Modeling and Control of Cable Driven Parallel Manipulators with Elastic Cables: Singular Perturbation Theory

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Abstract. This paper presents a new approach to the modeling and control of cable driven parallel manipulators and particularly KNTU CDRPM. First, dynamical model of the cable driven parallel manipulator is derived considering the elasticity of the cables, and then this model is rewritten in the standard form of singular perturbation theory. This theory used here as an effective tool for modeling the cable driven manipulators. Next, the integrated controller, applied for control of the rigid model of KNTU CDRPM in previous researches, is improved and a composite controller is designed for the elastic model of the robot. Asymptotic stability analysis of the proposed rigid controller is studied in detail. Finally, a simulation study performed on the KNTU CDRPM verifies the closed-loop performance compared to the rigid model controller.

1 Introduction

Cable driven parallel robots are a special kind of parallel robots in which rigid links are replaced by cables. This has produced some advantages for cable driven ones that has attracted the attention of researches [1,2,3]. High acceleration due to the reduced mobile mass, larger workspace, transportability and ease of assembly/disassembly, economical structure and maintenance are among these advantages. The most important limitation of cable driven robots is that, the cables suffer from unidirectional constraints that can only be pulled and not pushed. In this class of robots, the cables must be in tension in the whole workspace. Cables are sagged under compression forces, and therefore, to enable tension forces in the cables throughout the whole workspace, the mechanism must be designed over-constrained [4]. KNTU CDRPM is an over-constrained parallel manipulator that uses a novel design to achieve high stiffness, accurate positioning for high-speed maneuvers [5]. Controller must ensure that the cables are always in positive tension by using an appropriate redundancy resolution scheme, [5].

The major challenge in the controller design of these robots is deformation of the cables under tension. Elongation is one kind of these deformations that causes position and orientation errors. Moreover, the flexibility of the cables may

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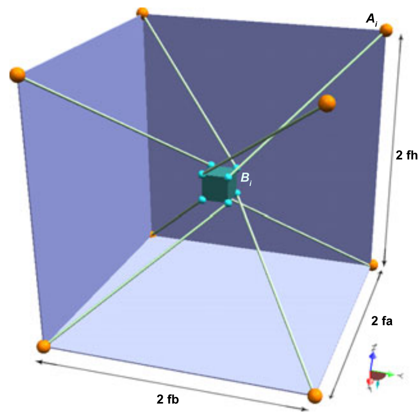

Fig. 1. The KNTU CDRPM, a perspective view

lead the system to vibration, and cause the whole system to be uncontrollable [6]. Although cable behavior has been the subject of researches in civil engineering but different use of them in parallel robots requires new studies. Cables in parallel robots are much lighter than one used in civil engineering and usually we have large changes in cable length and the tension exerted to them. Reported studies on the effect of cable flexibility on modeling, optimal design and control of such manipulators are very limited and usually neglected.

It should be noticed that a complete dynamic model of cable robots is very complicated. Furthermore, such complicated models are useless for controller design strategies, although they can accurately describe dynamic intrinsic characteristics of cables. Thus, in practice it is proposed to include only the dominant effects in the dynamics analysis. For this reason in many robotics applications, cables mass have been neglected and cable has been considered as a rigid element [7,8]. With those assumptions the dynamics of cable driven robot is reduced to the end-effector dynamics, that will lead to some inaccuracies in tracking error and especially the stability of the manipulator. In this paper a more precise model of the cable driven robot considering cable flexibility is derived and being used in the controller design and stability analysis. Using natural frequencies of system, Diao and Ma have shown in [9] that in fully constrained cable driven robots the vibration of cable manipulator due to the transversal vibration of cables can be ignored in comparison to that of cable axial flexibility. By this means, this model can describe the dominant dynamic characteristics of cable and can be used in the dynamic model of cable robot. Based on this observation, in this paper axial spring is used to model cable dynamics.

In this paper, considering axial flexibility in cables, a new dynamical model for cable driven robots is presented. This model is formulated in the standard form of *singular perturbation theory*. The most contribution of this theory in solving the control problems of the systems is in the modeling part [10]. By using the obtained model, the control of the system is studied. Next, the stability of the system is analyzed through Lyapunov second method and it is proven that the closed–loop system with the proposed control algorithm is stable. Finally the performance of the proposed algorithm is examined through simulation.

2 Singular Perturbation Standard Model

The singular perturbation model of a dynamical system is a state space model where the derivatives of some of the states are multiplied by a small positive scalar ε, that is [11]

$$\dot{x} = f(x, z, \varepsilon, t) \;\; x \in R^n \tag{1}$$

$$\varepsilon \dot{z} = g(x, z, \varepsilon, t) \;\; z \in R^m \tag{2}$$

It is assumed that f, g have continuous derivatives along $(t, x, z, \varepsilon) \in [0, t_1] \times D_1 \times D_2 \times [0, \varepsilon_0]$, on their domains $D_1 \subset R^n$ and $D_2 \subset R^m$. Putting $\varepsilon = 0$, the dimension of the standard model reduces from $m + n$ to n, since the differential equation (2) changes to

$$g(x, z, \varepsilon, t) = 0 \tag{3}$$

The model (1) and (2) is an standard model, if and only if, the equation (3), has $k \geq 1$ distinct real solutions:

$$z = h_i(t, x) \;\; \forall [t, x] \in [0, t_1], i = 1, 2, 3, \ldots \tag{4}$$

This assumption ensures that the reduced model with appropriate order of n is related to the roots of equation (3). For achieving the i-th reduced order model, substitute (4) in (1) and assume $\varepsilon = 0$, then:

$$\dot{x} = f(t, x, h(t, x), 0) \tag{5}$$

This approximation is a wise simplification of the dynamic system in which the high frequency dynamics is neglected, which is sometimes called a quasi-steady model. Since the velocity of variable z i.e. $\dot{z} = g/\varepsilon$ can be a large number while ε is small and $g \neq 0$, therefore, variable z converges rapidly to the roots of equation $g = 0$, the quasi-steady form of (2). The equation (5) is often called slow model.

3 Dynamics

Due to redundancy characteristic of KNTU CDRPM and other over–constrained cable driven parallel manipulators, the sagging of the cables is neglected. A simple model that can hold elastic characteristic of the cable and also can be used in controller design procedure, is to model the cable as a spring. This simple model can be well included in singular perturbation theory in order to derive a dynamic model for KNTU CDRPM considering elasticity of the cables. In what follows, we will first describe the dynamics of rigid robot briefly and then dynamic equations of the elastic system are derived using rigid ones. In the next step the dynamics equations are formulated in the standard form of singular perturbation theory.

3.1 Dynamics with Ideal Cables

The rigid model of parallel robots can be formulated into the general form of [12]:

$$M(x)\ddot{x} + C(x,\dot{x})\dot{x} + G(x) = J^T\tau \tag{6}$$

in which, x is the vector of generalized coordinates showing the position and orientation of the end-effector, $M(x)$ is a 6×6 matrix called mass matrix, $C(x,\dot{x})$ is a 6×6 matrix representing the *Coriolis* and *centrifugal* forces, $G(x)$ is a 6×1 vector of gravitational forces, J $n\times6$ denotes the Jacobian matrix, τ $n\times1$ is the cable tension vector. n is equal to the number of cables and for KNTU CDRPM it is equal to 8. The actuator dynamics can be represented as

$$M_m\ddot{L} + D\dot{L} + \tau = u \tag{7}$$

in which, L is the $n\times1$ cable length vector, M_m is a diagonal $n\times n$ inertia matrix of actuators, D a diagonal $n\times n$ matrix including viscous friction coefficients for actuators (pulleys), τ $n\times1$ cable tension vector, $u : n\times1$ actuator input vector. Use equations (6) and (7) to derive

$$M_{eq}(x)\ddot{x} + C_{eq}(x,\dot{x})\dot{x} + G_{eq}(x) = J^Tu \tag{8}$$

in which,

$$\begin{aligned} &M_{eq}(x) = M(x) + J^TM_mJ \\ &C_{eq}(x,\dot{x}) = C(x,\dot{x}) + J^TM_m\dot{J} + J^TDJ \\ &G_{eq}(x) = G(x) \end{aligned} \tag{9}$$

3.2 Dynamics with Real Cables

In parallel manipulators with elastic cables, actuator position is not directly related to end-effector position, and therefore, both the actuator and the end-effector positions must be taken into state vector. In other words both the cable length in the unloaded state and the cable length under tension are taken as state vector. For modeling a parallel manipulator with n cables, we assume $\hat{L}_{1i} : i = 1,2,...,n$ indicate the length of i-th cable under tension and $\hat{L}_{2i} : i = 1,2,...,n$ indicate the i-th cable without tension. In the case of rigid system, we have: $\hat{L}_{1i} = \hat{L}_{2i}(\forall\, i)$. In vector representation

$$L = (\hat{L}_{11},...,\hat{L}_{1n},\hat{L}_{21},...,\hat{L}_{2n})^T = (L_1^T|L_2^T) \tag{10}$$

The kinetic energy of the system is

$$T = \frac{1}{2}\dot{x}^TM(x)\dot{x}^T + \frac{1}{2}\dot{L}_2^TM_m\dot{L}_2 \tag{11}$$

The sum of total potential energy of the system is

$$P = P_1 + P_2(L_1 - L_2) \tag{12}$$

In which P_1 is the potential energy of the rigid robot and the second term, the potential energy of the i-th cable which its elasticity is approximated with a linear spring, is as follows

$$P_2 = \frac{1}{2}(L_1 - L_2)^T K (L_1 - L_2) \tag{13}$$

and K is the matrix of the stiffness coefficients of cables. Now the Lagrangian of the system is derived by $L = T - P$, as

$$L = \frac{1}{2}\dot{x}^T M(x)\dot{x} + \frac{1}{2}\dot{L}_2^T M_m \dot{L}_2 - P_1 - \frac{1}{2}(L_1 - L_2)^T K (L_1 - L_2) \tag{14}$$

The total dynamic equations of the system is derived simply by applying the Lagrange equations

$$\begin{cases} M(x)\ddot{x} + C(x,\dot{x})\,\dot{x} + G(x) = J^T K(L_2 - L_1) \\ M_m \ddot{L}_2 + K(L_2 - L_1) + D\dot{L}_2 = u \end{cases} \tag{15}$$

in which, the relation between x and L_1 is obtained by $\dot{L}_1 = J\dot{x}$. Furthermore, in eq. (15), K is the $n \times n$ diagonal stiffness matrix of the cables, $M(x)$ the 6×6 inertia matrix, $C(x, \dot{x})$ a 6×6 matrix with *Coriolis* and *centrifugal* terms, $G(x)$ the 6×1 vector of gravitational forces, J the $n \times 6$ Jacobian matrix, M_m the diagonal $n \times n$ inertia matrix of actuators(pulleys), D the diagonal $n \times n$ matrix including viscous friction coefficients for actuators, and $n = 8$ for KNTU CDRPM.

3.3 Singular Perturbation Model

The spring stiffness matrix K which connects two equations in (15) enables us to formulate these equations in singular perturbation form. /without loss of generality, assume that all of the cables stiffness are equal. Then write the elastic forces in the cables in the form $z = k(L_1 - L_2)$, $K = kI$. Since the singular perturbation theory is defined usually for small terms, define $\varepsilon = 1/k$, therefore $\varepsilon \to 0$ as $k \to \infty$. Multiplying two sides of the first line of equation (15) by M^{-1} and consider $z = k(L_1 - L_2)$, we have

$$\begin{cases} \ddot{x} = -M^{-1}(x)J^T z - M^{-1}(x)(C(x,\dot{x})\dot{x} + G(x)) \\ -\varepsilon\ddot{z} = M_m^{-1} z - M_m^{-1} D\dot{L}_2 + M_m^{-1} u - \ddot{L}_1 \end{cases} \tag{16}$$

Considering the following equations,

$$\begin{aligned} \dot{L}_2 &= \dot{L}_1 - \varepsilon\dot{z} \\ \dot{L}_1 &= J\dot{x} \\ \ddot{L}_1 &= J\ddot{x} + \dot{J}\dot{x} \end{aligned} \tag{17}$$

We can summarize equation (16), which is in the standard form of singular perturbation theory in the form

$$\begin{cases} \ddot{x} = a_1(x,\dot{x}) + A_1(x)z \\ \varepsilon\ddot{z} = a_2(x,\dot{x},\varepsilon\dot{z}) + A_2(x)z + B_2 u \end{cases} \tag{18}$$

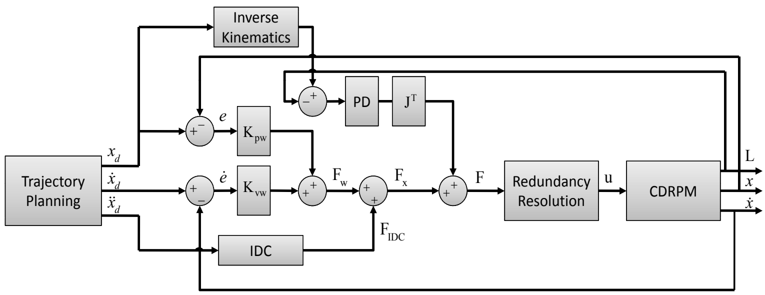

Fig. 2. The cascade control scheme

In which

$$\begin{aligned} A_1 &= -M^{-1}(x)J^T \\ a_1 &= -M^{-1}(x)(C(x,\dot{x})\dot{x} + G(x)) \\ a_2 &= -\varepsilon M_m^{-1}D\dot{z} + M_m^{-1}DJ\dot{x} - JM^{-1}(x)(C(x,\dot{x}) + G(x)) + \dot{J}\dot{x} \\ A_2 &= -(J(x)M^{-1}(x)J^T(x) + M_m^{-1}), \\ B_2 &= -M_m^{-1} \end{aligned}$$

Note that the rigid model is the marginal mode of the elastic model of eq. (6), when the stiffness of the cables tends to infinity or $\varepsilon \to 0$.

4 Control

4.1 Control Law for the Rigid Model

The controller applied to the rigid model is a combination of two control loops with an inverse-dynamic controller. The first control loop is a PD controller in joint-space and the second one in work space (Fig. 2). It is shown that this controller can improve the performance of the control system up to 80% compared to conventional single loop controllers [5]. The structure of this controller is illustrated in Fig. 2 and the control law is defined as:

$$\begin{aligned} F &= F_j + F_x \\ F_j &= J^T(K_{pj}(L_d - L) + K_{vj}(\dot{L}_d - \dot{L})) \\ F_x &= K_{pw}(x_d - x) + K_{vw}(\dot{x}_d - \dot{x}) + M_{eq}\ddot{x}_d + G_{eq} + C_{eq}\dot{x}_d \\ u &= P + P_n = (J^T)^\dagger F + (I - J^{T^\dagger}J^T)k_e \end{aligned} \tag{19}$$

in which, $(\cdot)^\dagger$ denotes the pseudo inverse and $(\cdot)_d$ denote the desired values. P and P_n are defined as

$$\begin{aligned} F &= J^T P \\ 0 &= J^T P_n \end{aligned}$$

and k_e is an n dimensional vector which is optimized through redundancy resolution scheme, [5]. K_{pj}, K_{vj}, K_{pw} and K_{vw} are diagonal positive definite matrices.

Stability Analysis of the Closed-loop System. First, let us derive the error dynamics to prove the stability of the closed-loop system using the controller in equation (19). According to the robot dynamic equations (8) and control law we can write

$$M_{eq}\ddot{x} + C_{eq}\dot{x} + G_{eq} = K_{pw}(x_d - x) + K_{vw}(\dot{x}_d - \dot{x}) + M_{eq}\ddot{x}_d + G_{eq} + C_{eq}\dot{x}_d + J^T(K_{pj}(L_d - L) + K_{vj}(\dot{L}_d - \dot{L})) \tag{20}$$

Or,

$$M_{eq}\ddot{e} + (K_{vw} + J^T K_{vj} J)\dot{e} + K_{pw}e + J^T K_{pj} e_L + C_{eq}\dot{e} = 0 \tag{21}$$

in which, $e_L = L_d - L$ and $e = x_d - x$. Now, introduce a Lyapunov candidate to prove the stability of the system under control.

$$V = \frac{1}{2}\dot{e}^T M_{eq}\dot{e} + \frac{1}{2}e^T K_{pw} e + \frac{1}{2}e_L^T K_{pj} e_L \tag{22}$$

in which, M_eq, K_{pw} and K_{pj} matrices are positive definite, therefore V is positive definite. The derivative of Lyapunov function is:

$$\dot{V} = \dot{e}^T M_{eq}\ddot{e} + \frac{1}{2}\dot{e}^T \dot{M}_{eq}\dot{e} + e^T K_{pw}\dot{e} + e_L^T K_{pj}\dot{e}_L \tag{23}$$

Substitute the term $M_{eq}\ddot{e}$ from the dynamic equations of the system.

$$\dot{V} = \dot{e}^T(-(K_{vw} + J^T K_{vj} J)\dot{e} - K_{pw}e - J^T K_{pj} e_L - C_{eq}\dot{e}) + \tfrac{1}{2}\dot{e}^T \dot{M}_{eq}\dot{e} + e^T K_{pw}\dot{e} + e_L^T K_{pj}\dot{e}_L \tag{24}$$

Hence,

$$\begin{aligned}\dot{V} &= -\dot{e}^T(K_{vw} + J^T K_{vj} J)\dot{e} + \frac{1}{2}\dot{e}^T(\dot{M}_{eq} - 2C_{eq})\dot{e} \\ &= -\dot{e}^T(K_{vw} + J^T K_{vj} J + 2J^T D J)\dot{e} \le 0 \end{aligned} \tag{25}$$

note that $J^T K_{vj} J$ is a positive semi-definite (PSD) matrix, because K_{vj} is PD and

$$y^T(J^T K_{vj} J)y = y^T(J^T K_{vj}^{1/2} K_{vj}^{1/2} J)y = z^T z \ge 0. \tag{26}$$

Therefore, $K_{vw} + J^T K_{vj} J + 2J^T D J$ which is sum of two PSD matrices and a PD matrix, is a PD matrix. Then we can conclude $\dot{V} \le 0$. Therefore, we know that the motion of the robot will converge to the largest invariant set that satisfies $\dot{V} = 0$. In this case, $\dot{V} = 0$ results in $\dot{e} = 0$. Therefore, from equation (21) the largest invariant set is

$$K_{pw}e + J^T K_{pj} e_L = 0 \tag{27}$$

It is shown in Appendix that $J.e$ has the same sign of e_L , hence, we can write $e_L = \alpha J e$, $\alpha > 0$ and then we can rewrite equation (27) in this form:

$$(K_{pw} + \alpha J^T K_{pj} J).e = 0, \;\; \alpha > 0 \tag{28}$$

According to the above equation and positive definiteness of $(K_{pw} + \alpha J^T K_{pj} J)$ it is concluded that $e = 0$. Therefore, as time tends to infinity we have $x = x_d$ and this means the end-effector position converges to the desired trajectory.

Table 1. Geometric and Inertial Parameters of the KNTU CDRPM

Description	Quantity
K: Spring stiffness matrix	$100I_{8\times8}$
M_m: Inertia matrix of actuators	$0.006I_{8\times8}$
D: Viscous friction coefficients for actuators	$0.244I_{8\times8}$
The parameters of controllers:	
$\tilde{K}_p = 13500,\ \tilde{K}_v = 700$	
$K_{pj} = 10^5\mathrm{I}_{8\times8},\ K_{dj} = 10^4\mathrm{I}_{8\times8}$	
$K_{pw} = 10^7\mathrm{diag}(80, 50, 1000, 77.5, 14, 19.5)$	
$K_{dw} = 10^7\mathrm{diag}(24, 9, 600, 16.5, 1.14, 5.7)$	

4.2 Control Law for the Elastic Model

Control of the systems with real cables can be done using a composite control scheme that is a well-known technique in the control of singularly perturbed systems [10]. In this framework the control effort u_{tot} consists of two main parts, i.e. u the control effort for slow subsystem, the model in eq. (8), and u_f the control effort for fast subsystem. Here we use a control law that is combination of rigid model control and a PD controller for the fast dynamics

$$u_t = u + \tilde{K}_p(L_1 - L_2) + \tilde{K}_v(\dot{L}_1 - \dot{L}_2) \tag{29}$$

As a practical point of view, it must be said that L_1 can be measured by an encoder and L_2 by a *string pot*. In next section, it is shown through simulation that this controller can stabilize the closed-loop system with real cables and reach to a desired tracking error. Stability analysis of the system with this composite controller will be discussed in later researches.

4.3 Simulation Study

In this section, the performance of the proposed controller is demonstrated through simulating the KNTU CDRPM. The dynamic equations of the CDRPM

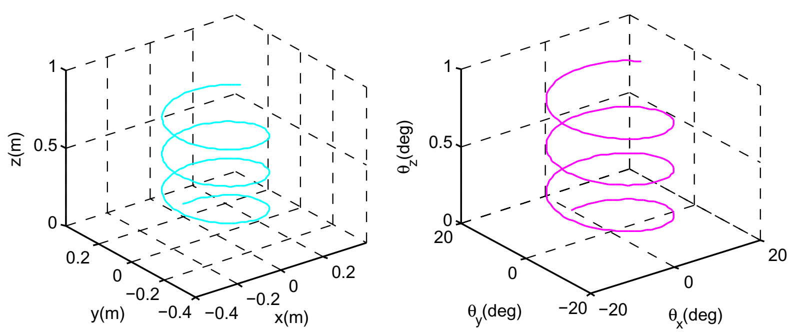

Fig. 3. Desired path in the workspace

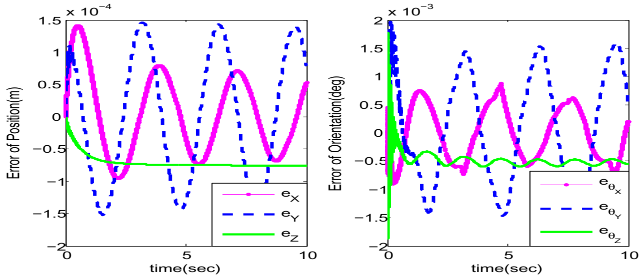

Fig. 4. The tracking error of the controller for elastic model

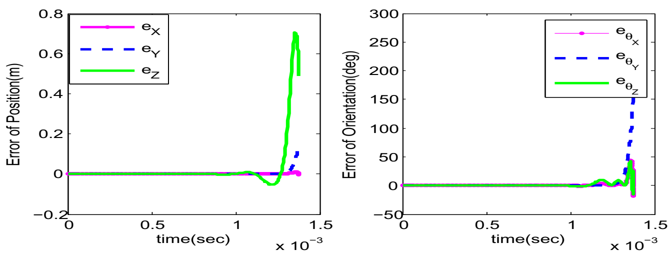

Fig. 5. The tracking error of the rigid model controller on the elastic model

considering the elasticity of the cables are shown in eq. (15).These equations in the standard form of singular perturbation theory are shown in eq. (18). Table 1 shows robot and controller specifications, other parameters are the same as what is given in [5]. The desired path of the manipulator in 3D is cylindrical and is shown in Fig. 3. The tracking performance of the CDRPM using the proposed controller is shown in Fig. 4. As seen in this figure, the proposed control topology is capable of reducing the tracking errors less than 0.15 millimeters in position and less than 2×10^{-3} degrees in orientation. The tracking error of a single controller for the rigid model i.e.u in eq. (19) is shown in Fig. 5 for comparison. It is obvious that this controller cannot stabilize the cable driven manipulator.

5 Conclusions

A dynamical model for cable driven manipulators considering the flexibility of the cables is proposed using cable model as a linear axial spring. The model is formulated in standard form of singular perturbation theory. A composite control is employed for control of cable driven manipulators, which is composition of the controller for the rigid model and a PD controller for controlling the fast dynamics. It is shown that the rigid control law can stabilize the system with ideal and inflexible cables asymptotically. The efficiency of the proposed controller is verified through simulations on KNTU CDRPM.

A Appendix

Here, we will show that $\boldsymbol{J}\boldsymbol{e}_x = \boldsymbol{J}(\boldsymbol{x}_d - \boldsymbol{x})$ has the same sign of $\boldsymbol{e}_l = (\boldsymbol{\ell}_d - \boldsymbol{\ell})$, the proof will be done *by reduction to the absurd* (or *contradiction*). Therefore, assume that they have different sign:

$$l_d - l = \alpha \boldsymbol{J}(x_d - x)\,, \alpha < 0 \tag{30}$$

Therefore, $\exists M \gg \frac{1}{\varepsilon} \Rightarrow \frac{\Delta l}{M} = \frac{\alpha}{M} J \Delta x$.
$\frac{\Delta l}{M} = dl$ and we know that $dl \simeq \boldsymbol{J}dx$, so from equation (30) we have:

$$\boldsymbol{J}dx = dl \simeq \frac{\alpha}{M} \boldsymbol{J} \Delta x \tag{31}$$

$$dx \simeq \frac{\alpha}{M} \Delta x \tag{32}$$

Which is a wrong expression when $\alpha < 0$. Thus by contradiction, we can conclude that $\alpha > 0$, i.e. $\boldsymbol{J}(x_d - x)$ and $(l_d - l)$ have the same sign.

□

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