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Forward Kinematic Analysis of A Planar Cable Driven Redundant Parallel Manipulator Using Force Sensors

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***Abstract*— Newly developed cable driven redundant parallel manipulators (CDRPM) have numerous advantages compared to that of the conventional parallel mechanisms. However, there exist some challenging issues in over-constrained mechanisms like CDRPMs. In contrast to serial manipulators, complexity of parallel manipulator forward kinematics (FK) is one of the main issues being under study in the control of such manipulators. Moreover, using extra sensory data is a common approach in the FK solution of rigid–linked parallel manipulators, which is considered by fewer researchers for CDRPMs. In this paper, tension force sensors of the cables are used as an extra sensor to simplify analytical solution of the FK for a planar CDRPM. To find a suitable solution, geometrical and physical characteristics of the robot are analyzed. It is shown that the proposed method provides the required accuracy and significantly improves the process time compared to the conventional methods.**

I. INTRODUCTION

New designs of parallel manipulators are gaining more attraction among researchers and practitioners in a many applications. A closed chain mechanism causes a stiffer structure capable of performing in high accelerations for a fully constrained manipulator [1]. In a parallel mechanism, each limb contributes in the movement of the payload; and therefore, it can carry more payload to moving mass ratio. This characteristics of parallel manipulator (PM) make them suitable for special applications such as the popular Stewart-Gough platform in flight simulators [2]. However, additional to the complexity of production [3], and control [4] of such manipulators, there are some challenges to the structures of conventional parallel manipulators to accommodate stringent requirements for a wide range of applications. The main limitations of the parallel manipulators are their limited workspace [5] and existence of singularity regions within its workspace [6]. Using an electric powered cable–driven actuator, as an alternative for the massive and stroke–limited linear actuator, the workspace of the manipulator can be inevitably extended even within the size of a football stadium [7], or a platform of large adaptive reflector with two square kilometer footprint [8]. By installing the driver units on the fixed platform, only light-weight cables mass is added to the mass of the end-effector. Therefore, manipulators such as a RoboCrane can carry large forces as the weight of a shipping cargo with the use of a CDRPM structure [9]. However, a cable can only carry tension forces, and to guarantee that the cables are always under tension, different methods are being used in the manipulator structure or control system. A suitable solution for high acceleration applications, is to use redundant actuator(s), and to use a redundancy resolution technique in the manipulator control system, in order to ensure that all the cables are always under tension. This can be performed in a fully–constrained or over–constrained moving platform [10], with the payoff of more complex geometrical [11], or force feasible workspace analysis [12].

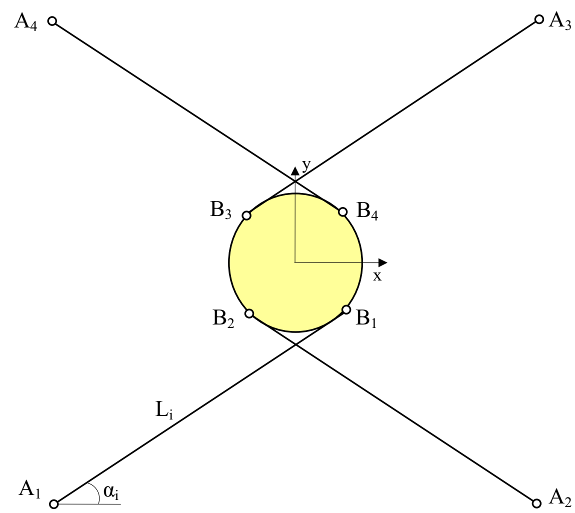

Fig. 1. The schematics of a $4R\underline{P}R$ planar cable driven redundant parallel manipulator

In CDRPMs, forward kinematics defines the problem of finding the pose of the moving platform as a function of the cable lengths. The Planar CDRPM has four actuator variables and forward kinematic solution should calculate the position and orientation of the end-effector in a three dimensional workspace as a function of these four variables. Forward kinematic solution is known as a challenging problem in the parallel manipulators [13], and various researches are dedicated to find the solution of FK problems for different parallel manipulators. However, most of the studies in the literature focused on the fully actuated manipulators in which the number of actuator variables are equal to the dimension of the workspace. Moreover, other alternatives of solutions to this problem has been proposed by using additional sensors for the passive joints [14] or solving

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the problem as an optimization problem [15]. Another solution to the forward kinematic equations is to simplify relations between coordinates of the attachment points and inverse kinematics by means of a closed form solution[16]. This solution can be fused with a linear sensor data in order to increase the precision of the forward kinematic solution[17]. In another research the nonlinear equations of forward kinematics is converted into two groups of linear matrix equations[18], while [14] proposes to solve algebraic polynomial relations for the attachment points instead of solving complicated trigonometric equations.

Most of above mentioned semi–analytical solutions highly depend on the measurement of the actuator length and orientation. Therefore, they cannot be used in the forward kinematics analysis of the Planar CDRPM. In this paper, a quasi–analytical method is used to satisfy the required performance for the purpose of kinematics analysis and online implementation in control. The general structure consist of the four actuated three degrees of freedom planar cable driven redundant parallel manipulator shown in figure 1. This manipulator is a basic stage for the development of a forward kinematics solution for a six dimensional CDRPM which is under investigation for possible high speed and wide workspace applications such as virtual acceleration generator. In this paper, inverse and forward kinematics of this manipulator are fully analyzed. Then, geometrical relations of joint space variables are combined with the other physical relationships to simplify the FK solution by means of tension force sensors data. The results show that the proposed method provides the required performance, and moreover, significantly improves the process time compared to conventional method.

II. INVERSE KINEMATICS

A. Mechanism Description

The planar CDRPM under study is illustrated in figure 1. This robot is a planar three degrees of freedom manipulator with one degree of actuator redundancy. This robot has four identical cable limbs. The cable driven limbs are modeled as revolute-prismatic-revolute ($R\underline{P}R$) joints, since the cables can only bear tension force and are not exposed to radial or bending forces. Two cartesian coordinate systems A and B are attached to the fixed base and moving platform. Points A_1, A_2, A_3, A_4 lie on the fixed circular frame with an R_A radius and B_1, B_2, B_3, B_4 lie on the moving circular frame with R_B radius. The origin O of the fixed coordinate system is located at the centroid of the circular frame. Similarly, the origin G of the moving coordinate system is considered to be located at centroid of the circular moving platform. The transformation from the moving platform to the fixed base frame can be described by a position vector named $\overrightarrow{g} = \overrightarrow{OG}$ and the angle of rotation between two coordinate system denoted by ϕ. Consider a_i and ${}^B b_i$ denote the position vectors of points A_i and B_i in the coordinate system A and B, respectively. Moreover, α_i denotes the angle between each cable with respect to the x axis and

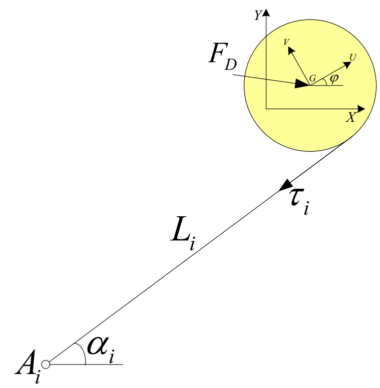

Fig. 2. ith Attachment point on the moving platform and related vectors

ϕ_i is the absolute rotation angle of $\boldsymbol{E}_i = \overrightarrow{GB_i}$ vector. Furthermore, it is assumed tha the tension force of the cables, $\boldsymbol{\tau} = [\tau_1, \tau_2, \ldots, \tau_4]^T$, are measured at B_i points. Although in the FK analysis of the Planar CDRPM, all the attachment points, are considered to be arbitrary on a circle, the geometric parameters given in table I are used in the simulations.

B. Inverse Kinematics

The inverse kinematic analysis is the first and simplest step in the kinematics analysis of PMs, which is used in the dynamics analysis [19] and control [20] of such manipulators. For inverse kinematic analysis of the Planar CDRPM, it is assumed that the position and orientation of the moving platform $\boldsymbol{G} = [x_G, y_G]^T, \phi$ are given and the problem is to find the joint variable of the CDRPM, $\boldsymbol{L} = [L_1, L_2, L_3, L_4]^T$. From the geometry of the manipulator as illustrated in figure 2 the following loop closure equation can be derived:

$$ {}^A\overrightarrow{A_iB_i} + {}^A\overrightarrow{\boldsymbol{a}_i} = {}^A\overrightarrow{\boldsymbol{g}} + {}^A\boldsymbol{E}_i \tag{1} $$

The length of the i'th limb is obtained through taking the dot product of the vector $\overrightarrow{A_iB_i}$ with itself. Therefore, for $i = 1, \ldots, 4$:

$$ L_i = \left\{ [\boldsymbol{g} + \boldsymbol{E}_i - \boldsymbol{a}_i]^T [\boldsymbol{g} + \boldsymbol{E}_i - \boldsymbol{a}_i] \right\}^{\frac{1}{2}}. \tag{2} $$

Assume that the moving attachments are located at a distance of R_B from the origin of the moving coordinate and $\boldsymbol{E}_i$'s polar coordinate is written as (R_B, ϕ_i) with respect to the fixed coordinate. Therefore, length of the cables can be determined by the components of equation 2 :

$$ L_i = \Big[(x_G - x_{Ai} + R_B\cos(\phi_i))^2 + (y_G - y_{Ai} + R_B\sin(\phi_i))^2 \Big]^{\frac{1}{2}}. \tag{3} $$

C. Jacobian Analysis

Jacobian analysis plays a vital role in the study of robotic manipulators [21]. Let the actuated joint speed be denoted by a vector $\dot{\boldsymbol{L}}$ and the linear and angular velocity of the moving platform be described by a vector $\dot{\boldsymbol{x}} = \left[\dot{x}_G \quad \dot{y}_G \quad \dot{\phi}\right]^T$. Then the differential kinematics relation can be given by the following equation:, in which, $\boldsymbol{J}$ is the Jacobian matrix of the manipulator.

$$\dot{\boldsymbol{L}} = \boldsymbol{J} \cdot \dot{\boldsymbol{x}}, \tag{4}$$

The Jacobian matrix not only reveals the relation between the joint velocities $\dot{\boldsymbol{L}}$ and the moving platform velocities $\dot{\boldsymbol{x}}$, but also constructs the transformation needed to find the actuator forces $\boldsymbol{\tau}$ from the forces acting on the moving platform $\boldsymbol{F}$. When $\boldsymbol{J}$ becomes singular, there will be a non-zero twist $\dot{\boldsymbol{x}}$ for which the active joint velocities are zero, and this singularity is called forward kinematics singularity. In this section we investigate the Jacobian of the CDRPM platform shown in figure 1. For this manipulator, the input vector is given by $\boldsymbol{L} = [L_1, L_2, L_3, L_4]^T$, and the output vector can be described by the velocity of the centroid G and the angular velocity of the moving platform as follows:

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \boldsymbol{V}_G \\ \boldsymbol{\omega}_G \end{bmatrix}, \tag{5}$$

in which, $\boldsymbol{\omega}_G = \dot{\phi}\hat{Z}_i$ is the angular velocity of the moving platform in $\hat{Z}$ axis direction. The Jacobian matrix can be derived by formulating a velocity loop-closure equation for each limb.

$$\boldsymbol{V_G} + \boldsymbol{\omega}_G \times \boldsymbol{E_i} = \dot{L}_i\hat{\boldsymbol{S}}_i + L_i\left(\boldsymbol{\omega}_i \times \hat{\boldsymbol{S}}_i\right) \tag{6}$$

where, the $\hat{S}_i$ vector is the unit vector of the i'th cable from A_i to B_i with respect to the fixed coordinate. Furthermore $\boldsymbol{\omega}_i$ denotes the angular velocity of i'th limb with respect to the fixed frame A. To eliminate $\boldsymbol{\omega}_i$, dot-multiply both sides of equation 6 by $\hat{\boldsymbol{S}}_i$.

$$\dot{L}_i = \hat{\boldsymbol{S}_i}^T \boldsymbol{V}_G + (\boldsymbol{E}_i \times \hat{\boldsymbol{S}}_i)^T \boldsymbol{\omega}_G. \tag{7}$$

Using a matrix form of equation 7 for $i = 1, 2, 3, 4$, the CDRPM Jacobian matrix J is derived as following.

$$\boldsymbol{J} = \left[\begin{array}{c|c} \hat{\boldsymbol{S}}_1^T & (\boldsymbol{E}_1 \times \hat{\boldsymbol{S}}_1)^T \\ \hat{\boldsymbol{S}}_2^T & (\boldsymbol{E}_1 \times \hat{\boldsymbol{S}}_2)^T \\ \hat{\boldsymbol{S}}_3^T & (\boldsymbol{E}_1 \times \hat{\boldsymbol{S}}_3)^T \\ \hat{\boldsymbol{S}}_4^T & (\boldsymbol{E}_4 \times \hat{\boldsymbol{S}}_4)^T \end{array}\right] \tag{8}$$

Note that equation 8 denotes the general form of the jacobian matrix of the manipulator in terms of the full vector of linear and angular velocities. However, for a planar manipulator only the first two columns of this matrix corresponding to the planar velocities $\dot{x}_G$ and $\dot{y}_G$ and the last column of the matrix corresponding to the angular velocity $\dot{\phi}$ is needed. By this means the CDRPM Jacobian matrix $\boldsymbol{J}$ becomes a non–square 4×3 matrix, since the manipulator is a redundant manipulator, as follows:

$$\boldsymbol{J} = \left[\begin{array}{c|c|c} \cos(\alpha_1) & \sin(\alpha_1) & R_B\sin(\phi_1 - \alpha_1) \\ \cos(\alpha_2) & \sin(\alpha_2) & R_B\sin(\phi_2 - \alpha_2) \\ \cos(\alpha_3) & \sin(\alpha_3) & R_B\sin(\phi_3 - \alpha_3) \\ \cos(\alpha_4) & \sin(\alpha_4) & R_B\sin(\phi_4 - \alpha_4) \end{array}\right] \tag{9}$$

III. Forward Kinematics

Finding the Cartesian position of the end–effector by means of joint variable measurements is called forward kinematics. This problem is a challenging issue for the parallel manipulators. In this research, the cables tension forces is used as an extra sensory data in the solution of FK. Note that tension forces of the CDRPMs cables are usually measured for the purpose of control. Hence they may be used for forward kinematics solution with no additional cost. Furthermore, including the force sensor data significantly improves the forward kinematic solution performance and process cost. For the rigid-linked PMs, a rotational sensor is positioned on the revolute, universal or spherical joints to measure absolute angle of the actuator, and these can be used for the purpose of forward kinematics analysis. However, in CDRPMs, the angle of cables, α_i, cannot be easily measured at fixed attachment point. Therefore, the FK solution discussed in this section consists of both geometrical and statical force balancing properties of the manipulator to extract a polynomial solution for the FK problem.

A. Forward Kinematics Formulation

The basic statical force balancing equation can be given as below:

$$\boldsymbol{J}^T\boldsymbol{\tau} = \boldsymbol{W}_e \tag{10}$$

in which, $\boldsymbol{W}_e$ is the moving platform exerted wrench to the environment, consisting of the following forces and moment:

$$\boldsymbol{W}_e = [f_x \quad f_y \quad m_z]^T \tag{11}$$

Substitution of parametric amounts of the Jacobian matrix equation 9 into equation 10 leads to:

$$\begin{bmatrix} \cos(\alpha_1) & \dots & \cos(\alpha_4) \\ \sin(\alpha_1) & \dots & \sin(\alpha_4) \\ R_B\sin(\phi_1 - \alpha_1) & \dots & R_B\sin(\phi_4 - \alpha_4) \end{bmatrix} \boldsymbol{\tau} = \boldsymbol{W}_e \tag{12}$$

Since PM's forward kinematic equations are usually complex and cannot be solved in realtime control systems, α_i angles are usually measured by extra sensors. This extra sensory method is not useful for the cable driven PMs because their fixed attachment points are on the cable driver units and the angle is hardly measurable. However, equation 12 presents the relation between α_i angles and the force vectors:

$$\begin{aligned} \sum_{i=1}^{4} \tau_i\cos(\alpha_i) &= f_x, \\ \sum_{i=1}^{4} \tau_i\sin(\alpha_i) &= f_y, \\ R_B\sum_{i=1}^{4} \tau_i\sin(\phi_i - \alpha_i) &= m_z. \end{aligned} \tag{13}$$

Fortunately, the forces can be measured quite accurately. Moreover, for the purpose of control and redundancy resolution in these types of manipulators, force feedback of the cables is very popular and force sensors usually exist in the system control loop [20]. On the other hand, geometrical relation between position vector of the moving platform centroid, x_G, y_G, and the joint variables can be extracted from equation 2, for $i = 1, \ldots, 4$, as following:

$$L_i \cos(\alpha_i) = x_G - x_{A_i} + R_B \cos(\phi_i) \tag{14}$$
$$L_i \sin(\alpha_i) = y_G - y_{A_i} + R_B \sin(\phi_i) \tag{15}$$

Hence, the relationship between end–effector pose, (x_G, y_G, ϕ) and the trigonometric functions of α_i angles can be found as following.

$$\cos(\alpha_i) = \frac{x_G + x_i}{L_i} \tag{16}$$
$$\sin(\alpha_i) = \frac{y_G + y_i}{L_i} \tag{17}$$

in which, x_i and y_i are two intermediate variables defined as following:

$$x_i = -x_{A_i} + R_B \cos(\phi_i) \tag{18}$$
$$y_i = -y_{A_i} + R_B \sin(\phi_i) \tag{19}$$

Now, using the set of equations 13, and the latter four equations, the relation between the position of the end–effector to its orientation is derived as:

$$x_G = \left(f_x - \sum_{i=1}^{4} \frac{\tau_i x_i(\phi)}{L_i}\right) / \left(\sum_{i=1}^{4} \frac{\tau_i}{L_i}\right) \tag{20}$$
$$y_G = \left(f_y - \sum_{i=1}^{4} \frac{\tau_i y_i(\phi)}{L_i}\right) / \left(\sum_{i=1}^{4} \frac{\tau_i}{L_i}\right) \tag{21}$$

In order to simplify the calculations, reconsider the square of equation 3 as:

$$L_i^2 = (x_G + x_i)^2 + (y_G + y_i)^2 \tag{22}$$

First try to solve for x_G and y_G. This can be accomplished by reordering the equation 22 into:

$$x_G^2 + y_G^2 + r_i x_G + s_i y_G + u_i = 0 \tag{23}$$

in which, for $i = 1, \ldots, 4$,

$$\begin{cases} r_i = 2x_i \\ s_i = 2y_i \\ u_i = x_i^2 + y_i^2 - L_i^2 \end{cases} \tag{24}$$

Equation 23 provides four quadratic relations for x_G and y_G for each limb. Subtracting each two equations from each other results into a linear equation in terms of x_G and y_G.

$$A \begin{bmatrix} x_G \\ y_G \end{bmatrix} = b \tag{25}$$

in which,

$$A = \begin{bmatrix} r_1 - r_2 & s_1 - s_2 \\ r_2 - r_3 & s_2 - s_3 \\ r_3 - r_4 & s_3 - s_4 \\ r_4 - r_1 & s_4 - s_1 \end{bmatrix}; \quad b = \begin{bmatrix} u_2 - u_1 \\ u_3 - u_2 \\ u_4 - u_3 \\ u_1 - u_4 \end{bmatrix} \tag{26}$$

Note that both of the A and b matrices are functions of x_i and y_i and only depends on the end–effector orientation ϕ with respect to the fixed frame. Numerical solution of this equation is discussed in [8]. However, by substitution of equations 20 and 21 into equation 23, results into an equation in which the inverse kinematics equation is related to the measured actuator forces τ_i and the indeterminate variables are changed to f_x, f_y, and the ϕ angle. Two cases are therefore, distinguished for the solution of the forward kinematics. First consider the case where no external forces are applied to the moving platform $f_x = f_y = 0$. In this case the inverse kinematics equations are only a function of ϕ, which can be easily solved. Next, Consider the case where external forces exists $f_x, f_y \neq 0$. In this case, such forces can also be determined in the process of forward kinematics solution. In order to do that, equation 25 is expanded and rearranged by a symbolic manipulator software to factor f_x, f_y, and ϕ. The rearrangement leads to the following system of equations for $i = 1, \ldots, 4$

TABLE I

STRUCTURAL PARAMETERS OF THE PLANAR MANIPULATOR

Description	Quantity
R_A: Radius of the fixed attachment points	90 m
R_B: Radius of the moving attachment points	10 m
θ_{A_i}: Angle of the fixed attachment points	$\left[-\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}\right]$
θ_{B_i}: Angle of the moving attachment points	$\left[-\frac{\pi}{4}, -\frac{3\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{4}\right]$

$$\begin{aligned} 0 = & \ (e_{i,1} + e_{i,2} f_x + e_{i,3} f_y) \\ + & \ (e_{i,4} + e_{i,5} f_x + e_{i,6} f_y) \cos(\phi) \\ + & \ (e_{i,7} + e_{i,8} f_x + e_{i,9} f_y) \sin(\phi) \end{aligned} \tag{27}$$

in which, $e_{i,j}$s are coefficients of indeterminate parameters derived by such manipulations. These parameters depend on the manipulator geometrical properties, $x_{A_i}, y_{A_i}, R_B, \theta_B$, and joint variables, L_i and τ_i. The details of these components are quite involved, and are not elaborated for the sake of simplicity.

Now consider the system of equations in 27, and notice that f_x and f_y must be first determined by one equation in the set 27 then substituted in the others. In fact by carefully examining the details of each pair of equation 27, f_x and f_y are symbolically determined from pairs $i = 1, 2$, respectively and then substituted in the next two pais. Finally, through these substitution and the standard change of variable $t = \tan(\phi/2)$, the set of trigonometric equation will be reduced to two sixth–order polynomials as following:

$$c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4 + c_5 t^5 + c_6 t^6 = 0 \tag{28}$$

$$d_0 + d_1 t + d_2 t^2 + d_3 t^3 + d_4 t^4 + d_5 t^5 + d_6 t^6 = 0 \tag{29}$$

in which, c_i's and d_i's are coefficients that are determined as a linear combinations of $e_{i,j}$s. Finally, combination of these two polynomials results into a fifth–order polynomial which can be solved numerically. In order to verify the accuracy and integrity of the solution a simulation study is performed which is detailed in the next section.

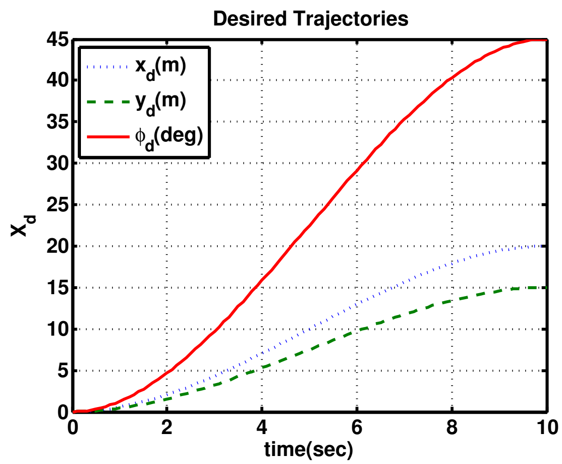

Fig. 3. Desired Trajectory devised for the Planar CDRPM

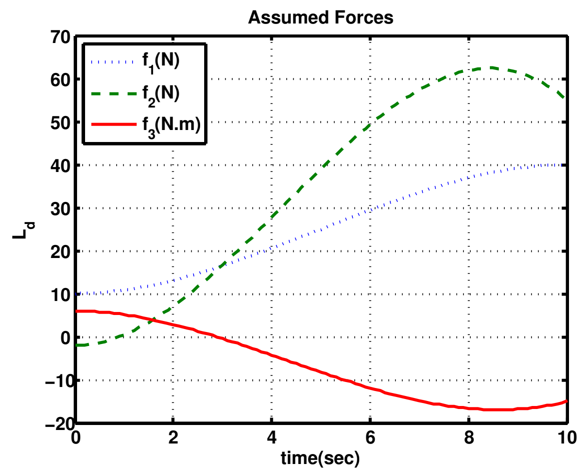

Fig. 4. Assumed exerted forces on the Planar manipulator moving platform

IV. Simulation Results

The Planar CDRPM with three degree of freedom and one degree of redundancy is analyzed here. Assume that the planar parallel manipulator has four fixed attachment points which are distributed equally on a circle with radius of $R_A = 90m$ while moving attachment points are distributed on $R_B = 10m$ radius circle in a way that insures cross configuration of cables related to each other. A fixed coordinate system is considered at the center of the manipulator and a moving coordinate system is attached to the moving platform which initially coincides with the fixed coordinate system. Figure 1 represents this configuration. Based on the assumed coordinate systems, position of the fixed attachment points in the fixed coordinate system and the moving attachment points in the moving coordinate system are represented in table I. Also a set of trajectories as shown in figure 3 is considered for the moving platform to follow during the simulation.

The method presented in this paper to solve for the forward kinematics requires the actuator force at any time. In real robot these forces is measured by the force sensors installed at each moving attachment point but for simulations a set of three trajectories shown in figure 4 is assumed. Two of these trajectories are assumed to be the sum of inertial and disturbance forces exerted on the moving platform in x and y directions and the third one is assumed to be the moment exert on moving platform in z direction. Furthermore, by means of manipulator's jacobian these forces is mapped to forces in joint space which in this case are the forces exert by the cables on moving platform. These forces are also shown in figure 5. Finally, by using inverse kinematic relation of manipulator, the desired trajectories of the moving platform can be used to find the cable lengths as shown in figure 6. To verify the accuracy and integrity of the obtained solution, these length and joint space forces are used in the proposed forward kinematic solution to determine the final location of moving platform, and the results are compared to the original trajectories. Figure 7 shows the error between calculated forward kinematics solution using the proposed method to that of the original trajectories. This simulation took 0.3793 sec in an Intel core2 Duo (T7500) machine with 1 gigabyte of RAM.

In order to compare efficiency of this method to conventional forward kinematics solutions, the results proposed in [8] is shown in figure 8. This numerical solution took 0.7018 sec on the same computer. As it can be seen from figure 8 and 7 it is obvious while the error is nearly the same for both method, the proposed method in this paper is twice faster.

V. Conclusions

In this paper, forward kinematics of a planar cable driven parallel manipulator is derived. Since parallel manipulator's forward kinematics equations are usually complex and cannot be solved in realtime control systems, in conventional methods the actuators rotation angles, α_i, are measured by extra sensors. This extra sensory method is not useful for the cable driven PMs because their fixed attachment points are on the cable driver units and the angle is hardly measurable. The proposed method in this paper, shows that the data of cables tension force sensors can be used instead of the extra passive joint sensor. Moreover, for the purpose of control and redundancy resolution in these types of manipulators, force control of the cables is very popular and force sensors usually exist in the system control loop. The method examined on a given planar cable driven parallel manipulator. It is shown that the proposed method not only provides the sufficient performance, but also significantly reduces the process time compared to a similar method. This method is under development for further applications in the spatial cable driven manipulators.

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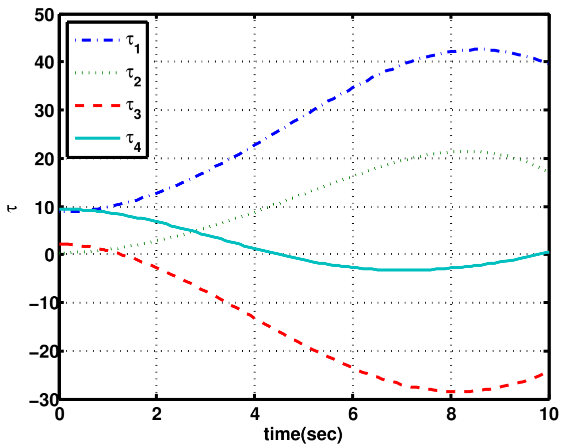

Fig. 5. The projection of the moving platform forces on actuators.

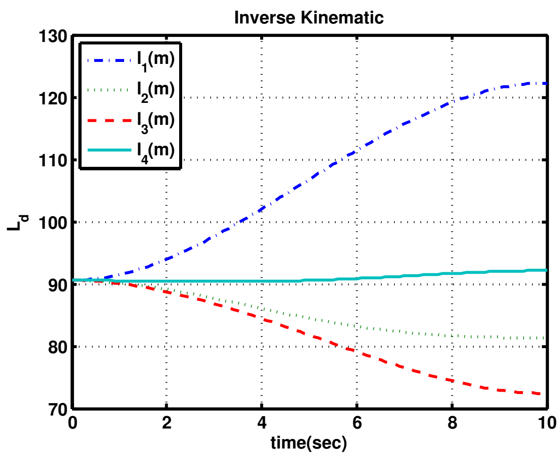

Fig. 6. Results of the inverse kinematics for the given trajectory

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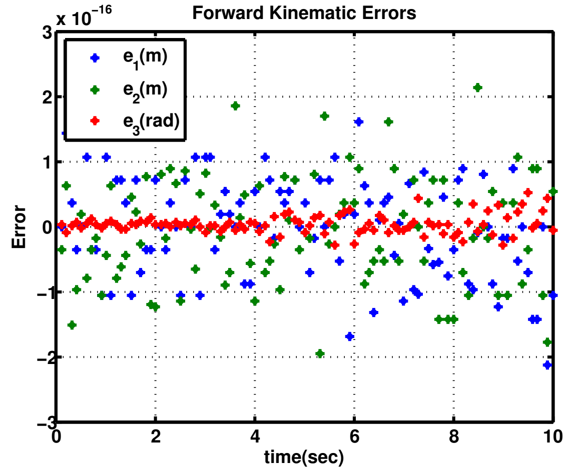

Fig. 7. Performance of the proposed method

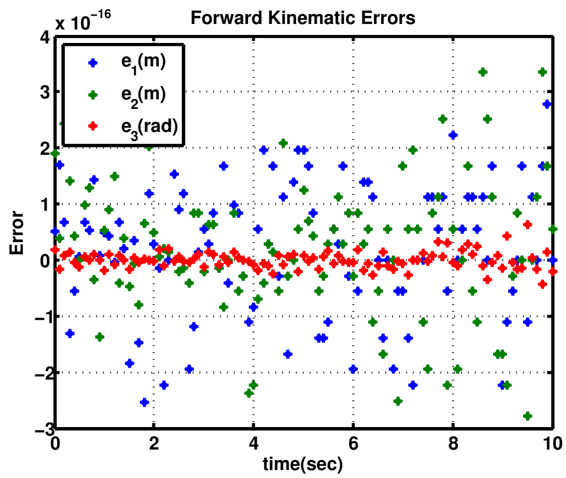

Fig. 8. Performance of the conventional method

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