

Identifying a two linked robot with non-symmetrical modified friction and Backlash-flexibility models

J. Bahrami, Me. Keshmiri, Mo. Keshmiri and H. D. Taghirad

Abstract— Identifying a robotic system, especially in presence of some nonlinear phenomena, such as friction and backlash, is a troublesome problem. In this paper, initially single link parameter identification is discussed. Parameters such as actuators parameter, link inertia, friction and backlash are identified in this section. A combination of actuator's model and a model of two flexible masses in attendance of friction and backlash is utilized for this purpose. Identifying two link robot parameters will be under debate afterwards and identification results for the second link are presented in three distinct forms. Furthermore, a method is introduced for selecting best estimated parameters set, in order to be exerted on systems controller. The effects of identified friction torques and backlash modeling on controller improvement are presented. Grey box structure model, available in Matlab's system identification tool box is utilized in this research.

I. INTRODUCTION

System identification for robots can be studied in three major levels. First level is assigned to identifying kinematic parameters. In the second level dynamics model parameters are identified such as inertia's parameters. During third level, in addition to previous identifications, friction model parameters and backlash are identified [1].

Friction is a nonlinear phenomenon which can be found in every object movements that are in contact with each other. Friction plays a substantial rule in every stages of motion in every machine. In high precision positioning systems, it is beneficial to be aware of the friction magnitudes, and it can be helpful in preventing undesirable effects such as limit cycles and constant errors [2].

Backlash exists in most mechanical systems with actuator just like friction. Controlling the load after a backlash is very troublesome, especially where high precision motion is essential [3].

DC motors are some electromechanical parts with a wide range of use in industries due to their ease of position and velocity control abilities [4]. DC motor Identification in presence of nonlinear friction behavior is highly under consideration [5].

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Several surveys have been accomplished in the field of robotic arms identification. Bompos et al. worked on a problem of modeling, identifying and controlling a robot in [6]. Discovering dynamic parameters along with nonlinear friction modeling, they were able to increase the accuracy of target tracking of an arbitrary trajectory and proved the certitude of identification. Kara and Eker [7], used the Hemerestian model for motor DC identification. To identify mechanical arms, Wernholt and Gunnarsson [8, 9], used flexible two and three masses models. They didn't consider any backlash in their models. Kostic et al. [10], demonstrated the importance of choosing an appropriate friction model and applying the estimated parameters to a control system. Radkhah et al [11], moreover, estimated 32 inertia parameters for a serial robot.

Within this survey a grey box model is used for identifying mechanical arms. To determine the nonlinear gray box, model structure is considered as a continuous time space state as follow;

$$\begin{aligned} \dot{x}(t) &= f(t, x(k), \theta, u(t)) \\ y(t) &= h(t, x(k), \theta, u(t)) + e(t), \end{aligned} \quad (1)$$

where f and h are nonlinear functions, $x(t)$ is a variable vector and $u(t)$ and $y(t)$ are input and output signals of the system, respectively. $e(t)$ is the white disturbance signal of measurement and t represent the time. θ is also a parameter vector. Based on measured input and output test, purpose is to determine parameters in a way that minimize the criterion,

$$V_N(\theta) = \frac{1}{N} \sum_{t=1}^N \varepsilon^2(t, \theta), \text{ where } \varepsilon(t, \theta) = y(t) - \hat{y}(t, \theta). \quad (2)$$

Gray box structure model, available in Matlab system identification tool box is utilized within this research [12].

II. COMBINED MODEL OF ROBOTIC ARMS AND DC MOTOR

Combining two masses flexible model with DC motor model, a combined model of planar single link is generated [7,8], where motor's voltage is the input and motor's angular position is the output (Figure 1).

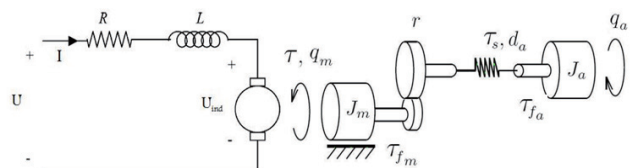


Fig. 1. Combined model of two flexible masses and DC motor

A. Model of a DC motor

For the electrical circuit of a DC motor U is voltage, I is

current, R , L are resistance and inductance, respectively. U_{ind} is armature's voltage and τ and \dot{q}_m are torque and angular velocity of motor, respectively.

For the Maxon DC motor [13], inductance value is neglected due to its insignificance.

By defining k_n and k_m are velocity and torque constants respectively, following equation can be derived for i^{th} motor torque:

$$\tau_{mi} = \frac{k_{mi}}{R_i} U_i - \frac{k_{mi}}{R k_{ni}} \dot{q}_{mi} = P_{m1mi}(U_i - \dot{q}_{mi} / P_{m2mi}), \quad (3)$$

Motor parameters are given in table 1.

TABLE I. MOTORS CONSTANTS

Parameters	Value	Parameters	Value
P_{m1m1}	19.059	P_{m1m2}	42.62
P_{m2m1}	0.0254	P_{m2m2}	0.0107

B. First link model of a single link robot

In figure 1 τ_s is the backlash-flexibility model which is modeled as a nonlinear spring. Gearbox reduction constant is $r_1 = 156$. τ_{fm} is friction in the motor side and τ_{fa} is friction for first link side.

Defining the variable vector as,

$$\begin{aligned} x_1 &= q_{m1}, x_2 = \dot{q}_{m1}, x_3 = q_{m1}/156 - q_{a1}, \\ x_4 &= q_{a1}, x_5 = \dot{q}_{a1}, \end{aligned} \quad (4)$$

nonlinear state-space model is obtained as follow;

$$\begin{aligned} \dot{x}_1 &= \dot{q}_{m1} \\ \dot{x}_2 &= (1/J_{m,g}) \times (P_{m1m1}(U - \dot{q}_{m1}/P_{m2m1}) - \tau_s/r_1 - \tau_{fm}) \\ \dot{x}_3 &= \dot{q}_{m1}/r_1 - \dot{q}_{a1} \\ \dot{x}_4 &= \dot{q}_{a1} \\ \dot{x}_5 &= (1/J_{a1}) \times (\tau_s - \tau_{fa}), \end{aligned} \quad (5)$$

where $J_{m,g}$ and J_{a1} are moments of inertia of motor gearbox rotors and the first link, respectively. τ_{fm} and τ_{fa} are frictions exerted on motor and link sides, respectively. q_{m1} and q_{a1} demonstrates angular positioning for motor and the link, respectively. Finally, τ_s is a model for backlash-flexibility.

1) Friction model for first link model of a single link robot

One of the most predominated models for general state of static friction is the model stated in [14],

$$F(v) = (F_c + (F_s - F_c) e^{-|v/v_s|^{p_s}}) \text{sgn}(v) + F_v v, \quad (6)$$

In the above equation following definitions are used. F_c is coulomb friction, F_s is Stribeck friction, v_s is Stribeck velocity and F_v is viscous friction. Considering that friction can emerge non-symmetrically, this model can also be expressed non-symmetrically. The proposed method for Stribeck model can be achieved by performing some changes in the above model. This model generates suitable results for the corresponding link. One prevalent dynamic model is Dahl's model which is expressed as equation (7).

$$\frac{dF_{mi}}{dt} = \sigma \dot{q}_{mi} - F_{mi} \left| \frac{\dot{q}_{mi}}{F_{cmi}} \right|, \quad (7)$$

In this model the friction model in motor 1 side, F_{cm} , is

$$\begin{aligned} F_{cm} &= (f_{11} + (f_{12} - f_{11}) e^{-\frac{(|\dot{q}_{m1}| - f_{13})^2}{f_{14}}}) \times \text{Pos}(\dot{q}_{m1}) \\ &+ (f_{21} + (f_{22} - f_{21}) e^{-\frac{(|\dot{q}_{m1}| - f_{23})^2}{f_{24}}}) \times \text{Neg}(\dot{q}_{m1}), \end{aligned} \quad (8)$$

and the friction model in first link side of the single link robot is

$$\begin{aligned} \tau_{fa} &= f_{a1} \times \tanh(70 \times \dot{q}_{a1}) \times \text{Pos}(\dot{q}_{a1}) \dots \\ &+ f_{a2} \times \tanh(70 \times \dot{q}_{a1}) \times \text{Neg}(\dot{q}_{a1}), \end{aligned} \quad (9)$$

$\text{Pos}(v)$ and $\text{Neg}(v)$ are defined as

$$\text{Pos}(v) = \begin{cases} 1 & \text{if } v > 0 \\ 0 & \text{if } v \leq 0 \end{cases}, \text{ and } \text{Neg}(v) = -\text{Pos}(v) \quad (10)$$

Adding the effect of viscous friction torque, friction torque equation can be written as,

$$\tau_{fm} = F_{cm} + f_3 \times \dot{q}_{m1}, \quad (11)$$

2) Backlash model for first link of a single link robot

Dead zone model is known as a classic model for modeling backlash [15]. Spring hardness, k , can be used in nonlinear form. Proposed backlash model is considered as follow;

$$\begin{aligned} \tau_s &= k \times (q_{m1}/r_1 - q_{a1}) \times \dots \\ &(1 - \sec h(\beta \times (q_{m1}/r_1 - q_{a1}))^\alpha), \end{aligned} \quad (12)$$

Backlash has been taken into account in this model. While backlash occurs, $(q_{m1}/r_1 - q_{a1})$ will fall in the backlash space and τ_s will possess a zero value. Proposed model demonstrate a nonlinear spring with backlash.

Assuming $q_{m1} = r_1 \times q_{a1}$ a solid body model can be obtained without flexibility and backlash [6] (Figure 2).

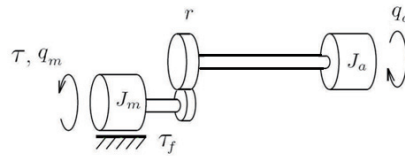


Fig. 2. Solid Body Model

C. Two link robot modeling

To derive a planar two link robot equations, Lagrange method is used.

By defining τ_{g1} and τ_{g2} as torques generated from gearboxes of link 1 and 2, respectively,

$$\begin{aligned} \tau_{g1} &= (\tau_{m1} - \tau_{f1} - J_{m1,g1} \ddot{q}_{m1}) \times r_1, \\ \tau_{g2} &= (\tau_{m2} - \tau_{f2} - J_{m2,g2} \ddot{q}_{m2}) \times r_2. \end{aligned} \quad (13)$$

Dynamic equations will be in the general form of

$$\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{bmatrix} \ddot{q}_{m1} \\ \ddot{q}_{m2} \end{bmatrix} + \begin{bmatrix} C_1(q_m, \dot{q}_m) \\ C_2(q_m, \dot{q}_m) \end{bmatrix} = \begin{bmatrix} \tau_{m1} \\ \tau_{m2} \end{bmatrix}, \quad (14)$$

Where q_{mi} is angular position of motor i , τ_{m1} and τ_{m2} are first and second motors output torque. Gearbox reductions used in these equations are $r_1 = 156$ and $r_2 = 84$ [12]. Finally τ_{f1} and τ_{f2} are friction torques on first and second motors side. Mass matrix is considered as the combination of 4 parameters P_1, \dots, P_4 .

For motor sides friction, F_{cmi} , $i = 1, 2$ is considered as

$$F_{cmi} = (f_{11mi} + (f_{12mi} - f_{11mi})e^{-\left(\frac{|\dot{q}| - f_{13mi}}{f_{14mi}}\right)^2}) \times \text{Pos}(\dot{q}_{mi}) \dots + (f_{21mi} + (f_{22mi} - f_{21mi})e^{-\left(\frac{|\dot{q}| - f_{23mi}}{f_{24mi}}\right)^2}) \times \text{Neg}(\dot{q}_{mi}), \quad (15)$$

By adding effect of viscous friction, friction torque for motors can be written like equation (16).

$$\tau_{fi} = F_{mi} + f_{3mi} \times \dot{q}_{mi}, \quad i = 1, 2 \quad (16)$$

D. Data collection

Choosing an input excitation signal as a step for model identifying matters is in a high level of importance. This signal depends on the selected identified model. For instance, excitation signals for solid body models are choose so that backlash and flexibility will affect them least. On the other hand, for two masses models including backlash, the backlash ought to have the highest affect on model. Signals are chosen in form of sine functions which their crest factors are minimized [16].

E. Data preprocessing

Whereas, motors angular position is measured as the output, and knowing that the signals are acquired periodically, motors angular velocity can be numerically calculated as shown in figure 3, [17].

In this technique noises are omitted through a procedure of exerting Discrete Fourier Transform to the data and transmitting them from time domain to frequency domain and omitting undesired frequencies. Desired frequencies are then multiplied to $j\omega(k)$ and finally the result will be transformed to time domain by exerting the Inverse Discrete Fourier Transform.

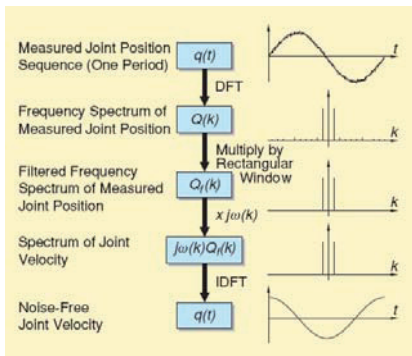


Fig. 3. Preprocessed procedure performed on experiment datas

III. APPLYING SOLID BODY MODEL TO FIRST LINK IDENTIFICATION OF A SINGLE LINK ROBOT

A. First link parameters estimation for a single link robot

To identify the parameters of a solid body model ten sets of experiments are implemented. In order to analyze the variation measure criterion is used [18]. If estimated parameters variation were small, estimated parameters will give suitable results for different examinations, and therefore estimated parameters can declare system's dynamic precisely.

In order to quantify the variation, the percentage ratio between standard deviation and average of each estimated parameter is calculated for every ten set of parameters. If variations fall under 30 percent, results are acceptable. Estimation results are shown in table 2.

TABLE 2. VARIATION MEASURE CRITERION FOR SOLID BODY MODEL

Parameters	(Std/Ave) %	Best Parameters Set	Parameter s	(Std/Ave) %	Best Parameters Set
J	16.345	1.97E-05	f_{14}	11.688	53.005
P_{m1}	6.436	19.249	f_{21}	5.071	0.010
P_{m2}	6.888	0.025	f_{22}	6.998	0.031
f_{11}	3.590	0.010	f_{23}	6.666	11.313
f_{12}	7.742	0.030	f_{24}	8.857	58.093
f_{13}	8.402	10.541	f_3	5.760	5.04E-05

B. electing best set of parameters for first link of single link robot.

To carry out a survey on results of simulation and comparing them with computed results, fitness criterion can be used [19].

In order to find the best set of parameters for controller following procedure is proposed. Estimated parameters for each experiment are used to simulate other set of experiments. Fitness values are acquired this way. Percentage ratio for standard deviation and average of fitness values are computed subsequently. Parameters set with the least percentage will be chosen to be applied to system's controller.

According to table 3 best set of parameters is regarded to estimated parameters yielded from test 3 which are given in table 4.

TABLE 3. PRECISION EVALUATING CRITERION FOR IDENTIFIED PARAMETERS IN SOLID BODY MODELS

Estimated Parameters		Fitness average value	Standard Deviation	Standard Deviation to Average Ratio %
Test 3	Position	93.38	5.08	5.44
	Velocity	95.29	2.16	2.26

C. Exerting solid body model in controlling first link of a single link robot

Identified model's proficiency is evaluated through applying it to system's controller. Computed torque method, which is based on systems model, is used as system controller [20]. Result comparison is performed in three

different states by using different model in each state. For the first state mathematical model derived from robots dynamic and calculated parameters is used. In second state, controller utilizes a model similar to previous state which the inertia and motor estimated parameters is used in it. Finally, in third state, a completely identified model is used.

Presented pictures are explained using some abbreviations as follow;

- MM: mathematical model
- IFLRM: identified model, without considering friction estimations
- IFRM: complete identified model

Desired trajectory tracking results for motor's angular position in time is shown in figures 4. It is obvious that tracking errors are reduced by changing model parameters from mathematical parameters to the identified parameters.

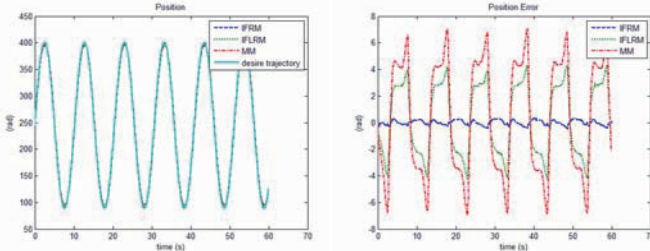


Fig. 4. Proficiency of three different models for pursuing a sine trajectory (left) and tracking position errors (right)

D. Exerting backlash-flexible model in identifying first link of a single link robot

Parameters estimation is performed, similar to previous section and results are given in table 6. Motor and viscous friction parameters are used as obtained in previous stage, constantly. Utilizing evaluation criterion, parameters set accuracy is calculated and best parameters set is selected which is presented in table 5. Best set of parameters related to estimated parameters in test 3 are given in table 4.

Finally, identified model in this stage and solid model identified in previous stage are compared through applying them to system controller (figure 5). Comparing errors, shows that errors generated from backlash-flexibility model is half the size of errors generated from solid body model.

TABLE 4. VARIATION MEASURE CRITERION FOR FLEXIBLE MODEL WITH BACKLASH

Parameters	(Std/Ave) %	Best Parameters Set	Parameters	(Std/Ave) %	Best Parameters Set
$J_{m,g}$	23.16	1.32E-05	f_{11}	7.65	0.0089
J_a	22.0	0.090	f_{12}	2.32	0.0285
P_{m1}	0	19.25	f_{13}	15.83	12.52
P_{m2}	0	0.025	f_{14}	13.0	59.53
f_{a1}	9.34	0.0063	f_{21}	8.75	0.0081
f_{a2}	8.65	0.0056	f_{22}	2.45	0.030
k	4.32	380.16	f_{23}	13.29	14.10
α	6.65	0.31	f_{24}	8.99	63.73
β	8.09	169.34	f_3	0	6.94E-05

TABLE 5. PRECISION EVALUATING CRITERION FOR IDENTIFIED PARAMETERS IN FLEXIBLE WITH BACKLASH MODELS

Estimated Parameters	Fitness average value	Standard Deviation	Standard Deviation to Average Ratio %
Test 3			
Position	92.64	4.93	5.32
Velocity	94.26	3.46	3.67

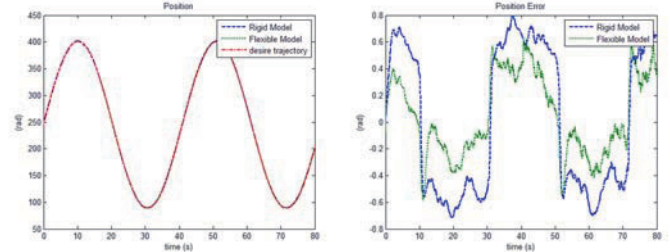


Fig 5. Comparing two identified models in tracking the trajectories(left) and tracking position errors (right)

IV. IDENTIFYING SECOND LINK OF THE ROBOT

In this stage, first link is fixed and analyses are performed on the second link. Identifications are executed for high and low velocity states.

A. Parameter estimation for high and low velocities

In order to identify parameters for the second link in high speed condition 8 experiments and in low speed condition 9 experiments are implemented. Results for these estimations are presented in table 6 and 7. Variation measure criterion is ostensible in these results.

TABLE 6. VARIATION MEASURE CRITERION FOR HIGH VELOCITY MODEL IN SECOND LINK

Parameters	(Std/Ave) %	Best Parameters Set	Parameters	(Std/Ave) %	Best Parameters Set
P_2	2.62	0.135	f_{21m2}	8.99	0.0029
f_{11m2}	5.64	0.0024	f_{22m2}	4.04	0.0097
f_{12m2}	3.18	0.0103	f_{23m2}	5.52	17.95
f_{13m2}	10.49	17.74	f_{24m2}	5.03	108.30
f_{14m2}	6.07	113.17			

TABLE 7. PRECISION EVALUATING CRITERION FOR IDENTIFIED PARAMETERS IN LOW VELOCITY MODELS IN SECOND LINK

Parameters	(Std/Ave) %	Best Parameters Set	Parameters	(Std/Ave) %	Best Parameters Set
P_2	29.36	0.185	f_{22m2}	1.57	0.0112
f_{12m2}	1.76	0.0111	f_{23m2}	23.65	19.28
f_{13m2}	20.70	28.54	f_{24m2}	21.75	91.07
f_{14m2}	11.28	125.96			

B. Selecting best set of parameters

Table 8 and 9 exhibits the best results for standard deviation to parameters average ratio and the values for best parameters sets are given in table 6 and 7.

TABLE 8. PRECISION EVALUATING CRITERION FOR IDENTIFIED PARAMETERS OF SECOND LINK FOR HIGH SPEED

Estimated Parameters	Fitness average value	Standard Deviation	Standard Deviation to Average Ratio %
Test 3 Motor 2 Position	86.13	9.24	10.73
Motor 2 Velocity	96.18	1.02	1.06

TABLE 9. PRECISION EVALUATING CRITERION FOR IDENTIFIED PARAMETERS OF SECOND LINK FOR LOW SPEED

Estimated Parameters	Fitness average value	Standard Deviation	Standard Deviation to Average Ratio %
Test 3 Motor 2 Position	82.76	4.04	4.88
Motor 2 Velocity	83.05	2.20	2.65

V. IDENTIFYING A TWO LINK ROBOT

A. estimating and selecting best parameters set.

In this stage, identification is performed by exciting both links of the robot. 12 experiments are used to create the data sets which a combination of low and high speed excitation is used. Results for using variation measure criterion are given in table 10 and the criterion's validity is discernible. Using precision evaluation proposed method for each parameter set, best set of parameters for using in controller is extracted from 7th test (table 11). Estimated parameters values are given in table 10.

TABLE 10. VARIATION MEASURE CRITERION FOR TWO LINK ROBOT

Parameters	(Std/Ave) %	Best Parameters Set	Parameters	(Std/Ave) %	Best Parameters Set
f_{11m1}	24.11	0.011	f_{12m2}	2.49	0.010
f_{12m1}	4.430	0.038	f_{13m2}	12.14	24.146
f_{13m1}	12.93	10.182	f_{14m2}	12.70	114.718
f_{14m1}	13.62	51.088	f_{22m2}	2.80	0.011
f_{21m1}	29.25	0.023	f_{23m2}	27.10	25.934
f_{22m1}	8.300	0.039	f_{24m2}	12.45	97.418
f_{23m1}	10.48	16.318	P_1	23.18	1.101
f_{24m1}	18.08	41.463	P_2	10.69	0.136
f_{3m1}	11.57	2.932E-	P_3	15.00	0.148
			P_4	26.61	0.068

TABLE 11. PRECISION EVALUATING CRITERION FOR TWO LINK ROBOT

Estimated Parameters	Fitness average value	Standard Deviation	Standard Deviation to Average Ratio %
Motor 1 Position	77.59	8.35	10.76
Motor 1 Velocity	85.50	6.24	7.30
Motor 2 Position	88.00	6.70	7.62
Motor 2 Velocity	85.50	3.49	4.08

A. Exerting identified model in system controller

1) Motors trajectory tracking

Results for exerting identified model in tracking desired trajectories for motor 1 and 2 is presented in figure 8. Errors caused during tracking are exhibited in figure 9. It can be seen that using identified mass matrix in system's control model, pursuit errors will decrease for both motors and especially for motor 2.

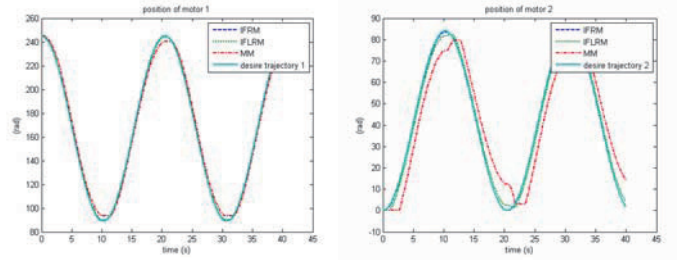


Fig. 8. Proficiency of three different models for pursuing trajectory by motor 1 (left) and motor 2 (right)

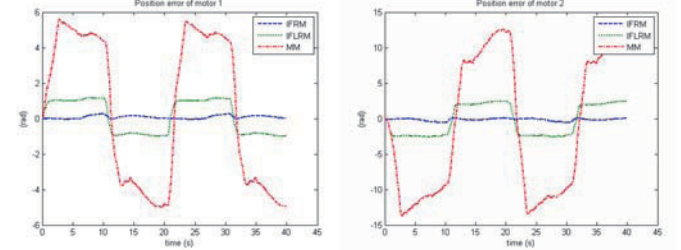


Fig 9. Position tracking errors for motor 1 (left) and motor 2 (right) in three states

VI. IDENTIFIED FRICTION MODELS FOR FIRST LINK OF A SINGLE LINK ROBOT

Considering proposed friction models and best estimated parameters set (table 2 and 4), estimated friction torques with respect to velocity are shown in figure 13, where τ_f is solid body model (Figure 2). τ_{fm} and τ_{fa} are flexibility model frictions (Figure 1). In order to demonstrate nonlinearities of estimated torques, an asymmetric model is presented in figure 13.

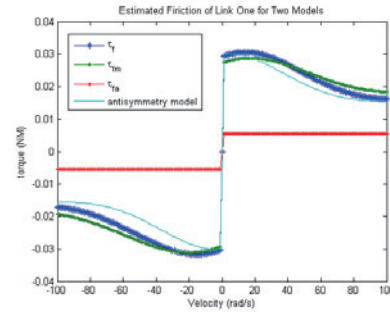


Fig. 13. Estimating friction torque for both models

VII. COMPARING RESULTS FOR IDENTIFYING SECOND LINK

Considering the proposed friction model and best estimated parameters set for second link (table 6, 7 and 10), estimated friction torques with respect to velocity is presented in picture 14. τ_f demonstrates identified friction for second link in low and high speed. τ_{f2} is also demonstrates identified friction for the second link of a two

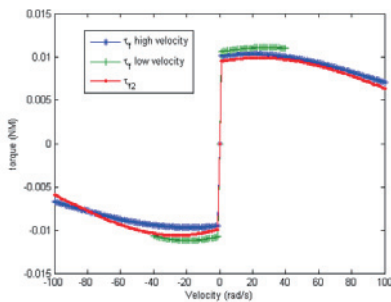


Fig. 14. Estimating friction torque for both models

link robot. It can be concluded that estimated torque friction is higher for low velocities than high velocities.

Identified inertia values for second link is given in table 12 for three different states. It can be seen that values for lower speed have more difference with respect to two other conditions.

TABLE 12. COMPARING IDENTIFIED VALUE FOR SECOND LINK INERTIA PARAMETERS

Identifying second link	value
High Speed State	0.135
Low Speed State	0.185
Two links arms	0.136

VIII. CONCLUSION

In this research, system identification procedure is introduced as well as a single link robot parametric identification. A combination of DC motor and mechanical arm is used in this paper. Proposed models for friction model and backlash-flexibility model are taken into account for the corresponding system. Models and identification procedure are verified through variation measure criterion. In addition, a new method for evaluating identified parameters sets precision was introduced and utilized in selecting the best set of parameters in order to apply to systems controller.

Furthermore, second link's identification was performed for a two link robot for three various states. Results for estimating parameters of this model and also estimated parameters for inertia were compared for three states.

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