Adaptive Robust Backstepping Control Design for A Non-minimum Phase Model of Hard Disk Drives

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***Abstract*—In this paper, a new adaptive robust approach for non-minimum phase systems is proposed, based on the synthesis algorithm of dynamical backstepping design procedure. The previously proposed adaptive robust backstepping method [1] has a limitation in stabilization of non-minimum phase systems, which is removed in this paper. The dynamic model of the voice coil motor actuator, which is used in the read/write head of hard disk drive, is considered as a case study to apply the proposed method. A simple but accurate model of this system is presented the proposed control method is applied onto this model. Simulations are performed for the embedded control system of hard disk drives. The obtained results verify the effectiveness of the proposed control law in terms of transient performance, tracking errors, and disturbance rejection, in both track seeking and track following modes.**

Keywords- Dynamic Backstepping, Nonminimum Phase Model, Adaptive Robust Control, HDD Servo System

I. INTRODUCTION

Improvement of the head positioning performance in hard disk drives is necessary for increasing storage density, and reducing data access time. It is essential that the servo system achieve faster transition from start track to target track (seeking) and more precise positioning of the read/write head (following). Because of different objectives in seeking and following modes, mode switching control (MSC) is commonly used. In MSC, nonlinear controllers are frequently used for seeking task [1], and adaptive control [3], repetitive control and many other approaches are developed for following task [4][5]. A quick look at previous works shows a scamper from switching control to unified and smooth control. There are several attempts to develop such control algorithms to work for both seeking and following modes, such as [6] and [7].

Robust control is a well known control approach to overcome the uncertain dynamics and its unforeseen behavior. The system which is governed by a robust controller usually has a fast response and also a predetermined stability margin, but it takes a quite substantial control effort. The adaptive control as another major approach, deals mainly with the structural uncertainties such as parametric ones, and usually has neither a fast response nor a predetermined transient performance; however, it could reach a precise zero tracking error without any excess control effort. A systematic way to combine adaptive and robust control approaches to preserve the advantages of the both methods while overcoming their drawbacks has been proposed in [8]. It is very useful for applications like HDD servo system [9], in which two different objectives are essential. Within this line of research, an approach called IDCARC is developed in [10], while a combination of ARC control, with dynamic backstepping design is proposed in [11], in order to develop a unified controller for single and dual stage HDD servo system [1]. Although this approach is very promising in practice, it suffers from a stringent limitation that cannot be applied to non-minimum phase systems.

In this paper, we reclaim the ARC backstepping method to guarantee the stability of non-minimum phase systems. In order to accomplish this task a zero-placement algorithm is embedded in the design approach. In Section II the required theoretical preliminaries for the ARC backstepping design of [1] is reviewed. Section III is devoted to redesign approach for non-minimum phase systems. In Section IV the dynamic model for hard disk drives is described briefly, and high order linear models for HDD subsystems, and their experimentally verified uncertainty bounds are given. Then, two controllers are designed for the system using the conventional approach [1] and the proposed controller structure. Finally, the details of simulation studies and the comparison results are elaborated, and the concluding remarks are given in Section V.

II. ARC BACKSTEPPING FOR MINIMUM PHASE SYSTEMS

A. Problem Statement

Consider a SISO system described by

$$y(t) = \frac{B(s)}{A(s)}u(t) + \frac{D(s)}{A(s)}\Delta(y,t) + d(y,t) \tag{1}$$

in which $A(s) = s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0$, $B(s) = b_m s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0$, and $D(s) = d_l s^l + d_{l-1}s^{l-1} + \cdots + d_1 s + d_0$, where $l, m < n$. The plant parameters a_i's and b_i's are unknown constants while for simplicity d_i's are assumed to be known. Likewise, d_y is the output disturbance and $\Delta(y,t)$ represents any disturbance coming from the intermediate channels of the plant. In this representation, $\Delta(y,t)$ will be handled as follows [1]: first use the prior information about the nature of the disturbance $\Delta_n = q^T(y,t)\sigma$, where

$$q(y,t) = \left[q_p(y,t), q_{p-1}(y,t), \dots, q_1(y,t)\right]^T \in \mathbb{R}^{\boldsymbol{p}}$$

represents the known basis shape functions, and $\sigma = \left[\sigma_p, \sigma_{p-1}, \dots, \sigma_1\right]^T$ represents the unknown magnitudes. This nominal model will be explicitly used in the controller design to improve achievable performance. The disturbance modeling error, $\tilde{\Delta} = \Delta - \Delta_n$, will be dealt via robust feedback to manage a robust performance. With this approach of disturbance modeling, the state space representation of the plant (1) is given as follows: (Without loss of generality, assume $m < l$)

$$\dot{x}_1 = x_2 - a_{n-1}x_1 \tag{2}$$

$$\vdots$$

$$\dot{x}_{n-l} = x_{n-l+1} - a_l x_1 + d_l q^T \sigma + d_l \tilde{\Delta}$$
$$\vdots$$
$$\dot{x}_{\rho} = x_{\rho+1} - a_m x_1 + d_m q^T \sigma + d_m \tilde{\Delta} + b_m u$$
$$\vdots$$
$$\dot{x}_n = a_0 x_1 + d_0 q^T \sigma + d_0 \tilde{\Delta} + b_0 u$$
$$y = x_1 + d_y$$

The following standard assumptions indicate the framework of the system and nonlinearities in which the system is incorporate:

Plant is *minimum phase* with order n, relative degree ρ, and the sign of b_m is known. The extent of parametric uncertainties, $\theta \triangleq \left[-a_{n-1}, \dots, -a_0, b_m, \dots, b_0, \sigma_p, \dots, \sigma_1\right]$, and uncertain nonlinearities, $\tilde{\Delta}$, d_y and $\dot{d}_y$, are known, i.e.,

$$\begin{aligned} &\theta \in \Omega_\theta \triangleq \{\theta | \theta_{min} < \theta < \theta_{max}\} \\ &\tilde{\Delta} \in \Omega_\Delta \triangleq \{\tilde{\Delta} | \|\tilde{\Delta}\| < \delta(t)\} \\ &d_y \in \Omega_d \triangleq \{d_y | \|d_y\| < \delta_d(t)\} \\ &\dot{d}_y \in \Omega_f \triangleq \{\dot{d}_y | \|\dot{d}_y\| < \delta_f(t)\} \end{aligned} \qquad (3)$$

where θ_{max}, θ_{min}, δ, δ_d, δ_f are assumed to be known. Given the reference trajectory, $y_r(t)$, the objective of the controller design is to synthesize a control signal, $u(t)$ such that outpu,t $y(t)$ tracks the reference trajectory as closely as possible, in spite of various model uncertainties. The reference trajectory and its derivatives up to n are assumed to be known, bounded, and piecewise continuous. Since only the output, $y(t)$, is available for measurement, requiring the full state information of the system, we may design a *Kreisselmeier* observer [12], whose details are in [1].

B. Parameter Projection

The discontinuous projection based ARC design can be used to solve the stability problem of parameter adaptation and robustness for system (1), in spite of various uncertainties including uncertain dynamics. The parameter estimate $\hat{\theta}$ is updated through a parameter adaptation law which is given by $\dot{\hat{\theta}} = Proj_{\hat{\theta}}(\Gamma\tau)$, where the projection mapping $Proj_{\hat{\theta}}(\cdot)$ is defined by equation (4).

$$Proj_{\hat{\theta}}(\cdot) = \begin{cases} 0 & if\ \hat{\theta}_i = \hat{\theta}_{i,max}\ , \quad \cdot > 0 \\ 0 & if\ \hat{\theta}_i = \hat{\theta}_{i,max}\ , \quad \cdot < 0 \\ \cdot & otherwise \end{cases} \qquad (4)$$

The regressor τ, will be defined within the design procedure. This discontinuous projection method, used in developments of ARC controllers, guarantees that $\hat{\theta}$ stay in a pre-defined bounded region all time.

C. Controller Design Procedure

In [1] a systematic algorithm to design a Dynamic ARC output tracking controller from the backstepping perspective is described. Recursive backstepping procedure interlaces the choice of a *Lyapanov* function with the design of feedback control. It divides a design problem for the full system into a sequence of design problems for lower-order, even scalar, systems. The *Lyapanov* function encapsulates whole state of closed-loop system and also taking in account the whole uncertainties and disturbances; consequently, the performance of the system will be improved, and the robust stability of the system will be more reliable. In ARC control scheme parameter adaptation acts as a mechanism to select a value to each unknown parameter to reduce the uncertainty as much as possible, consequently, reduce the control effort exerted by the robust portion of controller. This leads to a more accurate model for the system in low frequency domains, and consequently, more precise tracking performance. The design procedure has been explained in [1].

III. Redesign For Non-minimum Phase Systems

To reclaim the method for non-minimum phase systems, we use a special zero placement method to first make the system minimum phase, and then apply ARC backstepping method. This controller can stabilize the non-minimum phase system, with the cost of affecting the transient response.

A. Zero Placement

For a strictly proper plant a method is represented in [13] for zero placement. Consider the system in the form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} u \qquad (5)$$

$$y = [C_1 \quad C_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad (6)$$

in which C_2 is a non-singular $\mathrm{m} \times \mathrm{m}$ matrix, in which m is number of system outputs, and accordingly other vectors and matrices have proper dimensions. The transmission zeros are given from

$$|\lambda I - A_{11} + A_{12} C_2^{-1} C_1| = 0 \qquad (7)$$

Now, a virtual output can be defined as below

$$w = y + M\dot{x}_1 \qquad (8)$$

in which M is a $\mathrm{m} \times (\mathrm{n} - \mathrm{m})$ matrix called *measurement matrix*. According to (5) and (6) we can write

$$w = [F_1 \quad F_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad (9)$$

where,

$$F_1 = C_1 + MA_{11} \qquad (10)$$

$$F_2 = C_2 + MA_{12} \qquad (11)$$

Therefore, the assigned transmission zeros with the virtual output can be calculated from this equation

$$|\lambda I - A_{11} + A_{12} F_2^{-1} F_1| = 0 \qquad (12)$$

This equation could be considered as a pole placement problem for controllable pair (A_{11}, A_{12}) with state feedback

$$K = F_2^{-1} F_1 \qquad (13)$$

Using equations (10) to (13) we can obtain

$$K = (C_2 + MA_{12})^{-1}(C_1 + MA_{11}) \qquad (14)$$

Therefore,

$$M = (C_2 K - C_1)(A_{11} - A_{12} K)^{-1} \qquad (15)$$

Finding a proper matrix K yields an appropriate matrix M. Hence the system with virtual output w will have transmission zeros assigned in the desired locations.

For proper systems the method is simpler. Suppose the system described by an arbitrary representation

$$\dot{x} = Ax + Bu \qquad (16)$$

$$y = Cx + Du \qquad (17)$$

in which $D \neq 0$. The inverse system equations can be written as

$$\dot{z} = (A - BD^{-1}C)z + BD^{-1}y \tag{18}$$

$$u = -D^{-1}Cz + D^{-1}y \tag{19}$$

Therefore, the transmission zeros are given from the equation

$$|\lambda I - A + BD^{-1}C| = 0 \tag{20}$$

Now the virtual output will be

$$w = y + M\dot{x} \tag{21}$$

in which, M is the measurement matrix with dimension $\mathrm{m} \times \mathrm{n}$. Using (16) and (17) the virtual output can be written as

$$w = Fx + Gu \tag{22}$$

Where,

$$F = C + MA \tag{23}$$

$$G = D + MB \tag{24}$$

Moreover, the new transmission zeros are given from

$$|\lambda I - A + BG^{-1}F| = 0 \tag{25}$$

This equation could be considered as a pole placement problem for controllable pair (A, B) with state feedback

$$K = G^{-1}F \tag{26}$$

Using equations (23) to (26) we can obtain

$$K = (D + MB)^{-1}(C + MA) \tag{27}$$

Therefore

$$M = (DK - C)(A - BK)^{-1} \tag{28}$$

By this means the system transmission zeros are placed to desired locations.

B. *Tracking problem with zero placement*

An important point in the method of zero placement is that the same steady state values for both real and virtual outputs can be achieved. We know that

$$\lim_{t\to\infty} \dot{x} = 0 \tag{29}$$

hence

$$\lim_{t\to\infty} w = \lim_{t\to\infty}(y + M\dot{x}) = \lim_{t\to\infty} y \tag{30}$$

In other words this strategy does not affect on the closed loop steady state tracking error.

C. *Augmentation of zero placement part to the controller*

Block diagram of the closed loop system with ARC backstepping controller for minimum phase plants is shown in Figure 1. By Augmentation of zero placement block, the closed loop system is shown in Figure 2. Since the states of the original system and the system with virtual output are equal, we can use the real output y and the original model for observation. Also the representation used in zero placement process is different from the representation used in backstepping controller design. Therefore, we use a similarity transform T to obtain appropriate states. In here, we will represent an algorithm to find this transformation matrix.

Note that if we have proper system we can use same representations for backstepping controller and zero placement blocks. So we should ignore the transformation block.

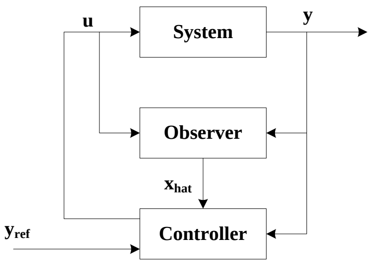

Figure 1. Block diagram of closed loop system for minimum phase plant

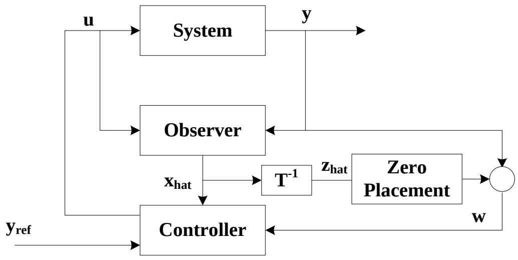

Figure 2. Block diagram of closed loop system for nonminimum phase plant

To find matrix T first rewrite two representations using new notations. Consider the representation used in backstepping controller design as

$$\dot{x} = Ax + Bu \tag{31}$$

$$y = Cx = [1 \quad 0 \quad \cdots \quad 0]x \tag{32}$$

and the representation using in zero placement as

$$\dot{z} = \bar{A}z + \begin{bmatrix} 0 \\ \bar{b}_2 \end{bmatrix} u \tag{33}$$

$$y = \bar{C}z \tag{34}$$

Construct the transformation matrix T as follows

$$x = Tz \tag{35}$$

in which,

$$\bar{A} = T^{-1}AT \tag{36}$$

$$T\bar{A} = AT \triangleq \mathcal{A} \tag{37}$$

Then

$$\begin{aligned} \mathcal{A}_{ij} &= t_{i1}\bar{a}_{1j} + t_{i2}\bar{a}_{2j} + \cdots + t_{in}\bar{a}_{nj} \\ &= a_{i1}t_{1j} + a_{i2}t_{2j} + \cdots + a_{in}t_{nj} \end{aligned} \tag{38}$$

From all $\mathcal{A}_{ij}$ elements we obtain the linear system

$$\sum_{k=1}^{m} \left(t_{ik}\bar{a}_{kj} - a_{ik}t_{kj}\right) = 0 \tag{39}$$

The equations in linear system (39) are not linearly independent; therefore, some equations should be removed. Fortunately we can obtain remaining parameters from these

$$\bar{B} = T^{-1}B \tag{40}$$

$$\bar{C} = CT \tag{41}$$

According to (32) and (33) and using (40) and (41) we have

$$B = T\bar{B} = T\begin{bmatrix} 0 \\ \bar{b}_2 \end{bmatrix} \Rightarrow T(:, n) = \frac{1}{\bar{b}_2}B \tag{42}$$

$$\bar{C} = CT = [1 \quad 0 \quad \cdots \quad 0]T \Longrightarrow T(1,:) = \bar{C} \tag{43}$$

Thus we can obtain all elements of matrix T.

An important analysis is obtained from equality of states of both minimum phase and non-minimum phase systems. The backstepping controller stabilizes the minimum phase system and its states would be bounded, so the non-minimum phase system's output would be also bounded.

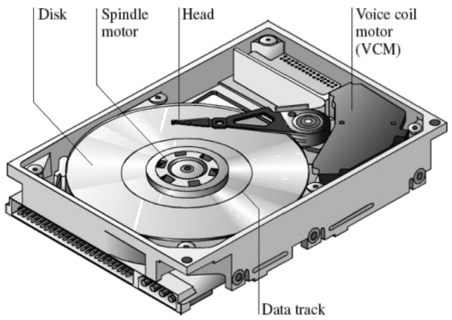

Figure 3. HDD components

IV. Simulation Results

A. HDD Servo System

A picture of a HDD servo system is illustrated in Figure 3. The voice coil motor moves the carriage, base plates and suspensions and heads all together, and moves the read/write head to achieve desired track on disk surface. In order to apply linear robust controllers to this problem, VCM is represented by a linear model and multiplicative uncertainty, using a systematic linear identification scheme. Such a linear model for VCM is found in [1], [9] and [14]. Suppose that the system is defined by using the following perturbation to the nominal plant P_0:

$$P(s) = \big(1 + \Delta(s)W(s)\big)P_0(s) \tag{44}$$

In this equation $W(s)$ is a stable transfer function indicating the upper bound of the uncertainty and $\Delta(s)$ indicates the admissible uncertainty block, which is a stable but unknown transfer function with $\|\Delta\|_\infty < 1$. In this general representation, $\Delta(s)$ describes the normalized perturbation of the true plant from nominal plant, and $W(s)$ represents the upper bound of the amplitude of the uncertainty profile with respect to frequency.

$$\left|\frac{P(j\omega)}{P_0(j\omega)} - 1\right| = |W(j\omega)|, \forall\omega \tag{45}$$

Nominal plant P_0 can be evaluated experimentally, through a series of frequency response estimates of the system in the operating regime. Linear identification for the system can be applied with different input amplitudes, while their outputs are measured and logged. By minimizing the least squares of the prediction error, from the set of input-output information, a set of linear models are estimated for the system. The uncertainty profile upper bound $W(s)$, is then obtained using Equation (45), while the nominal plant P_0 is selected from the average fit over all the individual identified plants. By this means, not only the nominal plant of the system is obtained, but also a measure of its perturbations, will be encapsulated by multiplicative uncertainty representation.

Using this technique the linear model is derived from experimental frequency response estimates of system. The nominal model of the VCM-actuator which is shown in Figure 4 is a stable 12th order system. This model has 3 RHP zeros. The uncertainty profile of the model is experimentally derived and using equation (45) the uncertainty profiles are calculated and illustrated in Figure 5.

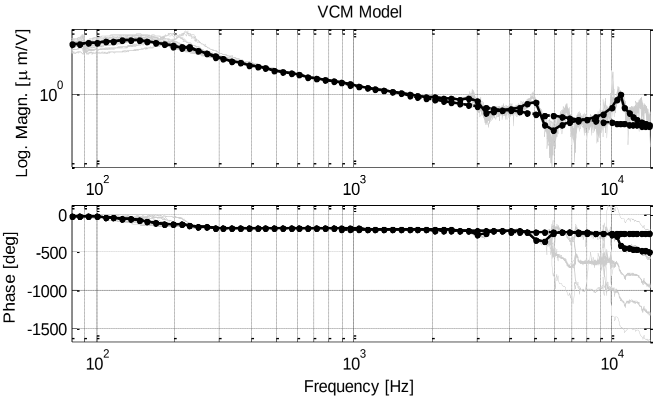

Figure 4. The frequency response estimates of the VCM (gray), high order nominal model (solid), and the reduced model (dashed).

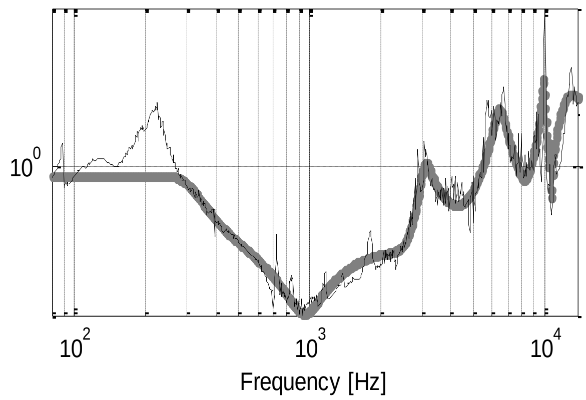

Figure 5. The multiplicative uncertainty profile of VCM actuator (gray line)

B. Model Order Reduction

Due to increasing demands on quality and productivity of industrial systems, more exact and complicated mathematical models are needed to demonstrate more exact system dynamics. Also, some of new design methods, like modern robust or adaptive ones, spontaneously lead to a high order controllers. On the other hand, in the implementation, high order system requires high computational cost, difficult commissioning, poor reliability and potential problems in the maintenance. The dynamic adaptive backstepping controller is designed based on a reduced order model of the system. Hence, the full order model of the system and its uncertainty profile is used in simulation to verify the robustness of the proposed algorithm in presence of modelling uncertainty.

We can reduce the model of VCM to a 4th order system. Referring back to Figure 4 the bode diagram of the reduced order model of VCM of 4th order is plotted versus the nominal high order model. The poles and zeros of the reduced-order system are shown in the TABLE I. The DC gain of this transfer function is 96.

TABLE I. Poles Zeros Pattern of VCM Reduced Order Model

Poles	Zeros
$-234.7 \pm 908.7i$	2.436×10^4
$-132.9 \pm 1437.8i$	$-196.611 \pm 1410.917i$

This model is non-minimum phase and will be used in the proposed controller synthesis. In order to apply conventional ARC method, we need to have a minimum phase model. By means of *Hankel Norm Order Reduction* one can obtain a minimum phase reduced model for the system. The poles and zeros of the reduced-order system are shown in the TABLE II and the DC gain of this transfer function is 95. Note that although this model can be used for the synthesis, the major effect of unstable zeroes of the system is neglected in the design procedure within the conventional approach.

TABLE II. POLES ZEROS PATTERN OF VCM REDUCED ORDER MODEL USING HANKEL NORM REDUCTION

Poles	Zeros
$-254.070 \pm 908.271i$	-5.1010×10^4
$-86.218 \pm 1480.880i$	$-128.547 \pm 1490.813i$

C. ARC Control of HDD Servo System

In this section the backstepping procedure is applied first on the minimum phase model, and then the proposed method is used for the non-minimum phase model. Also the effect of the choice of desired zero in zero placement routine is analyzed on the closed-loop performance.

In the simulations, the reference trajectory is generated based on the idea of Structural Vibration Minimized Acceleration Trajectory (SMART), in which the residual vibration of the suspension is minimal. The solution of this problem can be analytically obtained and optimal reference trajectory can be achieved [4]. Simulation studies have been performed to verify the effectiveness of the proposed controller in terms of tracking errors, and disturbance rejection. In order to compare simulation results for representatives of different controllers proposed for such system as in literature [9], and [15], the following performance indices are used:

- $e_M = \max_t\{|e(t)|\}$, is the maximum absolute value of the tracking error.
- $e_F = \max_{T_{f-1} \le t \le T_f}\{|e(t)|\}$, is the maximum absolute value of the tracking error during the last one millisecond.
- $t_{ac} = \max_t\{t: (|y(t) - y_r(\infty)| > 0.5 w_{tr})\}$ is the time that the absolute error of output from $y_r(\infty)$, final value of the reference trajectory, reaches to half of one track's width (w_{tr}) and remains within this limit until the end of simulation. This measure is an indication of the *access time*, the time that read/write head is considered to be located on the target track and it can begin to read/write, and shows how fast is the system in tracking mode.
- $\mathcal{L}_2[e] = \sqrt{\frac{1}{T_f}\int_0^{T_f}|e|^2 dt}$ is an average tracking performance index for the entire error curve $e(t)$. T_f represents the total simulation time here.
- $\mathcal{L}_2[u] = \sqrt{\frac{1}{T_f}\int_0^{T_f}|u|^2 dt}$, is the mean value of the control input.

In the simulations the trajectory to reach to track 648, with each track's width equal to $3.945\ \mu m$, is considered. The total rise time is 5 milliseconds. The goal is to reach to at most half of track width ($= 1.9725\ \mu m$) for tracking error, with the lowest possible access time. The faster the access time, the faster the HDD can perform.

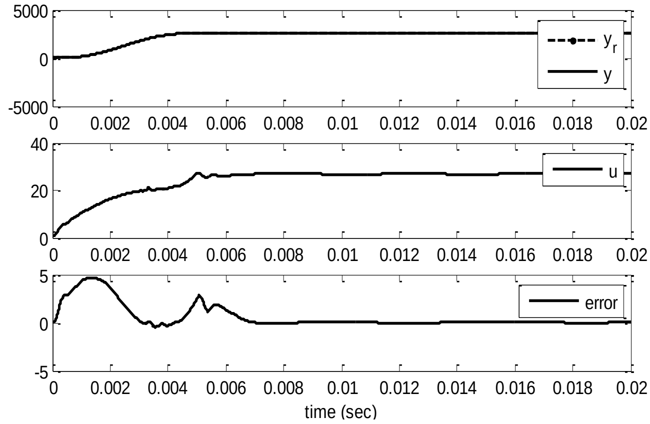

Figure 6. Closed loop system response (top), control signal (middle) and tracking error (bottom) for the controller design in minimum phase case

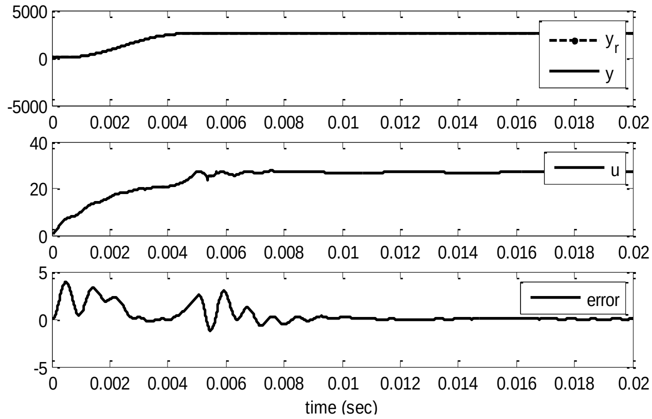

Figure 7. Closed loop system response (top), control signal (middle) and tracking error (bottom) for the controller design in non-minimum phase case (1st choice)

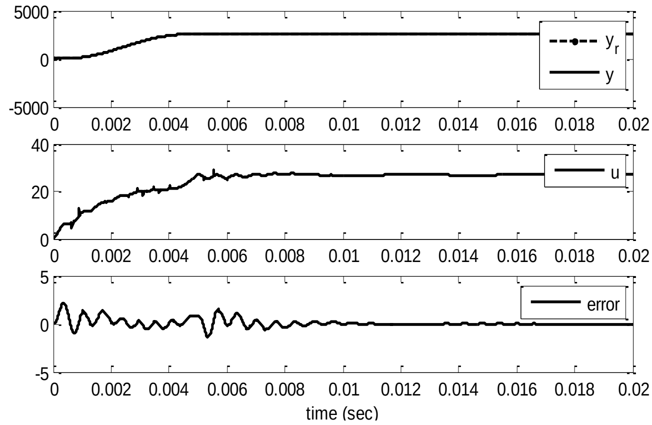

Figure 8. Closed loop system response (top), control signal (middle) and tracking error (bottom) for the controller design in non-minimum phase case (2nd choice)

First we employ the backstepping method for the minimum phase model. The simulation result is shown in Figure 6. Next we use the proposed method to control the non-minimum phase model. We need to choose a desired zero to replace the RHP zero according to it. In the first simulation the symmetry of the RHP zero are preferred. Result is shown in Figure 7. Then we choose another zero location to see whether a better response is achievable. Zero placement procedure can replace all zeros, so we can also change the LHP zeros' locations. It would give us larger range of responses. Through a trial and error exercise, the best response is almost obtained by transferring the RHP zero to a place two times farther from imaginary axis in left plan. Result of this simulation is shown in Figure 8. Note that because of the saturation on the input, you cannot choose any desired zeros. Your choices have not to affect the stability of the response of the system. The saturation bounds considered in here is $\pm 30\ volts$.

D. Discussions

As it is seen in Figure 6 the response is acceptable for both seeking and tracking modes. The tracking error converges to almost zero and the response has a smooth transient. Using zero placement approach leads to a decreased maximum error value, in the expense of increasing the oscillations of the response (Figure 7 and Figure 8). By choosing a better desired zero the closed loop response leads to a better tracking error and better access time (Figure 8). The significant improvement in response can be seen by comparing Figure 8 to that given in Figure 7.

The robustness of this method is verified by running the simulations for several different model uncertain samples. The performance indices for two worst uncertain models are shown in TABLE III. Note that the same uncertain models are used for all three scenarios, as the uncertain plant in the closed loop system. First examine the response to the two worst case uncertain models indicated as Set 1 and Set 2, for three controllers. As it is seen the difference of two consecutive rows are much, and the controllers are able to control the uncertain models quite well with almost the same behavior. Next examine the values of e_f for three controllers, and compare the values to the required steady state error of $1.9725\ \mu m$. As it is seen all three controllers are able to reduce the tracking error for the uncertain model to less than $1/50$ of the required precision, which is very promising for future design of HDD's. The effect of considering non-minimum phase model in the design of controller is better appreciated by examining the transient error e_M and access time t_{ac}. As it is seen in the result, the maximum error obtained in the last design, is twice better than conventional design, while access time of this controller is about 1.5 times faster than the conventional controller. Since the required access time is about the trajectory's rise time $(5\ ms)$, the proposed controller can certainly meet this requirement. This is obtained while the norm of control input is not changed much. This increase of performance can lead to a HDD with more than 50 times better precision, and about 1.5 times more speed.

TABLE III. PERFORMANCE INDICES FOR THE SIMULATIONS

Performance Index		e_M (μm)	e_f (μm)	t_{ac} (ms)	$\mathcal{L}_2[e]$ (μm)	$\mathcal{L}_2[u]$ $(volt)$
MP case	Set 1	4.6861	0.0100	6.71	1.3663	24.808
	Set 2	4.6963	0.0127	6.92	1.4583	24.874
NMP case (1st choice.)	Set 1	3.8478	0.0174	6.07	0.9874	24.810
	Set 2	3.8580	0.0192	6.21	1.0024	24.875
NMP case (2nd choice.)	Set 1	2.1346	0.0331	4.82	0.4338	24.812
	Set 2	2.1426	0.0350	4.93	0.5524	24.876

V. CONCLUSIONS

In this paper, an alternative algorithm for synthesis of dynamical backstepping design procedure is proposed to develop a new scheme for ARC controller of non-minimum phase systems. In the original method, the dynamic backstepping controller ensures the robustness of the tracking error performance, while parameter adaptation acts as a mechanism to select a value to each unknown parameter to reduce the uncertainty as much as possible; consequently, this combination reduces the control effort exerted by the robust part of the controller. Finally, the lack of stability for non-minimum phase systems is rectified using a zero placement procedure. Choosing the desired zeros provides a new design parameter to optimize the response in arbitrary criteria. Results of simulations, executed for the read/write head embedded control systems of hard disk drives (HDD), shows that the stable responses of non-minimum phase systems and effectiveness of this structure in terms of transient and steady state performance, tracking errors, and disturbance rejection, in both track seeking and track following modes. Especially the maximum error obtained in the final design is twice better than conventional design, while access time of this controller is about 1.5 times faster than the conventional controller. This increase of performance can lead to a HDD with more than 50 times better precision, and the required reachable speed.

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