

# Delay-Independent Stability Analysis of Internet-based Tele-operation

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**Abstract**— This paper presents a sufficient stability condition for internet-based tele-operation systems in terms of LMI. The tele-operation scheme is modeled in state-space as a time-delay system in retarded form and a delay-independent stability criterion is extracted. By choosing Lyapunov-Krasovski functional, we show that the internet-based tele-operation system is stable and has good performance under specific LMI condition. With the given controller design parameters, stability of system is guaranteed in the presence of any value of delay. Numerical simulations are performed to verify the theoretical results.

**Keywords**—Tele-operation; Delay; Stability; LMI

## I. INTRODUCTION

The Internet is used widely nowadays and is being wider and wider day to day. However, it is still primarily used for emailing, web surfing, online shopping, etc. Recently several researchers have tried to use it as a communication channel for tele-operation. Previously some researchers had tried to use the Internet to control physical systems, but it was mostly in an open-loop arrangement (just to send the command signal over the net). Bilateral tele-operation via the Internet requires closing the control loop through the network. This imposes the time delay of a packet moving over the net to the control signal. This delay can cause severe problems such as instability for tele-operation over the Internet [1]. Several techniques have been tried to compensate for this effect, such as an observer developed for a supervisory control over the Internet ([2] and [3]), a position-based force-feedback scheme [4], scattering theory [5] and the wave variable based technique [6].

Tele-operation has been applied more and more in many fields such as unmanned underwater vehicles [7], nuclear experiments [8], mobile robots [9], tele-surgery [10],

micro manipulation [11] and space robotics [12]. The goal of tele-operation system is to enable human beings to interact with, modify, and control a remote physical environment by supervising actions through a communication channel such as Internet. One of the most important aspects in this regard is to achieve transparency which means to provide correct haptic feeling of the remote environment to the operator [13].

A bilateral tele-operation system consists of the master which is manipulated by a human operator and the slave which is designed to track the master in a remote environment. Information is transmitted between master and slave via communication channels. Internet is the most common communication channel used in this field. An overall block diagram of a tele-operation system is shown In Fig. 1. So far many researchers have employed position, velocity, force or impedance information to propose a variety of control structures, but most of these controllers can't ensure both stability and transparency independent of time delay, due to there is a tradeoff between these two goals. In comprehensive surveys presented in [14] and [15], many control architectures are reviewed.

In this paper, we presents a delay-independent stability criteria for internet-based tele-operation which is robust against any value of network's time-delay and achieve the gains of PD controller that provide good performance for tele-operation system.

This paper is organized as follows: In section 2 an overall description and modeling of tele-operation system is introduced. In Section 3, we represent the system equations in state-space and design a robust controller against time-delay of network in LMI format. Simulation results are presented in section 4 to validate properties of the proposed framework. Section 5 contains summary and concluding remarks.

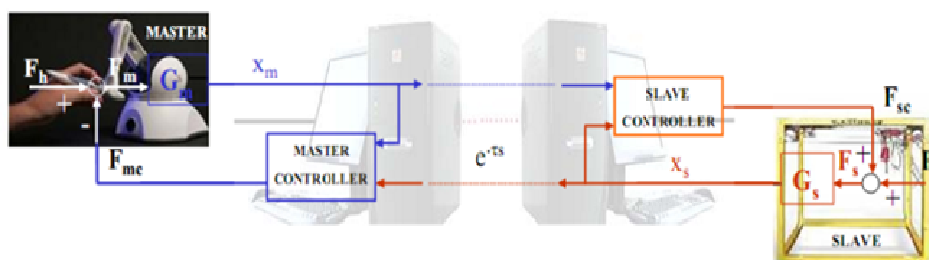


Figure 1. Dynamics of master and slave

## II. TELE-OPERATION MODELLING

For the sake of simplicity, the master and the slave have been modeled as mass-damper systems, as shown in Fig. 2.

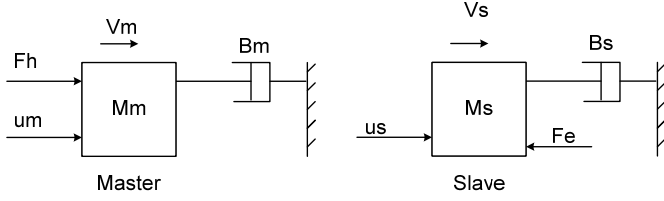


Figure 2. Dynamic of master and slave

$u_m, u_s$  are the control signals,  $f_h$  is the force applied to the master by the operator and  $f_e$  is the force exerted on the slave by environment. The master and slave dynamics can be described by

$$M_m \dot{v}_m + B_m v_m = u_m + f_h \quad (1)$$

$$M_s \dot{v}_s + B_s v_s = u_s - f_e \quad (2)$$

Where  $M, B$  and  $v$  denote inertia, damping coefficient and velocity, respectively. Subscript 'm' and 's' denote master and slave, respectively. The forces  $f_h$  and  $f_e$  are given by

$$f_h = f_h^* - Z_h v_m \quad (3)$$

$$f_e = Z_e v_s \quad (4)$$

Where  $f_h^*$  is the operator exogenous force.  $Z_h$  and  $Z_e$  are human and environment impedances that are supposed to be as follows

$$\begin{aligned} Z_h &= M_h s + B_h + \frac{K_h}{s} \\ Z_e &= M_e s + B_e + \frac{K_e}{s} \end{aligned} \quad (5)$$

A block diagram of the bilateral tele-operation system is shown in Fig. 3. This framework has been first introduced by Spong in [17].

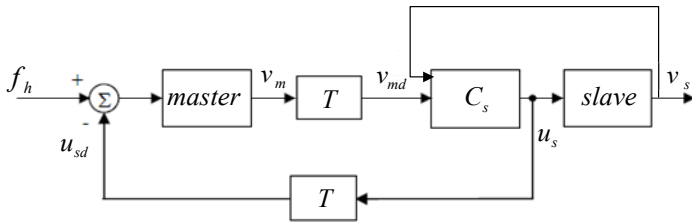


Figure 3. Block diagram of a bilateral control system

In the above structure, position of master ( $v_m$ ) is transmitted to the slave and at slave side, we use a PD controller for control position. Also, the force information from slave is transmitted to the master side. So the dynamic characteristics of the master is described as follows

$$M_m \dot{v}_m + B_m v_m = f_h - u_s(t - \tau) \quad (6)$$

Where  $u_s$  is the control input of the slave given by a PD controller as

$$u_s = k_P(x_{md} - x_s) + k_D(v_{md} - v_s) \quad (7)$$

In which  $x_{md} = x_m(t - T), v_{md} = v_m(t - T)$  and  $k_P, k_D$  are controller gains.

## III. CONTROLLER DESIGN

The goal of our design is to find gains of the PD controller using LMI framework such that the position and the force of master and slave track each other, in presence of time-delays.

Substituting (3) to (7) into the master and slave dynamics equations, (1) and (2), closed-loop state equations of the system is described as follows:

$$\dot{x}(t) = Ax(t) + A_d x_d(t) + Bw \quad (8)$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_h}{M_h + M_m} & -\frac{B_h}{M_h + M_m} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{k_P}{M_s + M_e} & -\frac{k_D}{M_s + M_e} \end{bmatrix},$$

$$A_d = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{k_P}{M_h + M_m} & -\frac{k_D}{M_h + M_m} \\ 0 & 0 & 0 & 0 \\ -\frac{k_P}{M_s + M_e} & -\frac{k_D}{M_s + M_e} & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 1 \\ \frac{1}{M_h + M_m} \\ 0 \\ \frac{1}{M_h + M_m} \end{bmatrix}$$

Where  $w = f_h^*$  and the state-space vector  $x(t) \in R^n$  is defined as:

$$x(t) = [x_m(t) \ v_m(t) \ x_e(t) \ v_e(t)]^T \quad (9)$$

In which  $x_e = x_{md} - x_s$

**Theorem:** The tele-operation system (8) with control gains given at (11) is stable and has good performance for any constant delay of communication channel, if there exists symmetric positive definite matrices  $Q', S$  and  $K$  such that the LMI shown in (10) holds

$$R = \begin{bmatrix} A_0 Q' + B_0 K + Q' A_0^T + K^T B_0^T + S & A_1 K \\ K^T A_1^T & -S \end{bmatrix} < 0 \quad (10)$$

where

$$\tilde{K} = K(Q')^{-1} = \begin{bmatrix} k_P & k_D & 0 & 0 \\ 0 & 0 & k_P & k_D \end{bmatrix} \quad (11)$$

**Proof.** Consider the following Lyapunov-Krasovski functional

$$V(x) = x^T(t)Px(t) + \int_{t-h}^t x^T(\tau)Qx(\tau)d\tau \quad (12)$$

It is clear that the Lyapunov function candidate is Positive Definite (PD). Taking the derivative of this function we have

$$\frac{dV(x)}{dt} = \dot{x}(t)Px(t) + x^T(t)P\dot{x}(t) + x^T(t)Qx(t) - x^T(t-h)Qx(t-h) \quad (13)$$

For nominal conditions ( $f_h^* = 0$ ), substituting (8) in (13) yields

$$\dot{V} = \begin{bmatrix} x^T \\ x^T(t-h) \end{bmatrix} R \begin{bmatrix} x \\ x(t-h) \end{bmatrix} \quad (14)$$

where

$$R = \begin{bmatrix} A^T P + PA + Q & PA_d \\ A_d^T P & -Q \end{bmatrix}$$

If  $R < 0$ , which guarantees  $\dot{V} < 0$ , then tele-operation system is asymptotically stable, but the inequality  $R < 0$  is not linear, so in order to decorate it in a linear format we write it first in its dual form [16]

$$R' = \begin{bmatrix} AQ' + Q'A^T + S & A_d Q' \\ Q'A_d^T & -S \end{bmatrix} < 0, \quad (15)$$

where

$$Q' = P^{-1}, S = P^{-1}QP^{-1}$$

The matrices  $A, A_d$  can be factorized as

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_h}{M_h+M_m} & -\frac{B_h}{M_h+M_m} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -\frac{1}{M_s+M_e} \end{bmatrix} \begin{bmatrix} k_p & k_D & 0 & 0 \\ 0 & 0 & k_p & k_D \end{bmatrix} = A_0 + B_0 \tilde{K} \quad (16)$$

$$A_d = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{M_h+M_m} \\ 0 & 0 \\ -\frac{1}{M_s+M_e} & 0 \end{bmatrix} \begin{bmatrix} k_p & k_D & 0 & 0 \\ 0 & 0 & k_p & k_D \end{bmatrix} = A_1 \tilde{K} \quad (17)$$

By substituting (16), (17) in (15) we have

$$R' = \begin{bmatrix} A_0 Q' + B_0 \tilde{K} Q' + Q' A_0^T + Q' (B_0 \tilde{K})^T + S & A_1 \tilde{K} Q' \\ Q' (A_1 \tilde{K})^T & -S \end{bmatrix} < 0 \quad (18)$$

By choosing  $K = \tilde{K} Q'$

$$R' = \begin{bmatrix} A_0 Q' + B_0 K + Q' A_0^T + K^T B_0^T + S & A_1 K \\ K^T A_1^T & -S \end{bmatrix} < 0, \quad (19)$$

This inequality is linear and can be treated by LMI solvers. the gains of the controller then can be found from

$$\tilde{K} = K(Q')^{-1} \quad (20)$$

This proves the theorem.

#### IV. NUMERICAL EXAMPLES

To show the feasibility of the proposed method, consider the tele-operation system with the following parameters,

$$\begin{aligned} M_m &= 1, B_m = 1.5, M_s = 1, B_s = 1.5 \\ M_h &= 1, B_h = 1, K_h = 25 \\ M_e &= 1, B_e = 1, K_e = 25 \end{aligned} \quad (21)$$

Using the LMI Toolbox of the MATLAB, the gains are calculated as follows

$$K_p = 60, K_D = 20 \quad (22)$$

First assume that communication time delay is 100 msec. Fig.4 and Fig.5 show the simulation results of the master and slave responses. As shown in Fig.4, by applying the designed control signal to the system, the slave position track the master position after determined time-delay. Fig.5 shows the human and environment forces. From Fig.5, we can see that the force tracking is very good. So we have achieved very good transparency

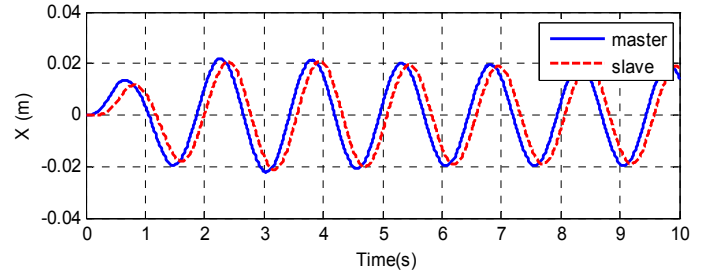


Figure 4. Position of the master and slave, ( $\tau = 100$  msec)

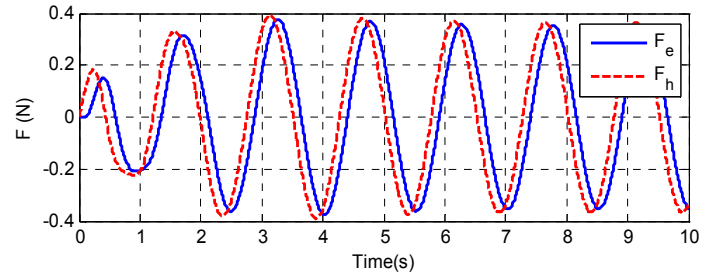


Figure 5. Human and environment forces, ( $\tau = 100$  msec)

Fig. 6 shows that position tracking is yet good, even when the time-delay is considered to be as much as 1000 msec. the same is true for force, shown in Fig. 7.

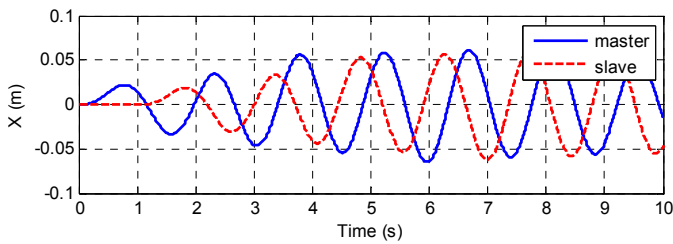


Figure 6. Position of the master and slave with, ( $\tau = 1000 \text{ msec}$ )

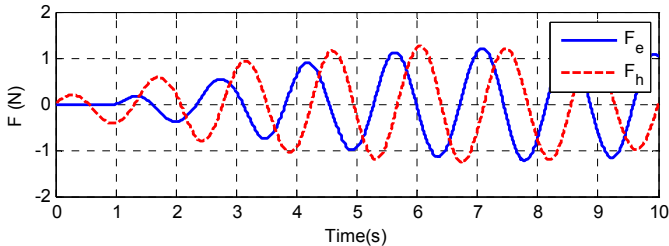


Figure 7. Human and environment forces, ( $\tau = 1000 \text{ msec}$ )

These simulations reconfirm the fact that the proposed method satisfies robust performance and stability in presence of time delay, regardless of its value.

## V. CONCLUSION

The problem of stabilizing a tele-operation system in presence of time delay in the communication channel is addressed. To achieve robust stability while acceptable tracking performance, a controller is designed via LMI. The LMI condition is independent of the time-delay of the network. Numerical example is used to show feasibility and performance of the proposed controller.

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