in which,

$$\boldsymbol{f}_{\tau_i} = \begin{bmatrix} \tau_i \hat{\boldsymbol{s}}_i \\ \tau_i \boldsymbol{E}^T \boldsymbol{b}_{i\times} \hat{\boldsymbol{s}}_i \end{bmatrix} \tag{78}$$

Add equation 72 for all limbs together and add them to 62 to eliminate $\boldsymbol{f}_{b_i}$ from these equations. By this means the explicit dynamic formulation for the whole manipulator is derived as in equation 1:

$$\boldsymbol{M}(\boldsymbol{\mathcal{X}}) = \sum_{i=1}^{i=6} \boldsymbol{M}_{li} + \boldsymbol{M}_{mp} \tag{79}$$

$$\boldsymbol{C}\left(\boldsymbol{\mathcal{X}}, \dot{\boldsymbol{\mathcal{X}}}\right) = \sum_{i=1}^{i=6} \boldsymbol{C}_{li} + \boldsymbol{C}_{mp} \tag{80}$$

$$\boldsymbol{G}(\boldsymbol{\mathcal{X}}) = \sum_{i=1}^{i=6} \boldsymbol{G}_{li} + \boldsymbol{G}_{mp} \tag{81}$$

$$\boldsymbol{F}(\boldsymbol{\mathcal{X}}) = \sum_{i=1}^{i=6} \boldsymbol{f}_{\tau_i} \tag{82}$$

These equations completely define the detail terms of the equations of motion of SGP given in 1 in an explicit form. Moreover, these terms consists of kinematic structures of the limbs and the moving platform in a matrix form, and therefore, they are very compact and tractable. It is worth mentioning that these equations can be systematically derived without use of any symbolic manipulation software.

V. Verification

In order to verify the obtained equations of motions for SGP, $\boldsymbol{M}_i$ is derived by means of Lagrange method, and the results are compared. Let $\dot{\boldsymbol{x}}_i$ denote a generalized coordinate and $\boldsymbol{T}_i$ denote the kinetic energy of the limb, $\boldsymbol{M}_i$ can be found from,

$$T_i = \frac{1}{2} \dot{\boldsymbol{x}}_i \boldsymbol{M}_i(\boldsymbol{x}_i) \dot{\boldsymbol{x}}_i \tag{83}$$

Furthermore, the kinetic energy of the limb is:

$$\begin{aligned} T_i =& \frac{1}{2} \boldsymbol{v}_{c_{i_1}}^T m_{i_1} \boldsymbol{v}_{c_{i_1}} + \frac{1}{2} \boldsymbol{\omega}_i^T \left({}^A\boldsymbol{I}_{c_{i_1}} + {}^A\boldsymbol{I}_{c_{i_2}} \right) \boldsymbol{\omega}_i \\ &+ \frac{1}{2} \boldsymbol{v}_{c_{i_2}}^T m_{i_2} \boldsymbol{v}_{c_{i_2}} \end{aligned} \tag{84}$$

By means of relations given in 54, $\boldsymbol{\omega}$, $\boldsymbol{v}_{c_{i_1}}$ and $\boldsymbol{v}_{c_{i_2}}$ can be transformed to:

$$\begin{aligned} \boldsymbol{\omega}_i &= \frac{1}{l_i} \hat{\boldsymbol{s}}_{i\times} \dot{\boldsymbol{x}}_i \;\; ; \;\; \boldsymbol{v}_{c_{i_1}} = \frac{-c_{i_1}}{l_i} \hat{\boldsymbol{s}}_{i\times}^2 \dot{\boldsymbol{x}}_i \\ \boldsymbol{v}_{c_{i_2}} &= \left(-\frac{(l_i - c_{i_2})}{l_i} \hat{\boldsymbol{s}}_{i\times}^2 + \hat{\boldsymbol{s}}_i \hat{\boldsymbol{s}}_i^T \right) \dot{\boldsymbol{x}}_i \end{aligned} \tag{85}$$

Moreover, it can be easily shown that for any arbitrary vector $\boldsymbol{a}$,

$$\begin{aligned} \hat{\boldsymbol{s}}_{i\times}^4 \boldsymbol{a} = -\hat{\boldsymbol{s}}_{i\times}^2 \boldsymbol{a} \;\; ; \;\; \hat{\boldsymbol{s}}_i \hat{\boldsymbol{s}}_i^T \boldsymbol{a} = (\boldsymbol{I}_3 + \hat{\boldsymbol{s}}_{i\times}^2) \boldsymbol{a} \\ \hat{\boldsymbol{s}}_{i\times}^2 \hat{\boldsymbol{s}}_i \hat{\boldsymbol{s}}_i^T \boldsymbol{a} = 0 \;\; ; \;\; \hat{\boldsymbol{s}}_i \hat{\boldsymbol{s}}_i^T \hat{\boldsymbol{s}}_{i\times}^2 \boldsymbol{a} = 0 \end{aligned} \tag{86}$$

Now, substitute $\boldsymbol{\omega}$, $\boldsymbol{v}_{c_{i_1}}$ and $\boldsymbol{v}_{c_{i_2}}$ from 85 into 84 and use equations 86, to simplify:

$$\begin{aligned} T_i =& \frac{1}{2} \dot{\boldsymbol{x}}_i^T \left(\frac{-1}{l_i^2} \left(\hat{\boldsymbol{s}}_{i\times} \left({}^A\boldsymbol{I}_{c_{i_1}} + {}^A\boldsymbol{I}_{c_{i_2}} \right) \hat{\boldsymbol{s}}_{i\times} \right. \right. \\ & \left. \left. + \left(m_{i_1} c_{i_1}^2 + m_{i_2} (l_i - c_{i_2})^2 \right) \hat{\boldsymbol{s}}_{i\times}^2 \right) + m_{i_2} \hat{\boldsymbol{s}}_i \hat{\boldsymbol{s}}_i^T \right) \dot{\boldsymbol{x}}_i \end{aligned} \tag{87}$$

comparing equation 87 together with 82 to equation 58 verifies identical derivation of $\boldsymbol{M}_i$ throughout two methods. Note that the other terms in the dynamics equations can be verified in a similar manner. Although for those terms Lagrange formulation will lead to an extensive manipulation which is not given here due to the limited space.

VI. Conclusions

Closed–chain kinematic structure of parallel manipulators causes the dynamic equation of such manipulators to be very bulky and intractable. On the other hand having an explicit formulation for the dynamic equations of such manipulator is essentially needed for model–based control routines. In this paper, A vector based Newton-Euler formulation is proposed, which preserves the inherent kinematic structure components of the manipulator in the final resulting equations. This method is applied to the most celebrated parallel manipulator, namely the Stewart–Gough platform. Key elements to derive the explicit dynamic equation in a tractable form are to define an intermediate variable from joint space and some matrix algebraic manipulation tools. In the proposed method the equations are not derived componentwise, and therefore, the resulting equations are reduced into a concise vector based representations. Furthermore, $\boldsymbol{M}$, $\boldsymbol{C}$, $\boldsymbol{G}$ matrices are extracted and fully given for both limbs and the end-effector in a concise form. The proposed methodology, and the simplification rules can be used to derive other manipulator dynamics.

References

[1] C.Gosselin, "Parallel computational algorithms for the kinematics and dynamics of planar and spatial parallel manipulators," *Trans. ASME Journal of Dynamic Systems, Measurement and Control*, pp. 22–28, 1996.

[2] N. Dasgupta and T. Mruthyunjaya, "A newton-euler formulation for the inverse dynamics of the stewart platform manipulator," *Mechanism and Machine Theory*, 1998.

[3] C. NGUYEN and F. POORAN, "Dynamic analysis of a 6 dof CKCM robot end-effector for dual-arm telerobot systems," *Robotics and Autonomous Systems*, vol. 5, pp. 377–394, 1989.

[4] G. Lebret, K. Liu, and F. Lewis, "Dynamic analysis and control of a stewart platform manipulator," *Journal of Robotic Systems*, vol. 10, no. 5, pp. 629–655, 1993.

[5] J. Wang and C. Gosselin, "A new approach for the dynamic analysis of parallel manipulators," *Multibody System Dynamics*, vol. 2, no. 3, pp. 317–334, 1998.

[6] M. Liu, C. Li, and C. Li, "Dynamics analysis of the Gough-Stewart platform manipulator," *IEEE Transactions on Robotics and Automation*, vol. 16, no. 1, pp. 94–98, 2000.

[7] B. Dasgupta and T. S. Mruthyunjaya, "a newton-euler formulation for the inverse dynamics of the stewart platform manipulator," *Mechanism and Machine Theory*, vol. 33, no. 8, pp. 1135–1152, 1998.

[8] W. Khalil and S. Guegan, "Inverse and direct dynamic modeling of Gough-Stewart robots," *IEEE Transactions on Robotics*, vol. 20, no. 4, pp. 754–761, 2004.

[9] J. He, H. Jiang, D. Cong, Z. Ye, and J. Han, "A Survey on Control of Parallel Manipulator," *Key Engineering Materials*, vol. 339, pp. 307–313, 2007.

[10] S. Ploen, "A skew-symmetric form of the recursive Newton-Euler algorithm forthe control of multibody systems," in *American Control Conference, 1999. Proceedings of the 1999*, vol. 6, 1999.

[11] F. Ghorbel, O. Chetelat, R. Gunawardana, and R. Longchamp, "Modeling and set point control of closed-chain mechanisms: Theory and experiment," *IEEE Transactions on Control Syst. Technol.*, vol. 8, p. 801815, September 2000.

[12] Y. Yiu, H. Cheng, Z. Xiong, G. Liu, and Z. Li, "On the dynamics of parallel manipulators," in *IEEE International Conference on Robotics and Automation*, vol. 4, pp. 3766–3771, Citeseer, 2001.

[13] T.-C. L. Hong-Chin Lin and K. H. Yae, "On The Skew-Symmetric Property Of The Newton-Euler Fo Mulation For Openchain Robot Manipulators," in *American Control Conference*, pp. 2322–2326, 1995.