Modelling and Control of Fully Constrained Cable Driven Robots with Flexible Cables

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Abstract: *In this paper modelling and control of cable-driven redundant parallel manipulators with flexible cables are studied in detail. In this type of manipulators the cables should remain in tension in the whole workspace. Based on new results, cables can be modelled as an axial spring. With this assumption the system is modelled using Lagrange's formulation. Furthermore, internal forces are introduced and incorporated in the proposed control algorithm. This algorithm is formed in cable length coordinates in which the internal forces play an important role. Finally, the closed loop system is proved to be stable, through Lyapunov analysis, and the performance of the proposed algorithm is studied through simulation.*

Keywords: Cable driven robot, Redundancy, Modelling, Vibration, Lyapunov stability.

1. Introduction

The first generation of industrial robots consists of rigid links connected in series by several joints. This is a general structure of serial manipulators. Nowadays, this type of mechanisms is widely used in industrial applications [1]. The extension of using robots in different industrial applications provides some necessities such as high acceleration and high accuracy. Based on structure of serial robots, these requirements cannot be fully satisfied, and to remedy this problem, parallel structures were proposed in the industries. In a parallel mechanism, all actuators are placed on the base of the robot, therefore, each actuator only carries part of the payload and does not need to support the weight of other actuators in comparison to serial robots. As a result, relatively small robots with small actuators can achieve high speed motion. Furthermore, since joints angle errors are not accumulated, this kind of robots will have high position accuracy. Though, parallel robots overcome some serial robot drawbacks, they suffer from a few problems, the most important one is their limited workspace. In general, parallel mechanisms workspace is mainly limited due to links conflicts [2]. Another drawback is singularity. Parallel robots lose their stiffness in singular positions and they may get one or more uncontrollable degrees of freedom, and as a result achieve undesirable motion [3].

To overcome these problems, from three decades ago a new attitude was formed in the design and implementation of parallel robots [4]. This attitude was based on using cables instead of rigid links, and to implement cable driven robots. A cable driven robot consists of a moving end-effector and a number of active cables connected to the end-effector. These cables are fixed on the base with actuating motors and pulleys. While the cable length is changing, the position and orientation of the end-effector is pulled toward its desired values. The most important limitation of cable robots is that the cables suffer from unidirectional constraints that can only pull and not push, while general parallel robots have actuators that can provide bi-directional tension.

In cable driven robots the cables must be in tension in the whole workspace. Based on this fact cable driven robots can be sorted in two types [5]; under constrained: which means gravity or a passive force is needed to makes the cables in tension and the position of end-effector is determined by this force. The second kind is fully constrained manipulators. In this type, the knowledge of cables lengths will determine the end-effector position and orientation. In addition, actuator redundancy is needed to ensure that all of the cables are in tension.

Cable driven robots have some advantages and drawbacks compared to conventional robots. Because of using cables, they have the great potential to use in very large workspace applications such as large adaptive reflectors and skycam [6,7]. Moreover, they have light moving components and low inertia, capable in heavy payloads similar to RoboCranes [8], ease in assembly/ disassembly and low cost. A major challenge is the nonlinear behaviour of the cables. Cables are usually flexible and have to encounter some unavoidable situations such as elongation because of the cable driven robot character. This flexibility may lead to position and orientation errors. Moreover, the system might be exposed to undesirable disturbances which may lead to vibration, and cause the whole system to be uncontrollable.

Because of the complication of this subject, research attempts on the effect of cable flexibility on modelling, optimal design and control are very limited and in most of reported research these effects are simply neglected. During last few decades, some researchers have studied cable behaviour in cable robots. Most of them have focused only on the kinematics and stiffness of the system. Kozak in [9], considers the mass of the cables and by using a static model of cables, shows how cable sagging affect the kinematics and stiffness of the system. In [10] a static model of cable is proposed and static deformation of cables is achieved. Behzadipour introduced a four springs model for cable and achieves necessary and sufficient conditions for stability of system based on positive definiteness of the robot stiffness matrix [11].

In this paper, considering axial flexibility in cables, we present a new dynamic model for cable driven robots, while all the cables are in tension. For the first time in this modelling both the cables lengths with and without tension are considered as the describing states in the modelling process. By using the obtained model, the control of the system is studied. Keeping the cables in tension is a critical point that must be noticed during control of the system. Next, the stability of the system is analysed through Lyapunov second method and it is proven that the system with proposed control algorithm is stable. Finally the performance of the proposed algorithm is examined through simulation.

2. Dynamics Analysis

Cable driven robots are closed kinematics chain mechanisms that use cable in their actuators. Though, the cable characteristics have been studied from long time ago, especially in civil engineering, using cables in parallel robots demonstrate a quite new application, compared to that studied in civil engineering. Generally, in civil engineering cables are heavy and bulky materials, whose static analysis is studied in order to stabilize the bridge type structures [12, 13]. However, in cable driven robots the cables are very light and change of length moves the end-effector toward its desired position. For this reason in many robotics applications, cables mass have been neglected and cable has been considered as a rigid element [14,15]. With these assumptions the dynamics of cable driven robot is reduced to the end-effector dynamics. However, in practice using this assumption will lead to some inaccuracies in control especially the stability of the manipulator. In this paper a more precise model of the cable driven robot considering cable flexibility is derived and being used in the controller design and stability analysis.

Stiffness analysis of the cable robots with flexible cables, may be seen in Behzadipour's and Kozak's works [9, 11]. Moreover, Zi and Merlet have studied the effect of flexibility on the system kinematics [10, 16]. To model the vibration due to flexibility in cables, Agrawal has used wave equation to model the cable vibration, providing the cables length are constant [17]. However in practice this assumption is also not true for cable driven robots. In the dynamic modelling of cable robots, this point should be noticed that a complete dynamic model of cable robots is very complicated. Furthermore, since the obtained model should be used in controller design strategies, such complicated models are useless for this objective, although they can accurately describe dynamic intrinsic characteristics of cables. Thus, in practices it is proposed to include only the dominant effects of flexible cables in the analysis.

Using natural frequencies of system, Diao and Ma have shown in [18], that in fully constrained cable driven robots the vibration of cable manipulator due to the transversal vibration of cables can be ignored in comparison to that of axial flexibility of cables. In other words, it has been justified to just model the cable as an axial spring in cable driven robots. By this means, this model can describe the dominant dynamic characteristics of cable and can be used in the dynamic model of cable robot. Based on this observation, in this paper axial spring are used to model cable dynamics.

2.1 Robot Dynamics with Ideal Cables

In this section let us first assume that the mass and the flexibility of the cables can be ignored, since they are much smaller and lighter than other mechanical parts. With this supposition the dynamics of cable driven robot reduced to that of the end-effector. Therefore, the dynamics of system can be expressed by the following vector equation [14, 15]:

$$M(x)\ddot{x} + N(x,\dot{x}) = J^T\tau \tag{1}$$

In which,

$$N(x,\dot{x}) = C(x,\dot{x})\dot{x} + G(x) \tag{2}$$

Where,

$M(x)$: Mass matrix of the system,

$C(x,\dot{x})\dot{x}$: Coriolis and centrifugal terms,

$G(x)$: Vector of gravity terms,

J : Jacobian matrix of system,

x : Vector of generalized coordinates.

On the other hand, the actuators dynamics is represented by

$$I\ddot{q} + D\dot{q} + r\tau = u \tag{3}$$

Where,

q : Angles vector of motors shaft

I : Actuator moments of inertia matrix

D : Actuator viscous friction matrix
r : The radius of drums
τ : Cable tension vector
u : Motor torque vector

As for the position reference, define all q to be zero when the end-effector centroid is located at the center of the frame; from this configuration positive angle q will cause a change ΔL in cable lengths, so we have:

$$rq = \Delta L = L - L_0 \Rightarrow q = r^{-1}(L - L_0) \tag{4}$$

By differentiating and using manipulator Jacobian definition $\dot{L} = J\dot{x}$:

$$\dot{q} = r^{-1}\dot{L} = r^{-1}J\dot{x} \tag{5}$$

$$\ddot{q} = r^{-1}J\ddot{x} + r^{-1}\dot{J}\dot{x} \tag{6}$$

Using equations (6), (5), (3) and (1) and some manipulations we can show that:

$$M_{eq}(x)\ddot{x} + N_{eq}(x,\dot{x}) = J^T u \tag{7}$$

in which,

$$M_{eq} = rM(x) + r^{-1}J^T IJ \tag{8}$$

$$N_{eq} = rN(x,\dot{x}) + r^{-1}J^T I\dot{J}\dot{x} + r^{-1}J^T DJ\dot{x} \tag{9}$$

As we see, actuator dynamics is transferred to Cartesian space by Jacobian matrix, which is a map from joint space to Cartesian space.

2.2 Robot Dynamics with Flexible Cables

In cable driven robots when the flexibility in cables is considered, position of actuator (motor rotation for opening the cable) is not directly related to end-effector position. Thus for such models, position of actuators and position of end-effector may be considered as system states vector. In other words, both the cable lengths before tension and the cable lengths after tension that are related to end-effector position by Jacobian matrix) may be used as independent variables into the dynamics analysis. New research results have shown that in fully constrained cable robots, dominant dynamics of cables are longitudinal vibration [18], therefore, axial spring model can suitably describe the effects of dominant dynamics of cable.

In order to model a cable driven robot with n cables assume that: $L_{1i}: i = 1,2,\ldots,n$ denotes the length of i'th cable with tension and $L_{2i}: i = 1,2,\ldots,n$ denotes the length of the i'th cable without tension. If the system is rigid, then $L_{1i} = L_{2i}$, $\forall i$. Use the notation:

$$L = (L_{11},\ldots,L_{1n},L_{21},\ldots,L_{2n})^T = (L_1|L_2)^T$$

Furthermore, if the flexibility is modeled with a linear axial spring with constant k_i, then the potential energy of system can be expressed by

$$P = P_0 + P_1 \tag{10}$$

In this equation P_0 is the potential energy of rigid robot and P_1 is the potential energy of cable. Using linear axial spring model for cable, the total potential energy of cables is

$$P_1 = \frac{1}{2}(L_1 - L_2)^T K(L_1 - L_2) \tag{11}$$

Where K is the stiffness matrix of cables. Kinetic energy of system is

$$K = \frac{1}{2}\dot{x}^T M(x)\dot{x} + \frac{1}{2}\dot{q}^T I_m \dot{q} \tag{12}$$

In which x is the generalized coordinate in cartesian space, q is the motor shaft position, $M(x)$ is the mass matrix and I_m is the actuator moments of inertia. The lagrangian is expressed by

$$\mathcal{L} = K - P = \frac{1}{2}\dot{x}^T M(x)\dot{x} + \frac{1}{2}\dot{q}^T I_m \dot{q} - P_0 - \frac{1}{2}(L_1 - L_2)^T K(L_1 - L_2) \tag{13}$$

The equations of motion of the system can be written using Euler-Lagrange formulation:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{y}_i}\right) - \frac{\partial \mathcal{L}}{\partial y_i} = Q_i \tag{14}$$

The final equation of motion can be written in the following form:

$$M(x)\ddot{x} + N(x,\dot{x}) = J^T K(L_2 - L_1) \tag{15}$$

$$I_m \ddot{q} + rK(L_2 - L_1) + D\dot{q} = u \tag{16}$$

in which,

$$N(x,\dot{x}) = C(x,\dot{x})\dot{x} + G(x) \tag{17}$$

$$L_2 - L_0 = rq \tag{18}$$

In these equations $\dot{L}_1 = J\dot{x}$, and furthermore,

x : Vector of generalized coordinates
q : Vector of angles of motors shaft
K : Stiffness matrix of cables
J : Jacobian matrix
$M(x)$: Mass matrix of the rigid system
$C(x,\dot{x})\dot{x}$: Coriolis and centrifugal terms
$G(x)$: Vector of gravity terms
I_m : Actuator moments of inertia matrix
D : Actuator viscous friction matrix
r : The radius of drums
u : Motor torque vector

Equations (15) and (16) represent cable driven robot as a nonlinear and coupled system. This representation includes both rigid and flexible subsystems and their interactions.

3. Controller Design

3.1 Internal Forces

It is well known in parallel robots that Jacobian transpose relates the resultant forces f applied on the end-effector to the cable tensions τ [3]:

$$f = J^T \tau \tag{19}$$

Since in cable driven robots actuator redundancy is a requirement, the number of the cable actuators are greater than the degrees of freedom, and therefore, the Jacobian matrix is not square. The inverse relation to calculate the tension in cables from the resultant force using pseudo inverse is given by [19]:

$$\tau = J^{T^\dagger} f + \left(I - J^{T^\dagger} J^T\right) c \tag{20}$$

In which, $J^{T^\dagger}$ denotes the pseudo inverse of the matrix J^T. The second term is generated from the null space of J^T which is denoted by internal forces among the cables. Notice that since the internal forces lie in the null space of J^Tit does not contribute into the driving force to the end-effector, and it only produce tension in cables in order to keep all the cables in tension. Internal force plays an important role in our proposed control Algorithm.

3.2 Control algorithm in cable length space

In this section proposed control algorithm in cable length space is discussed. In this algorithm we use internal force to ensure that all cables are in tension. A cable driven robot must satisfy this condition in its whole workspace. As in any parallel mechanisms, inverse kinematics of cable robots can be easily derived. As a result, a desired angle vector q_d which corresponds to desired position and desired orientation of end-effector in task space, may be easily obtained. By using desired set point vector q_d, the control input u is proposed to be:

$$u = K_p(q_d - q) - K_v\dot{q} + Q + rQ_G \tag{21}$$

where, $K_p(n \times n)$ [n is the number of cables] and $K_v(n \times n)$ denote feedback gain matrices. The term $Q(n \times n)$ denote internal force vector and satisfy

$$J^T Q = 0. \tag{22}$$

It is important to note that the vector Q does not contribute into motion of the end-effector, and only causes internal forces in the cables. This term ensures that all cables remain in tension in the whole workspace. Moreover, this term increases the stiffness of the system, and as a result, minimizes the vibration in transversal direction of the cables. The final term Q_G is added to compensate the gravitational force. This vector must satisfy

$$J^T Q_G = G(x) \tag{23}$$

In the following section we discuss on stability of system based on this proposed control algorithm.

3.3 Stability Analysis

To show that the control law given in equation (21) achieves set point tracking, consider the Lyapunov function as following:

$$V = \frac{1}{2}\dot{q}^T I_m \dot{q} + \frac{1}{2}\dot{x}^T M(x)\dot{x} + \frac{1}{2}(L_1 - L_2)^T K(L_1 - L_2)$$
$$+ \tfrac{1}{2}(q_d - q)^T K_p (q_d - q) \tag{24}$$

The Lyapunov function is generated using the total energy in the system. The first and second terms in equation (24) are the kinetic energies of the robot and actuators and the third term is the potential energy of the cables. Since q_d is constant, the time derivation of the Lyapunov function V is given by:

$$\dot{V} = \dot{q}^T I_m \ddot{q} + \dot{x}^T M(x)\ddot{x} + \frac{1}{2}\dot{x}^T \dot{M}(x)\dot{x}$$
$$+(L_1 - L_2)^T K(\dot{L}_1 - \dot{L}_2) + (q_d - q)^T K_p(-\dot{q}) \tag{25}$$

Substitute equation (15) and (16) in (25):

$$\dot{V} = \dot{q}^T[u - D\dot{q} - rK(L_2 - L_1)]$$
$$+\dot{x}^T[J^T K(L_2 - L_1) - C(x,\dot{x})\dot{x} - G(x)] + \frac{1}{2}\dot{x}^T \dot{M}(x)\dot{x}$$
$$+(L_1 - L_2)^T K(\dot{L}_1 - \dot{L}_2) + (q_d - q)^T K_p(-\dot{q}) \tag{26}$$

Moreover, substitute the proposed control effort (21):

$$\dot{V} = \dot{q}^T\left[K_p(q_d - q) - K_v\dot{q} + Q + rQ_G\right] - \dot{q}^T D\dot{q} -$$
$$\dot{q}^T rK(L_2 - L_1) + \dot{x}^T[J^T K(L_2 - L_1) - C(x,\dot{x})\dot{x} - G(x)]$$
$$+\frac{1}{2}\dot{x}^T \dot{M}(x)\dot{x} + (L_1 - L_2)^T K(\dot{L}_1 - \dot{L}_2)$$
$$-(q_d - q)^T K_p\dot{q} \tag{27}$$

To simplify this equation, use the robot mass matrix property [3]:

$$\dot{x}^T\left(\dot{M}(x) - 2C(x,\dot{x})\right)\dot{x} = 0 \tag{28}$$

Then

$$\dot{V} = -\dot{q}^T K_v\dot{q} - \dot{q}^T D\dot{q} + \dot{q}^T(Q + rQ_G) - \dot{x}^T G(x)$$
$$+[\dot{x}^T J^T K(L_2 - L_1) - \dot{q}^T rK(L_2 - L_1)$$
$$+(L_1 - L_2)^T K(\dot{L}_1 - \dot{L}_2)] \tag{29}$$

Using the Jacobian mapping between $\dot{L_1}$ and $\dot{x}$ and the kinematics relation between L_2 and q,

$$\dot{L}_1 = J\dot{x} \tag{30}$$

$$L_2 = rq + L_0 \Rightarrow \dot{L}_2 = r\dot{q} \tag{31}$$

Substitute these relations into (29) and simplify:

$$\dot{V} = -\dot{q}^T(K_v + D)\dot{q} + \dot{q}^T(Q + rQ_G) - \dot{x}^T G(x). \tag{32}$$

Assume that all spring constants are equal, then

$$K = kI \Longrightarrow \varepsilon = \tfrac{1}{k} \tag{33}$$

$$U = K(L_2 - L_1) \tag{34}$$

Then using (30) and (31) we have

$$\dot{q} = r^{-1}(\varepsilon\dot{U} + \dot{L}_1) \tag{35}$$

Using (22) and (35)

$$\dot{V} = -\dot{q}^T(K_v + D)\dot{q} + \varepsilon r^{-1}\dot{U}^T(Q + rQ_G) + \dot{x}^T J^T Q_G$$
$$-\dot{x}^T G(x) \Longrightarrow$$
$$\dot{V} = -\dot{q}^T(K_v + D)\dot{q} + \varepsilon r^{-1}\dot{U}^T(Q + rQ_G)$$
$$+\dot{x}^T(J^T Q_G - G(x)) \tag{36}$$

Substitute equation (23):

$$\dot{V} = -\dot{q}^T(K_v + D)\dot{q} + \varepsilon r^{-1}\dot{U}^T(Q + rQ_G) \tag{37}$$

For the stability, $\dot{V}$ must be negative semi-defenite:

$$\dot{V} \leq 0 \tag{38}$$

Since ε is a small parameter, there exist large enough K_v, to satisfy this condition can be simply met.

4. Simulations Studies

A simulation study has been performed in order to verify the effectiveness of the proposed control algorithm. In the following simulation study, the results of the closed loop performance of planar cable driven manipulator examined. Our model of a planar cable robot consists of a moving platform that is connected by four cables to a base platform shown in Fig. 1.

As it is shown in Fig. 1, A_i denote the fixed base points of the cables, B_i denote point of connection of the cables on the moving platform, L_i denote the cable lengths, and α_i denote the cable angles. The position of the center of the moving platform G, is denoted by $G = [x_G, y_G]$, and the orientation of the manipulator moving platform is denoted by ϕ with respect to the fixed coordinate frame. Hence, the manipulator posses three degrees of freedom $x = [x_G, y_G, \phi]$, with one degree of actuator redundancy.

4.1 Kinematics and Jacobian

For kinematic analysis, as it is shown in Fig. 1, a fixed frame *O : xy* is attached to the fixed base at the point *O*, the center of the base point circle which passes through A_i's. Moreover, another moving coordinate frame *G : UV* is located on moving platform at its centre of mass *G*. Assume that the point A_i lie at the radial distance of R_A from point *O*, and the point B_i lie at the radial distance of R_B from point G in the *xy* plane, when the manipulator is at central location. For inverse kinematics analysis, it is assumed that the position and orientation of the moving platform $X = [x_G, y_G, \phi]^T$ is given and the problem is to find the length variable of the manipulator, $l = [l_1, l_2, l_3, l_4]^T$. Let's define the instantaneous orientation angle of B_i's:

$$\phi_i = \phi + \theta_{B_i}$$

The loop closure equation for each cable (*i=1,2,3,4*), can be written as,

$$\overrightarrow{A_iG} = \overrightarrow{A_iB_i} + \overrightarrow{B_iG}$$

With some manipulation we can show that [20],

$$l_i = [\left(x_G - x_{A_i} + R_B\cos(\phi_i)\right)^2 +$$

$$+\left(y_G - y_{A_i} + R_B\sin(\phi_i)\right]^{\frac{1}{2}}$$

Jacobian analysis plays a vital role in the study of robotic manipulators. Jacobian matrix not only reveals the relation between the joint variable velocities $\dot{L}$ and the moving platform velocities $\dot{x}$, it constructs the transformation needed to find the actuator forces τ from the forces acting on the moving platform F. For the

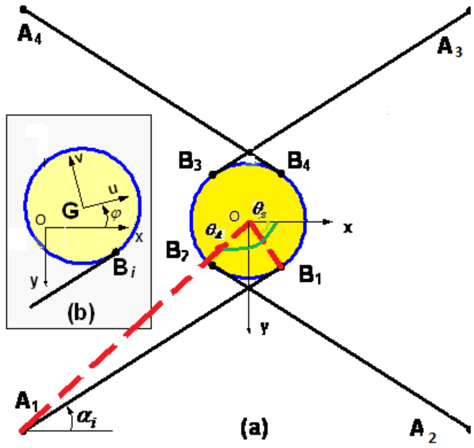

Figure 1: The schematics of Planar Cable Mechanism.

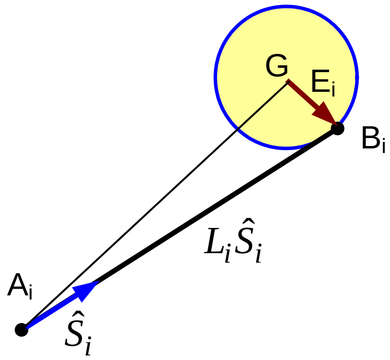

Figure 2: Vectors definitions for Jacobian derivation of the planar Cable manipulator.

geometry of the manipulator as illustrated in Fig. 2, the manipulator Jacobian matrix J is [20],

$$J = \begin{bmatrix} S_{1x} | S_{1y} | E_{1x}S_{1y} - E_{1y}S_{1x} \\ S_{2x} | S_{2y} | E_{2x}S_{2y} - E_{2y}S_{2x} \\ S_{3x} | S_{3y} | E_{3x}S_{3y} - E_{3y}S_{3x} \\ S_{4x} | S_{4y} | E_{4x}S_{4y} - E_{4y}S_{4x} \end{bmatrix}$$

Note that the Jacobian matrix J is a non-square 4×3 matrix, since the manipulator is redundantly actuated.

4.2 Control

The equations of motion for the end-effector can be written in the following form [21],

$$M\ddot{X} + \mathcal{G} = \mathcal{F}\ , \ X = [x_g, y_G, \phi].$$

In which, by consider flexibility in the cables, as have,

$$\mathcal{F} = J^T K(L_2 - L_1)\ \ ,\ L_2 = rq + L_0$$
$$I_m\ddot{q} + D\dot{q} + rK(L_2 - L_1) = \tau$$

In these equations

$$M = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_z \end{bmatrix} \quad \text{and} \quad \mathcal{G} = \begin{bmatrix} 0 \\ -mg \\ 0 \end{bmatrix}$$

Whose parametric values are given in Table 1. To show the effectiveness of the proposed control algorithm suppose that we want to move the system from initial position $L_2 = [2,1,3,4]$ to a fixed position $L_2 = [5,2,6,8]$. The controller is based on equation (21) and consists of three parts. PD term whose parameters are chosen as $K_P = 500, K_V = 50$, gravity compensation term based on equation (23), and internal forces term to ensure that all cables are in tension. As illustrated in Fig. (3), the results are satisfactory and the controller achieves the desired steady state values with small errors. This simulation verifies the stability guarantee of the proposed controller in presence of flexibility in the cables.

Table 1: System Parameters

Description	Symbols	Quantity
Mass of end-effector	m	2
Inertia	I_z	0.5
Cables stiffness	K	$100I_{4\times4}$
Motor inertia	I_m	$I_{4\times4}$
Damping coefficient	D	$0.3I_{4\times4}$
Drum radius	r	1

5. Conclusions

In this paper modelling and control of cable driven robots with cable flexibility are examined in detail. In the modelling of this kind of manipulators cables are modelled by linear axial spring, and the model of fully constrained cable driven robot is derived using Euler-Lagrange approach. Since in robots cables must remain in tension in the whole workspace, the notion of internal forces are introduced and directly used in the proposed control algorithm. The proposed control algorithm is designed in cable link space and consists of three parts. A simple PD control on the tracking error, the internal force that ensures us all of the cables are in tension and a gravity compensation term. Finally, the stability of the closed-loop system is analysed through Lyapunov second method, and it is shown that the proposed controller is capable to stabilize the system with flexible cables. Finally the performance of the proposed controller is examined through simulation.

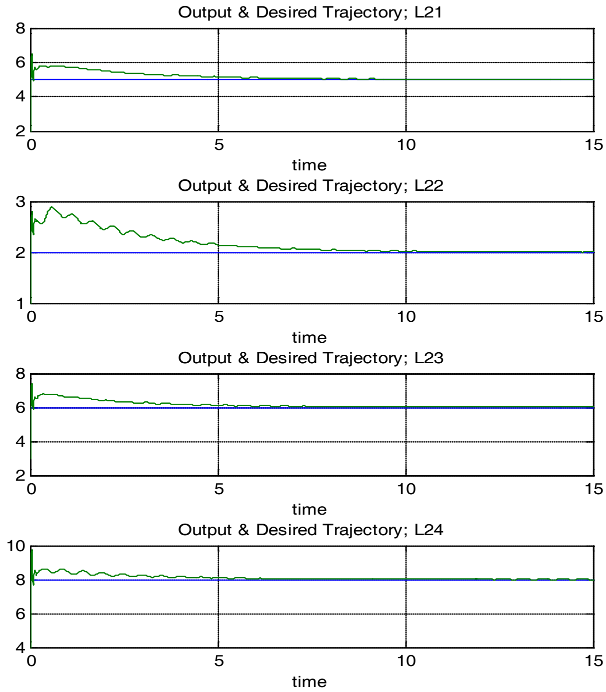

Figure 3: Plots of desired and actual position of L_2

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