers. In fault detection and diagnosis the problem of unknown input observer design is applied either in disturbance decoupling or in fault estimation [6 and 7]. The challenge there is usually to find appropriate matrices that make the error dynamics stable and insensitive to the unknown inputs [6, 7 and 8]. Another technique developed is to estimate the unknown input by integral variable(s). In this method the unknown input is estimated by a constant (or time polynomial) signal [9]. The estimation error can be arbitrarily reduced by increasing the observer bandwidth. However, when there are more than one unknown inputs in the system, they may interfere in the estimation of the other unknown input, and therefore, the error dynamics cannot be generated independent of the unknown inputs. Decoupling both unknown inputs in the estimation error dynamics requires very restrictive conditions as well as loss of unknown input information. Moreover, in fault detection and diagnosis problems it is desired to have an estimate of the unknown input. As a result when there are two or many fault signals, a disturbance decoupling scheme is not efficient. If one designs an integral observer for both unknown inputs, the resulting observer is unable to accurately estimate the states and the unknown inputs. Using two unknown input integral observers will increase the observer computational costs since it duplicates the order of the observer. In fact for reconstructing unknown inputs, it is not efficient to duplicate reconstruction of the state.

This paper analyzes this situation carefully, where there are at least two unknown inputs which need to be estimated, one unknown input is estimated by an unknown input integral observer and the decoupled unknown input is retrieved by the observer. The advantages of the presented observer are to provide robust estimation because it combines disturbance decoupling and integral estimation schemes and thus uses both methods advantages, because it doesn't use output derivative as in [5 and 11] and it yields to a normal observer like that proposed in [12 and 13], it can be implemented on practical systems [14]. Also it provides a promising scheme in order to retrieve decoupled signal(s) information. In this paper, after discussion on the structure of the observer, the existence conditions are derived. Then the effectiveness of the proposed observer is studied in detail and a design procedure is given.

II. BACK GROUANDS AND PRELIMINARIES

A. Problem Statement

In many practical situations, an observer should yield not only estimation of the state but also of an unknown input(s). Unknown inputs may represent model uncertainties, faults or failures, actual disturbance signals and other immeasurable inputs. If the unknown input dynamics is known as a priori, the problem of unknown input estimation is nothing more than a conventional observer design problem. This can be shown by augmenting the unknown input state space representation into that of the original system. However when the dynamics of the unknown input (i.e. its time shape) are not known, the only way to extract the information included in the state response is to examine the output estimation error. Therefore the unknown input can be estimated by integrating the output estimation error. This is equivalent to assuming an integrator as the unknown input dynamics as shown in [7]. When there are more than one unknown input vectors, an unknown input may enter the other unknown input estimation error dynamics, making the estimation of the latter inaccurate. Therefore it is logical to decouple the other unknown input when estimating the first and vice versa. Implementing an observer for decoupling both of the unknown inputs needs a very restrictive existence conditions and designing a separate observer for each unknown input is costly. This paper proposes two alternative methods for retrieving the decoupled unknown input with a lower order observer. Consider the following system:

$$E\dot{x}(t) = Ax(t) + Bu(t) + B_d d(t) + B_f f(t)$$
$$y(t) = Cx(t) \quad (1)$$

Vectors x, u, d, f, y are the system state, first and second unknown inputs and output respectively. It is assumed that input matrices for unknown inputs are of full column rank. This assumption is not restrictive as shown in [7]. Both unknown inputs are to be estimated and the system state should be estimated without steady state error. Though slowly changing time signals were assumed in this paper, for fast changing unknown inputs one can enhance the proposed observer performance by either increasing the observer bandwidth as suggested in [8] or the observer order and complexity as proposed in [9,7]. Note that through this paper, the roles of $d(t)$ and $f(t)$ are interchangeable.

Remark1: Compared to the normal state space description, (1) is a general description of a dynamic system. Therefore studying a descriptor system is preferable because setting $E = I$ the descriptor system is equal to a normal one. However a singular system may possess special features which are not present in a normal state space system.

B. *Definitions and preliminaries*

Definition1: System (1) is called regular if:

$$\exists\, s \in C \quad \det(sE - A) \neq 0 \quad (2)$$

Lemma1: System (1) is completely observable if and only if:

$$rank\begin{bmatrix} E \\ C \end{bmatrix} = rank\begin{bmatrix} sE - A \\ C \end{bmatrix} = n \quad (3)$$

Here C denotes the complex plane.

Lemma2: A descriptor system has an equivalent normal state space description if and only if:

$$rank\, E = n \quad (4)$$

And it has an equivalent standard description including a feed through term, namely D, if and only if:

$$\deg(\det(sE - A)) = rank\, E \quad (5)$$

Definition2: If a descriptor system satisfies (5), it is called Impulse free. The term indicates that the state response of the system doesn't include impulses.

Definition3: The following system is called a perfect unknown input proportional integral observer for system (1) if condition (7) is satisfied.

$$\dot{z}(t) = \hat{A}z(t) + \hat{B}y(t) + \hat{J}u(t) + \hat{H}w(t)$$
$$\hat{x}(t) = z(t) + \hat{D}y(t)$$
$$\dot{w}(t) = F(y(t) - C\hat{x}(t)) \quad (6)$$

$$\forall\, d(t) \in \Re^d, f(t) \in \Pi$$
$$\lim_{t\to\infty}\left(\begin{bmatrix} x(t) \\ f(t) \end{bmatrix} - \begin{bmatrix} \hat{x}(t) \\ w(t) \end{bmatrix}\right) = \lim_{t\to\infty}\begin{bmatrix} \varepsilon(t) \\ \varsigma(t) \end{bmatrix} = 0 \quad (7)$$

Here $d(t)$ is supposed to be the decoupled input and $f(t)$ is aimed to be estimated via integral variable(s).The symbol Π denotes a specific kind of signals. For example it can be the family of piecewise constant signals.

Remark2: Though the plant is assumed to be descriptor one, it is favorable to assume and design standard observers, because they can be easily implemented.

Lemma3: A normal, disturbance decoupling observer (6), which decouples $d(t)$ exists for (1) if:

$$rank\, CE^{\#}B_d = rank\, B_d = n_d$$
$$\forall s \in C,\ \mathrm{Re}(s) \geq 0$$
$$rank\begin{bmatrix} sE - A & B_d \\ C & 0 \end{bmatrix} = n + n_d \quad (8)$$

Matrix $E^{\#}$ is defined such that:

$$\begin{bmatrix} E^{\#} & C^{\#} \end{bmatrix} \times \begin{bmatrix} E \\ C \end{bmatrix} = I$$

Proof can be found in [6, 7]. Note that (8) requires that (1) is observable; therefore the observability requirement is not mentioned in the above lemma.

Lemma4: System (6) is an Unknown Input Proportional Integral observer for system (1), if there is a matrix U such that the following equalities are fulfilled:

$$\hat{A}UE + \hat{B}C = UA$$
$$\hat{J} = UB$$
$$UB_d = 0$$
$$UE + \hat{D}C = I$$
$$\hat{H} = UB_f$$

$$\mathrm{Re}\,\lambda_j\begin{bmatrix} \hat{A} & \hat{H} \\ -FC & 0 \end{bmatrix} < 0 \quad (9)$$

Proof is in [7].

Lemma5: If a descriptor system is regular according to definition1, there exists a similarity transformation which transforms (1) to the following system:

$$\dot{x}_1(t) = A_1x_1(t) + A_2x_2(t) + B_1u(t) + B_{d1}d(t) + B_{f1}f(t)$$
$$0 = A_3x_1(t) + A_4x_2(t) + B_2u(t) + B_{d2}d(t) + B_{f2}f(t)$$
$$y(t) = C_1x_1(t) + C_2x_2(t) \quad (10)$$

Vector $x_1(t)$ is of a dimension equal to $rank\ E$ .Also system (1) has another equivalent form of:

$$\dot{x}_1(t) = A_1x_1(t) + B_1u(t) + B_{d1}d(t) + B_{f1}f(t)$$
$$N\dot{x}_2(t) = x_2(t) + B_2u(t) + B_{d2}d(t) + B_{f2}f(t)$$

The latter equivalent form introduces the slow and fast subsystems. The fast subsystem is an impulsive subsystem possessing impulsive behavior and having an improper transfer function.

Proof is simple, based on a well known theorem [14].

Remark3: Equations (10) suggests a new understanding of a descriptor system as a differential algebraic system. Therefore a descriptor description is more general than the normal one because it provides an algebraic part. However one can make a good use of the information lie in the algebraic part of the system while this information is not provided in a standard state space system. Therefore in some applications it is logical to expect the observer not to work properly for a standard system.

Definition4: An input signal $d(t)$ is called retrievable if:

$$rank\, B_{d2} = n_d \tag{11}$$

Remark4: (11) means that B_{d2} is of full column rank i.e. has a left inverse. When B_{d2} is a nonzero matrix, (11) is not a restrictive condition for a system described by (10), as shown in [7].

Remark5: A normal state space system is in the form of (10) and any unknown input to this system is not retrievable. Therefore retrievability of an input is a special characteristic of a group of singular systems.

III. DISTURBANCE RETRIEVING OBSERVER AND THE PROBLEM OF FAULT DIAGNOSIS

Consider a system described by (1), with two different unknown input which models two types of faults. It is aimed to estimate two fault signals separately as well as the whole state vector utilizing the observer (6). The estimation error dynamics should not be perturbed by unknown inputs. Consider an observer as:

$$\dot{z}(t) = \hat{A}z(t) + \hat{B}y(t) + \hat{J}u(t) + \hat{H}w(t) + \hat{K}\omega(t)$$
$$\hat{x}(t) = z(t) + \hat{D}y(t)$$
$$\dot{w}(t) = F(y(t) - C\hat{x}(t))$$
$$\dot{\omega}(t) = G(y(t) - C\hat{x}(t)) \tag{12}$$

The first choice is to design an integral observer (12), for estimating both unknown inputs. It is evident that (12) cannot decouple unknown inputs perfectly but the more accurate the estimation is, the better decoupling is achieved. Two integral variables (vectors) are made to estimate two types of unknown inputs.

Lemma6: The following conditions are necessary for the existence of a proportional integral observer described by (12) for system (1):

1. The system is completely observable.

2. $rank\begin{bmatrix} A & B_d & B_f \\ C & & 0 \end{bmatrix} = n + n_d + n_f$ (13)

Proof: Augment the integral variables of (12) into the state vector of the observer. The augmented observer can be viewed as a proportional observer for the augmented plant, having the unknown inputs as auxiliary states. Assuming an estimated model for unknown inputs which is considered to be a constant signal and therefore having a zero state matrix, the observability conditions of the augmented plant can be stated as the following:

$$rank\begin{bmatrix} sE-A & -B_f & -B_d \\ 0 & sI & 0 \\ 0 & 0 & sI \\ C & 0 & 0 \end{bmatrix} = rank\begin{bmatrix} E & 0 \\ 0 & I \\ C & 0 \end{bmatrix} = n + n_d + n_f$$

Assuming the original plant to be observable, the above condition is automatically satisfied for a nonzero value for complex variable's'. In zero frequency, the observability condition is fulfilled if and only if (13) holds. This completes the proof. ■

It can be easily shown that observer (12) fails to yield a good estimation of both unknown inputs and it also fails to produce estimations with decoupled unknown inputs. It should be noted that (12) has a highly restrictive existence condition (13), the condition requires that the number of outputs is greater than or equal to the number of unknown inputs together. A more advanced scheme is suggested in the following lemma.

Lemma7: Two unknown input PI observers for decoupling and estimating the both unknown inputs, exist if the following holds:

$$rank\, CE^{\#}B_d = rank\, B_d = n_d$$

$$rank\, CE^{\#}B_f = rank\, B_f = n_f$$

$$rank\begin{bmatrix} sE-A & B_d \\ C & 0 \end{bmatrix} = n + n_d$$

$$rank\begin{bmatrix} sE-A & B_f \\ C & 0 \end{bmatrix} = n + n_f$$

The above lemma suggests a duplication of the observer order; two estimated state vectors will be produced in this method. The computational and implementation costs are doubled and the existence conditions are too restrictive. However a very good estimation of state can be obtained because a first estimation is independent of the first unknown input and second estimation is independent of the second unknown input.

Theorem1: A disturbance retrieving unknown input state observer exists for (10) if the following hold:

1. System (10) is completely observable.

2. The augmented unknown input $\begin{bmatrix} d^T & f^T \end{bmatrix}^T$ is retrievable.

3. $\forall s \in C,\ \mathrm{Re}(s) \geq 0$

$$rank\begin{bmatrix} sE-A & B_d & B_f \\ C & & 0 \end{bmatrix} = n + n_d + n_f$$

4. $rank\, CE^{\#}\begin{bmatrix} B_d & B_f \end{bmatrix} = rank\begin{bmatrix} B_d & B_f \end{bmatrix} = n_d + n_f$

Proof: Apply lemma3 conditions to the system (10) after augmenting two unknown inputs in a unified vector. If the augmented unknown input is retrievable, it can be retrieved from the algebraic part of (10). ■

The suggested method by theorem1 is a very effective observer scheme but it has a very restrictive existence condition. An enhanced version of the suggested scheme is proposed in the following theorem.

Theorem2: A disturbance retrieving unknown input PI observer exists for (10) if:

1. $d(t)$ is retrievable.

2. Conditions of lemma3 hold. (Equations 8)

Proof: The fulfillment of conditions (8) will guarantee the existence of a disturbance decoupled PI observer which yields an independent estimation of states and $f(t)$, then $d(t)$ can be retrieved using the algebraic part of (10) provided that it is retrievable. ■

Remark6: Theorem2 suggests an effective observation method with a much less restrictive existence conditions compared to the previous ones. The existence conditions are the same as the ones for a simple unknown input PI observer followed by the retrievability requirement. The observer order is equal to a simple PI observer, and the computational cost is almost the same. One unknown input is decoupled first and retrieved after the other is independently estimated by an integral (vector) variable.

Remark7: It is the designer choice to select an unknown input to be decoupled and the other to be estimated by integral observer. If the both unknown inputs are retrievable and satisfy (8), it is better to decouple the faster unknown input and estimate the one possessing slow dynamics. For instance a piecewise constant unknown input can be estimated much effectively than a sinusoidal one.

It should be noted that the existence condition of retrievability is stated in the form of description (10) and a similarity transformation is needed in order to check the existence of the observer as well as designing it. It is favorable to state the conditions with respect to system general matrices rather than a special canonical form like (10).

Lemma8: The following statements are equivalent.

1. $rank\begin{bmatrix}E & B\end{bmatrix} = n$ (15.a)

2. Matrix B_2 is of full row rank in (10). (15.b)

3. The fast subsystem of (1) is controllable. (15.c)

Proof is in [14].

Corollary1: The following statements together provide a sufficient condition for retrievability of an unknown input.

1. $rank\begin{bmatrix}E & B_d\end{bmatrix} = n$ (16.a)

2. $n_d \le n - rankE$ (16.b)

Proof: According to lemma8, condition 1 provides fast subsystem controllability. If condition 2 holds, number of rows in matrix B_{d2} is greater than the number of its columns. As a result fast subsystem controllability provides that B_{d2} is of full column rank and therefore $d(t)$ is retrievable. ■

(16.a) means that the fast subsystem is controllable by the unknown input while (16.b) suggests that the number of unknown inputs should be smaller than the number of algebraic variables ($x_2(t)$)in (10). Note that Corollary1 only provides sufficient conditions. In fact a small contribution to the algebraic part of the system is sufficient and the fast subsystem controllability by the unknown input is more than necessary.

Remark8: (16.b) is necessary but not sufficient; it illustrates remark3, more easily. If matrix E is of full rank (i.e. (1) is a normal system), none of unknown inputs can be retrieved. The necessity of (16.b) is obvious because if it doesn't hold, the unknown input cannot be retrieved.

Design procedure:

1. Check the existence conditions of observer in theorem2.

2. Find matrices $E^\#$ and $C^\#$ is such that:

$$\begin{bmatrix}E^\# & C^\#\end{bmatrix} \times \begin{bmatrix}E \\ C\end{bmatrix} = I_n$$

Note that based on the complete observability condition, theses matrices exist.

3. Compute the following matrices for the observer [6]:

$$\begin{aligned} L = {} & E^\# B_d (CE^\# B_d)^+ \\ & + G(I_q - CE^\# B_d (CE^\# B_d)^+) \end{aligned} \quad (17.a)$$

$$U = (I_n - GC)(I_n - E^\# B_d (CE^\# B_d)^+ C)E^\# \quad (17.b)$$

$$\hat{D} = C^\# + L(I_q - CC^\#) \quad (17.c)$$

$$\begin{aligned} \hat{J} &= UB \\ \hat{H} &= UB_f \end{aligned} \quad (17.d)$$

Matrix G denotes an arbitrary matrix and should be chosen such that $\hat{H}$ is nonzero. The scalar q is the number of outputs and I is the identity matrix.

4. Perform a pole placement algorithm or an optimization method to find K such that the following is stabilized:

$$\hat{A} = UA - KC \quad (18)$$

Note that observer existence conditions guarantee the observability of the pair (C, UA) for appropriate choices of $E^\#$ and G, as stated in [6].

5. Compute:

$$\hat{B} = \hat{A}\hat{D} + K \quad (19)$$

6. Having transformed the plant into the form of (10) in the first step of the design procedure, compute a left inverse for B_{d2} and set:

$$\hat{d}(t) = B_{ds}^\# (-A_3 \hat{x}_1(t) - A_4 \hat{x}_2(t) - B_2 u(t) - B_{f2} w(t)) \quad (20)$$

The state vectors of (10) can be computed easily via the transformation applied to the (1) to obtain (10). Note that $d(t)$ and $f(t)$ are interchangeable.

7. Construct the observer according to (6) and (20).

IV. NUMERICAL EXAMPLE

Example: Consider the following system which is already in the form of (10):

$$\dot{x}_1(t) = \begin{bmatrix}-2 & 0 \\ 0 & -1\end{bmatrix} x_1(t) + \begin{bmatrix}0 \\ 1\end{bmatrix} x_2(t) + \begin{bmatrix}1 \\ 2\end{bmatrix} u(t) + \begin{bmatrix}1 \\ 0\end{bmatrix} f(t) + \begin{bmatrix}1 \\ -1\end{bmatrix} d(t)$$

$$0 = \begin{bmatrix}0 & 1\end{bmatrix} x_1(t) + x_2(t) + d(t)$$

$$y(t) = \begin{bmatrix}1 & 0 \\ 1 & -1\end{bmatrix} x_1(t) + \begin{bmatrix}1 \\ 0\end{bmatrix} x_2(t)$$

System matrices are as follows:

$$E = \begin{bmatrix}I & 0 \\ 0 & 0\end{bmatrix}, A = \begin{bmatrix}-2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & 1\end{bmatrix}, B = \begin{bmatrix}1 \\ 2 \\ 0\end{bmatrix}$$

$$C = \begin{bmatrix}1 & 0 & 1 \\ 1 & -1 & 0\end{bmatrix}, B_d = \begin{bmatrix}1 \\ -1 \\ 1\end{bmatrix}, B_f = \begin{bmatrix}1 \\ 0 \\ 0\end{bmatrix}$$

Note that vector $x_1(t)$ consists of two states. Compute:

$$[E^\# \quad C^\#] = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 2 & 1 & -1 \end{bmatrix} \quad CE^\# B_d = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

Matrix $E^\#$ is not unique and should be found such that (8) holds. Here the third column of $E^\#$ can be chosen freely. By the above choice for this matrix, conditions of lemma3 hold. Now compute observer parameters according to (17)-(19):

$$L = \begin{bmatrix} 0.4828 & -0.2069 \\ 0.3448 & -0.8621 \\ 0.3103 & 0.7241 \end{bmatrix}, U = \begin{bmatrix} 0.7241 & 0.2759 & -0.4483 \\ 0.5172 & 0.4828 & -0.0345 \\ -1.0345 & 0.0345 & 1.0690 \end{bmatrix}$$

$$\hat{H} = \begin{bmatrix} 0.7241 \\ 0.5172 \\ -1.0345 \end{bmatrix}, \hat{J} = \begin{bmatrix} 1.2759 \\ 1.4828 \\ -0.9655 \end{bmatrix}, \hat{D} = \begin{bmatrix} 0 & 0.2759 \\ 0 & -0.5172 \\ 1 & 0.0345 \end{bmatrix}$$

$$\hat{B} = \begin{bmatrix} -0.4483 & 81.9624 \\ -0.0345 & 78.8544 \\ 1.0690 & -86.6157 \end{bmatrix}, K = \begin{bmatrix} -146.7954 & 176.1541 \\ -143.0502 & 166.6462 \\ 158.3909 & -181.4068 \end{bmatrix}$$

$$F = [-256.9922 \quad 291.8723]$$

The following values are used for simulation inputs:

$$u(t) = 1 \quad t > 0$$

$$d(t) = \sin(t) \quad t > 0$$

$$f(t) = \begin{cases} 1 & t < 5 \\ 0 & t > 5 \end{cases}$$

It is obvious that the latter unknown input is more suitable for being estimated by integral variable as presumed before. Figure1 shows the estimation error for x_2 which is the algebraic part of the state vector. The unknown inputs affect the estimation error steady state a bit and the estimation is acceptable.

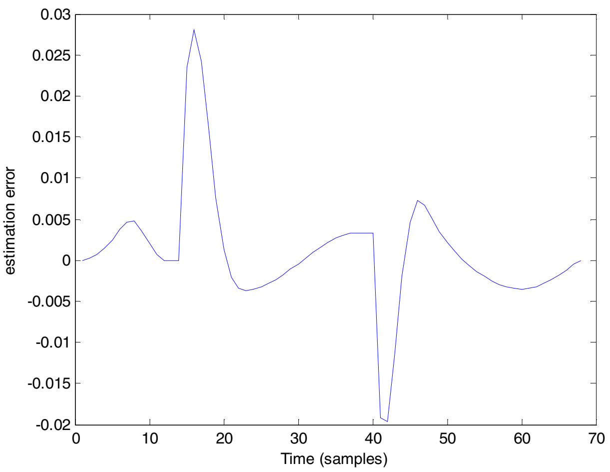

Figure 1: Estimation error for x_2

Figure 2, depicts the estimated disturbance. Because of the retrievability of disturbance, the estimation is purely algebraic and has no error.

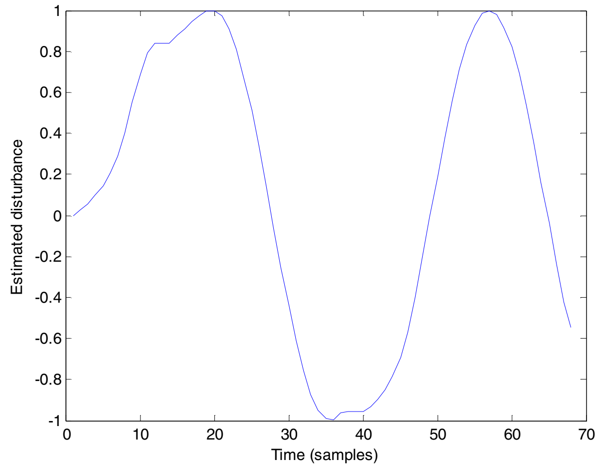

Figure 2: Disturbance estimation

Figure3 shows the estimated output as well as its actual value. Unknown inputs are decoupled perfectly and the estimation error is almost negligible. The initial condition mismatch faded in time, however there is a small error due to the error in estimating inputs, even when the initial conditions match which is shown in figure4.

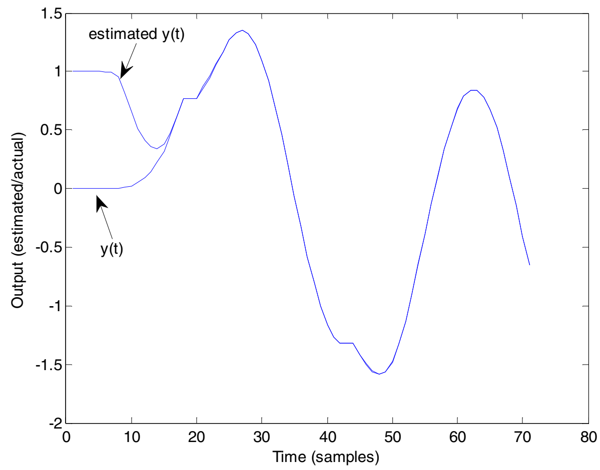

Figure 3: Actual and estimated output

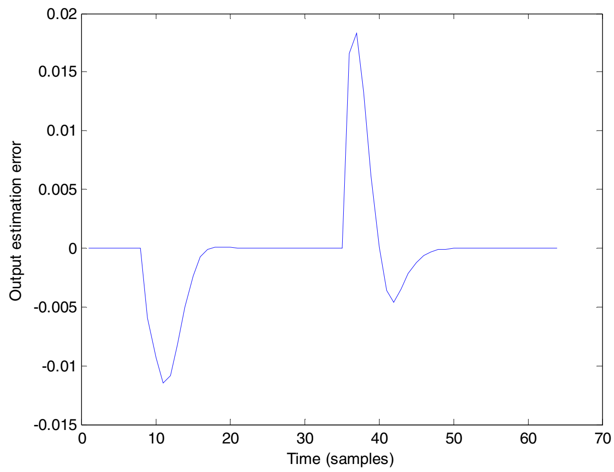

Figure 4: Output estimation error

The estimation for the other unknown input is shown in figure5. Compared to the observer dynamics the unknown input has a fast changing dynamic and the estimation settles after a while. By increasing the observer bandwidth one can make the observer faster.

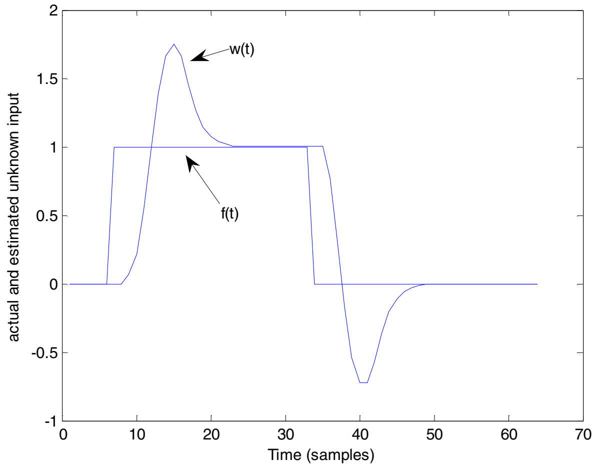

Figure 5: Actual and estimated unknown input

A PI observer for the above case, will yields a perturbed estimation of fault and disturbance which are shown in figures 6 and 7 respectively. Because the unknown inputs are not decoupled they interfere with the estimation and estimated unknown inputs are merely a combination of both signals. Therefore fault diagnosis is not possible with a PI observer. From the figures it is known that a sinusoidal unknown input is perturbing the plant, the frequency of this signal can be computed from figure7, but the step unknown input cannot be detected.

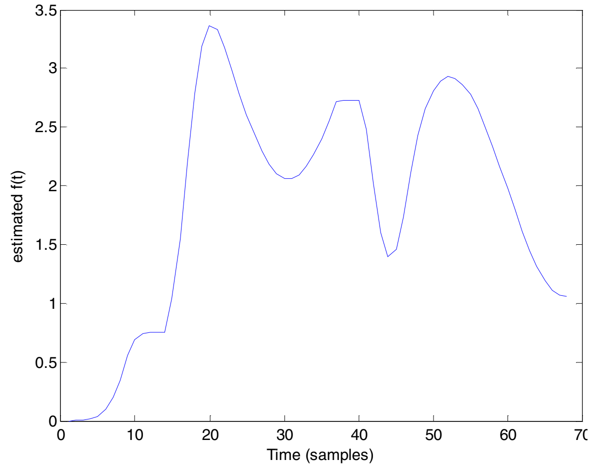

Figure 6: Estimated fault via PI observer

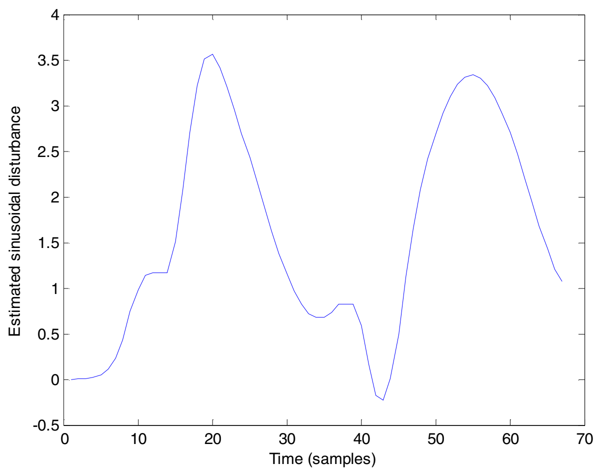

Figure 7: Estimated sinusoidal disturbance via PI observer

V. Conclusion

In this paper a retrieving observer scheme is proposed for singular systems. The observer is able to decuple an unknown input in the estimation and estimate it at the same time. This feature wasn't included in the existing methods which either decouple an input or estimate it. The main feature of the proposed scheme is that estimation can be made independent of the unknown inputs. An important application of this kind of observer can be found in fault diagnosis problem when two kinds of faults should be estimated separately. Also this algorithm reveals a special feature of a descriptor system which has not been focused on before; A descriptor system may provide more information about the unknown inputs than a standard state space system. This feature is used for retrieving the unknown input information from the algebraic part of system.

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