Delay-Independent Robust Stability Analysis of Teleoperation

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***Abstract*— This paper considers the robust stability of uncertain teleoperation systems. Sufficient stability conditions are derived in terms of LMI by representing the teleoperation scheme in retarded form of time-delay systems. By choosing Lyapunov-Krasovski functional, a delay-independent robust stability criterion is presented. We show that the teleoperation system is stable and has good performance under specific LMI condition. With the given controller parameters, stability of system is guaranteed in the presence of any value of delay and admissible uncertainty. To evaluate the theoretical analysis, Numerical simulations are performed.**

Key words: Teleoperation, Delay, Robust Stability, LMI

I. Introduction

THE Internet is now serving billions of users worldwide. It is now primarily used for communicating, information gathering, online shopping and other financial transactions, etc. But looking deeply, it is a good media to control physical systems from distance. This can be done in an open-loop arrangement in which the reference signal and not the control signal propagates through the net, or in a closed-loop arrangement for bilateral teleoperation, which requires transmitting the control signal through the network. This exposes the control loop to the time delay of packets in the network. Time delay, which is inseparable from the Internet, may be not noticeable in open-loop control, because it may just oppose a delay in action. Yet in closed-loop configuration it can render the system unstable or degrade its performance, as in the case of teleoperation [1]. Several techniques have been proposed to compensate for this effect, such as a time forward observer developed for a supervisory control over the Internet ([2] and [3]), a position-based force-feedback scheme [4], scattering theory by Anderson and Spong [5] and a wave variable based technique developed by Niemeyer and Slotine [6].

A bilateral teleoperation system consists of the master which is manipulated by a human operator and the slave which is designed to track the master in a remote environment. Information is transmitted between master and slave via communication channels. Internet is the most common communication channel used in this field. An overall block diagram of teleoperation system is shown In Fig. 1. So far many researchers have employed position, velocity, force or impedance information to propose a variety of control structures, but most of these controllers can’t ensure both stability and transparency independent of time delay, as there is a tradeoff between these two goals. In an extensive survey presented by [14] and [15], a large amount of control architectures are reviewed.

In this paper, we present a delay-independent robust stability criteria for teleoperation system which is robust against network’s time-delay and any admissible uncertainties. With the given controller parameters, stability of system is guaranteed in the presence of any value of delay and admissible uncertainties.

This paper is organized as follows: In section 2 an overall description and modeling of teleoperation system is introduced. In Section 3, we represent the system equations in state-space model and present a robust controller against time-delay of network by using LMI. In section 4, we extend the proposed criteria to the robust case. Simulation results are presented in section 5 to validate properties of the proposed framework. Section 6 contains summary and concluding remarks.

II. Tele-operation Modeling

For the sake of simplicity, the master and the slave have been modeled linear, as mass-damper systems, as shown in Fig. 2. This model is popular in this literature and is used in several articles (to name a few [19] and [20]).

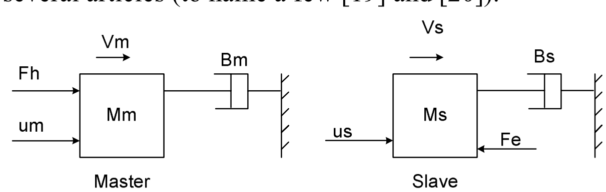

Figure 1. Dynamic of master and slave

u_m, u_s are the control signals, f_h is the force applied to the master by the operator and f_e is the force exerted on the slave by the environment. The master and slave dynamics can be described by

$$M_m \dot{v}_m + B_m v_m = u_m + f_h \quad (1)$$
$$M_s \dot{v}_s + B_s v_s = u_s - f_e \quad (2)$$

Where M, B and v denote inertia, damping coefficient and velocity, respectively. Subscript $'m'$ and $'s'$ denote

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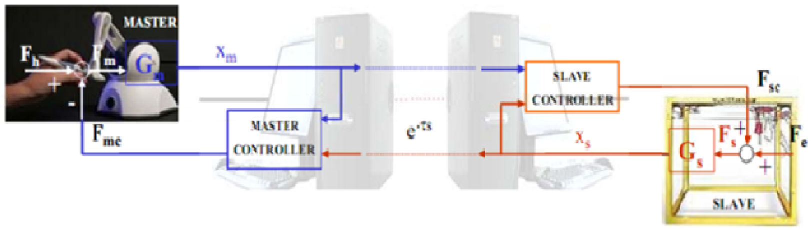

Figure 2. Dynamic of master and slave

master and slave, respectively. The forces f_h and f_e are given by

$$f_h = f_h^* - Z_h v_m \tag{3}$$
$$f_e = Z_e v_s \tag{4}$$

Where f_h^* is the operator exogenous force. Z_h and Z_e are human and environment impedances that are supposed to be as mass, damper and spring

$$\begin{aligned} Z_h &= M_h s + B_h + \frac{K_h}{s} \\ Z_e &= M_e s + B_e + \frac{K_e}{s} \end{aligned} \tag{5}$$

A block diagram of the bilateral teleoperation system is shown In Fig. 3. This framework has been first introduced by Spong in [17].

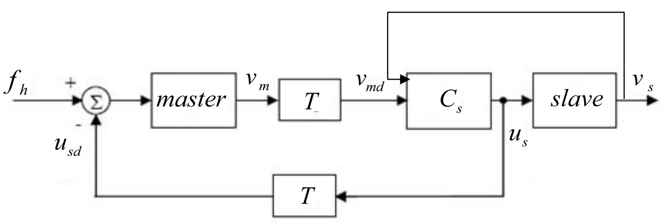

Figure 3. Block diagram of a bilateral control system

In the above structure, position of master (v_m) is transmitted to the slave and at slave side, we use a PD controller for control position. Also, the force information from slave is transmitted to the master side. So the dynamic characteristics of the master is described as follows

$$M_m \dot{v}_m + B_m v_m = f_h - u_s(t-\tau) \tag{6}$$

Where u_s is the control input of the slave given by a PD controller as

$$u_s = k_P(x_{md} - x_s) + k_D(v_{md} - v_s) \tag{7}$$

In which $x_{md} = x_m(t-T), v_{md} = v_m(t-T)$ and k_P, k_D are controller gains.

III. Controller Design

The goal of our design is to find gains of the PD controller using LMI framework such that the position and the force of master and slave track each other, in the presence of time-delay.
Substituting (3) to (7) into the master and slave dynamics equations, (1) and (2), closed-loop state equations of the system become as follows:

$$\dot{x}(t) = Ax(t) + A_d x_d(t) + Bw \tag{8}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_h}{M_h+M_m} & -\frac{B_h}{M_h+M_m} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{k_P}{M_s+M_e} & -\frac{k_D}{M_s+M_e} \end{bmatrix},$$

$$A_d = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{k_P}{M_h+M_m} & -\frac{k_D}{M_h+M_m} \\ 0 & 0 & 0 & 0 \\ -\frac{k_P}{M_s+M_e} & -\frac{k_D}{M_s+M_e} & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ \frac{1}{M_h+M_m} \\ 0 \\ \frac{1}{M_h+M_m} \end{bmatrix}$$

Where $w = f_h^*$ and the state-space vector $x(t) \in R^n$ is defined as:

$$x(t) = [x_m(t) \;\; v_m(t) \;\; x_e(t) \;\; v_e(t)]^{\mathrm{T}} \tag{9}$$

In which $x_e = x_{md} - x_s$

Theorem1: The teleoperation system (8) with control gains given at (11) is stable and has good performance for any constant delay of communication channel, if there exists symmetric positive definite matrices Q', S and K such that the LMI shown in (10) holds

$$R = \begin{bmatrix} A_0Q' + B_0K + Q'{A_0}^T + K^TB_0^T + S & A_1K \\ K^TA_1^T & -S \end{bmatrix} < 0 \tag{10}$$

where

$$\widetilde{K} = K(Q')^{-1} = \begin{bmatrix} k_P & k_D & 0 & 0 \\ 0 & 0 & k_P & k_D \end{bmatrix} \tag{11}$$

Proof. Consider the following Lyapunov-Krasovski functional

$$V(x) = x^T(t)Px(t) + \int_{t-h}^{t} x^T(\tau)\, Qx(\tau)d\tau \tag{12}$$

It is clear that the Lyapunov function candidate is Positive Definite (PD). Taking the derivative of this function we have

$$\frac{dV(x)}{dt} = \dot{x}(t)Px(t) + x^T(t)P\dot{x}(t) \tag{13}$$
$$+x^T(t)Qx(t) - x^T(t-h)Qx(t-h)$$

For nominal conditions ($f_h^* = 0$), substituting (8) in (13) yields

$$\dot{V} = \begin{bmatrix} x^T \\ x^T(t-h) \end{bmatrix} R \begin{bmatrix} x \\ x(t-h) \end{bmatrix} \tag{14}$$

where

$$R = \begin{bmatrix} A^TP + PA + Q & PA_d \\ A_d^TP & -Q \end{bmatrix}$$

If $R < 0$, which guarantees $\dot{V} < 0$, then tele-operation system is asymptotically stable, but the inequality $R < 0$ is not linear, so in order to decorate it in a linear format, by multiplying pre and post P^{-1}, we can rewrite it in its dual form [16]

$$R' = \begin{bmatrix} AQ' + Q'A^T + S & A_dQ' \\ Q'A_d^T & -S \end{bmatrix} < 0, \tag{15}$$

where

$$Q' = P^{-1}, S = P^{-1}QP^{-1}$$

To separate the matrices including controller gains which should be calculated, we factorized matrices A, A_d as

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_h}{M_h+M_m} & -\frac{B_h}{M_h+M_m} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} +$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -\frac{1}{M_s+M_e} \end{bmatrix} \begin{bmatrix} k_P & k_D & 0 & 0 \\ 0 & 0 & k_P & k_D \end{bmatrix} = A_0 + \mathrm{B}_0\widetilde{\mathrm{K}} \tag{16}$$

$$A_d = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{M_h+M_m} \\ 0 & 0 \\ -\frac{1}{M_s+M_e} & 0 \end{bmatrix} \begin{bmatrix} k_P & k_D & 0 & 0 \\ 0 & 0 & k_P & k_D \end{bmatrix}$$
$$= \mathrm{A}_1\widetilde{\mathrm{K}} \tag{17}$$

By substituting (16), (17) in (15) we have

$$R' = \begin{bmatrix} A_0Q' + \mathrm{B}_0\widetilde{\mathrm{K}}\,Q' + Q'{A_0}^T + Q'(\mathrm{B}_0\widetilde{\mathrm{K}})^T + S & \mathrm{A}_1\widetilde{\mathrm{K}}Q' \\ Q'(\mathrm{A}_1\widetilde{\mathrm{K}})^{\mathrm{T}} & -S \end{bmatrix} < 0 \tag{18}$$

By choosing $K = \widetilde{K}Q'$

$$R' = \begin{bmatrix} A_0Q' + B_0K + Q'{A_0}^T + K^TB_0^T + S & A_1K \\ K^TA_1^T & -S \end{bmatrix} < 0, \tag{19}$$

This inequality is linear and can be solved by LMI toolbox. The gains of the controller then can be found from

$$\widetilde{K} = K(Q')^{-1} \tag{20}$$

This proves the theorem.

IV. Robust stability analysis

Now, we consider uncertain teleoperation system as follows

$$\dot{x}(t) = (A + \Delta A)x(t) + (A_d + \Delta \mathrm{A_d})x_d(t) + Bw \tag{21}$$

Where matrices ΔA and $\Delta \mathrm{A_d}$ characterize the uncertainties in the system and satisfy the following assumption.

Assumption 1:

$$\Delta A = HF(t)E\ , \Delta \mathrm{A_d} = H_dF_d(t)E_d \tag{22}$$

Where $H, E, H_d,$ and E_d are known real constant matrices, and $F(t)$, $F_d(t)$ are unknown matrix functions with Lebesgue-measurable elements and satisfy $F(t)^TF(t) \leq I$ and $F_d(t)^TF_d(t) \leq I$, in which I is the identity matrix.

Before proceeding, we recall following lemma that will be used for the proof of the next theorem.

Lemma 1 [18]: For constant matrices H and E, and scalar $\varepsilon > 0$, the following inequality holds:

$$HFE + E^TF^TH^H \leq \varepsilon HH^T + \varepsilon^{-1}E^TE \tag{23}$$

Where F satisfies $F(t)^T F(t) \leq I$.

Theorem2: The uncertain teleoperation system (21) with given control gains is stable and has good performance for any constant delay of communication channel and any uncertainty satisfying assumption 1, if there exists symmetric positive definite matrices Q' and S, and positive scalars ε and ε_d such that the LMI shown in (24) holds

$$\begin{bmatrix} AQ' + Q'A^T + S + \varepsilon HH^T + \varepsilon_d H_d H_d^T & A_d Q' & EQ' & 0 \\ Q'A_d^T & -S & 0 & E_d Q' \\ Q'E^T & 0 & -\varepsilon & 0 \\ 0 & Q'E_d^T & 0 & -\varepsilon_d \end{bmatrix} < 0 \tag{24}$$

Proof. By replacing A and A_d with $A + \Delta A$ and $A_d + \Delta A_d$ in (15) we have

$$\begin{bmatrix} (A+\Delta A)Q' + Q'(A+\Delta A)^T + S & (A_d + \Delta A_d)Q' \\ Q'(A_d + \Delta A_d)^T & -S \end{bmatrix} < 0, \tag{25}$$

We can write (25) as follows

$$\begin{bmatrix} (A+\Delta A)Q' + Q'(A+\Delta A)^T + S & A_d Q' \\ Q'A_d^T & -S \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & \Delta A_d Q' \\ Q'\Delta A_d^T & 0 \end{bmatrix}}_{\alpha} < 0, \tag{26}$$

Substituting ΔA_d from (22), we have

$$\alpha = \begin{bmatrix} 0 \\ Q'E_d^T \end{bmatrix} F_d^T [H_d^T \quad 0] + \begin{bmatrix} H_d \\ 0 \end{bmatrix} F_d [0 \quad E_d Q'] \tag{26}$$

By applying lemma 1 we have

$$\alpha \leq \varepsilon_d \begin{bmatrix} H_d^T \\ 0 \end{bmatrix} [H_d \quad 0] + \varepsilon_d^{-1} \begin{bmatrix} 0 \\ Q'E_d^T \end{bmatrix} [0 \quad E_d Q'] \tag{27}$$

in the other hand by substituting ΔA from (22) and applying lemma 1 we have

$$\Delta A Q' + Q'\Delta A^T \leq \varepsilon HH^T + \varepsilon^{-1} Q'E^T EQ \tag{28}$$

By combination of (25) to (28) and applying Schur complement, we can write (24) and the theorem will be proved. The rest stages for designing controller, is similar to the previous section.

V. Numerical Examples

To illustrate the validation of our proposed method, consider the teleoperation system with the following parameters,

$$\begin{gathered} M_m = 1, B_m = 1.5, M_s = 1, B_s = 1.5 \\ M_h = 1, B_h = 1, K_h = 25 \\ M_e = 1, B_e = 1, K_e = 25 \\ H = [0\ 0.1\ 0\ 0]^T, E = [1\ 0\ 0\ 0] \\ H_d = [0\ 0.1\ 0\ 0]^T, E_d = [1\ 0\ 0\ 0] \end{gathered} \tag{29}$$

In the following simulation, we assume $F(t) = sint$, and it can be seen that $\|F(t)\| \leq 1$. Using the LMI Toolbox of the MATLAB, the gains are calculated as follows

$$K_P = 60, K_D = 20 \tag{30}$$

First assume that communication time delay is 100 ms. Fig.4 and Fig.5 show the simulation results of the master and slave responses. As shown in Fig.4, by applying the designed control signal to the system, the slave position track the delayed master position. Fig.5 shows the slave and delayed master forces. From Fig.5, we can see that the force tracking is very good too. So we have achieved acceptable transparency

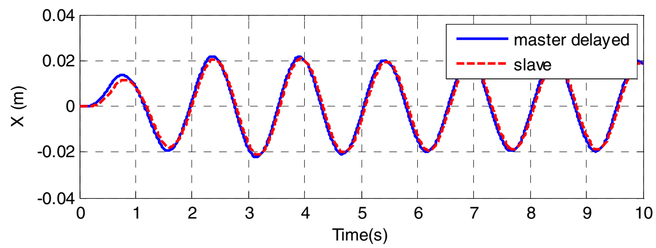

Figure 4. Position of the master with determined delay and slave, $(\tau = 100\ ms)$

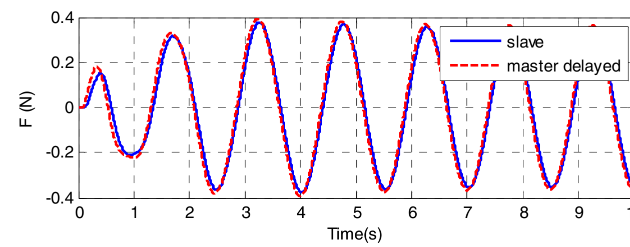

Figure 5. Force of the master with determined time delay and slave, $(\tau = 100\ ms)$

Fig. 6 shows that position tracking is yet good, even when the time-delay is considered to be as much as 1000 ms. the same is true for force, shown in Fig. 7.

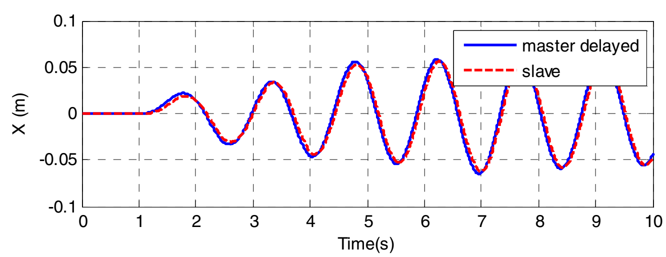

Figure 6. Position of the master with determined delay and slave, $(\tau = 1000\ msec)$

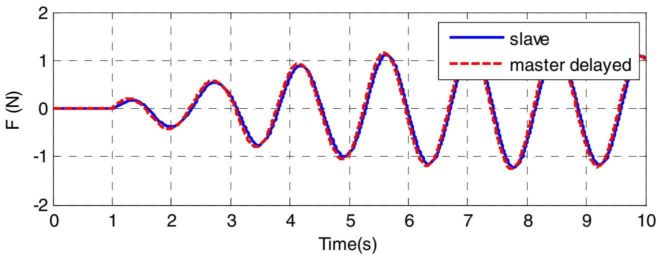

Figure 7. Force of the master with determined time delay and slave, $(\tau = 1000\ msec)$

It can be seen that the teleoperation system is stable and has good performance against any value of time delay and admissible uncertainties. The simulation results demonstrate the validity of the methodology presented by this paper.

VI. Conclusion

The problem of stabilizing an uncertain teleoperation system in the presence of time delay in the communication channel is addressed. To achieve robust stability while acceptable tracking performance, a controller is designed via LMI. The LMI condition is independent of the time-delay of the network. Numerical example is used to show feasibility and performance of the proposed controller.

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