PD Controller Design with H_∞ Performance for Linear Systems with Input Delay

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ABSTRACT

This paper presents H_∞ control problem for input-delayed systems. A neutral system approach is considered to the design of PD controller for input delay systems in presence of uncertain time-invariant delay. Using this approach, the resulting closed-loop system turns into a specific time-delay system of neutral type. The significant specification of this neutral system is that its delayed coefficient terms depend on the controller parameters. This condition provides challenging issues in theoretical research and provides new horizons in applications. In the present paper, new delay-dependent sufficient condition is derived for the existence of H_∞ controller in terms of matrix inequalities. The resulting H_∞ controller stabilizes the closed-loop neutral system and assures that the H_∞ performance norm to be less than a prescribed level. An application example is presented to illustrate the effectiveness of the proposed method.

KEYWORDS

H_∞ Control, Neutral Systems, Uncertain Time-Invariant Delay, PD Control, Linear Matrix Inequality

1. INTRODUCTION

Time-delays appear in many systems and processes, such as chemical and thermal processes [1], population dynamic model [2], rolling mill [3] and systems with long transmission line [4]. In many systems, time-delay is a source of instability. Hence, rich attention has been paid to the control design of time-delay systems of retarded or neutral type. Robust H_∞ state feedback control of uncertain neutral system has been considered in [5], and an optimization problem has been formulated with linear matrix inequality constraints to obtain an H_∞ state feedback controller. Observer-based H_∞ state feedback control for a class of uncertain neutral systems is another topic which has been considered in [6]. H_∞ output feedback control of neutral systems has also been the centre of attention in some literature such as [7] and [8]. In the time-varying delay case, Suplin et al., proposed delay-dependent sufficient conditions for H_∞ control of a neutral system in presence of time-varying state delay [9]. Recently, the stability problem of a class of linear switched systems with time-varying input delay is also addressed [10]. Furthermore, Liu et al. [11] presented some necessary and sufficient stability conditions for continuous-time positive systems with time-varying delay. A review on these results shows that few papers have focused on the special neutral time-delayed systems where the delayed coefficient terms depend on the control parameters. Recently this class of time-delay systems of neutral type was investigated using a proportional-derivative state feedback H_∞ controller [12].

On the other hand, it can be seen that applying a PD or PID controller to a linear system with input delay leads to a time-delay system of retarded or neutral type. Up to the present, many papers have considered the design of PID family controllers. Recently, new results on the synthesis of PID controllers for first-order plants with time delay are given [13], while Xu, et al. extended this idea for arbitrary order plants with time-delay [14]. Wang used a graphical approach to find the stabilizing region of PID controllers for high-order all pole plants with time-delay [15]. Furthermore, Hohenbichler [16] studied a method to compute the set of stabilizing PID controller parameters for arbitrary linear input-delay systems which leads to a closed-loop system of retarded or neutral type. Moreover, many results have been appeared in the literature using PID family controllers in particular applications [17-19]. In addition to the above methods, Smith predictor structure is used for the design of PI/PID controller for time-delay systems which is the most popular controller in industrial processes. A predictive PI (PIP) controller with the same structure as smith predictor is presented by Hägglund [19] which is suitable for processes with long dead-time. In order to improve the robustness of this controller, Normey-Rico et al. [20] proposed a modified

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Smith predictor using an additional filter to the structure of the PIP controller (FPIP). Recently, Normey-Rico et al. [22] proposed a unified approach for designing dead-time compensators on the base of FPIP.

Although some of the aforementioned works improve the robustness of the closed-loop system, they cannot guarantee the stability of the closed-loop system in presence of unknown delays, which may occur inevitably in many applications. Moreover, as we mentioned earlier, applying a PD or PID controller to a linear system with input delay generally leads to a time-delay system of neutral type. To this aim in the present work, using a neutral system approach design of a PD controller with uncertain input delay is focused to achieve a prescribed level of H_∞ performance.

This paper is organized as follows. Problem formulation is introduced in Section 2, and in Section 3, the proposed PD controller with H_∞ performance is designed for the time-varying delay case. This is accomplished in terms of some matrix inequalities for the closed-loop time-delay system of neutral type. An illustrative example is provided in section 4 to show the effectiveness of the proposed method in a case study compared to conventional methods.

2. Problem Formulation

In this paper, we consider the following time-delay system with input delay: line, as follows:

$$\begin{aligned} \dot{x}(t) &= Ax(t)+Bu(t-\tau)+Ew(t) \\ z(t) &= C_z x(t)+D_z w(t) \\ y_p(t) &= C_y x(t) \end{aligned} \tag{1}$$

where x is the state, $w\in\Re^p$ is the disturbance input of system that belongs to Sobolev space $\mathcal{W}^{1,2}(0,\infty,\Re^p)\cap\mathcal{L}^2(0,\infty,\Re^p)$, τ is the constant time-delay of the system and is assumed to satisfy $0<\tau\le\bar{\tau}$, $u\in\Re^m$ denotes the system input and $z\in\Re^q$ is the controlled system output. The matrices $A\in\Re^{n\times n}$, $B\in\Re^{n\times l}$, $F\in\Re^{n\times p}$, $C_z\in\Re^{q\times n}$, $C_y\in\Re^{r\times n}$, $D_z\in\Re^{q\times p}$ are assumed to be known. Furthermore, it is assumed that all the state variables are measured. Considering the PD law (2) as

$$u(t)=K_p y+K_d\dot{y} \tag{2}$$

the state space equations of the closed-loop system is given by

$$\begin{aligned} \dot{x}(t) &= Ax(t)+BK_pC_yx(t-\tau)+BK_dC_y\dot{x}(t-\tau)+Ew(t) \\ z(t) &= C_zx(t)+D_zw(t) \end{aligned} \tag{3}$$

Therefore, the resulting closed-loop system (3) is a time-delay system of neutral type which both coefficients of $x(t-\tau)$ and $\dot{x}(t-\tau)$ depending on the controller parameters. Here, we state two well known lemmas which will be used further in the main result of the paper.

Lemma 1 [24]: Assume $a(.)\in\Re^{na}$, $b(.)\in\Re^{nb}$ and $N\in\Re^{na\times nb}$ are defined on the interval Ω, then for any matrices $X\in\Re^{na\times nb}$, $Y\in\Re^{na\times nb}$ and $Z\in\Re^{na\times nb}$, the following holds:

$$-2\int_\Omega a^T(\alpha)Nb(\alpha)d\alpha \le \int_\Omega \begin{bmatrix} a(\alpha) \\ b(\alpha)\end{bmatrix}^T \begin{bmatrix} X & Y-N \\ Y^T-N^T & Z\end{bmatrix}\begin{bmatrix} a(\alpha) \\ b(\alpha)\end{bmatrix} d\alpha, \tag{4}$$

where

$$\begin{bmatrix} X & Y \\ Y^T & Z\end{bmatrix}>0 \tag{5}$$

Remark 1: The above inequality can be extended to the similar inequalities with multiple integrals.

Lemma 2 [25]: Consider the neutral system (3) and assume $d(t)=[w^T(t)\ \ \dot{w}^T(t)]^T$. If $\|T_{zd}\|_\infty<\gamma$, then the inequality $\|T_{zw}\|_\infty<\gamma$ is satisfied.

3. H_∞ Control Design

In many developed theories, conventional PD controller has been used for obtaining stability as well as performance objectives of the closed-loop system. On the other hand, H_∞ control is an effective method which guarantees asymptotic stability as well as performance objectives. This is why H_∞ control for time-delay systems has been among the most challenging topics in recent years. All the aforementioned facts motivate us to elaborate on the following Theorem as the main result of this paper.

Theorem 1: Given scalar $\bar{\tau}>0$, the closed-loop system (3) is asymptotically stable and $\|T_{zw}\|_\infty<\gamma$ for any constant time-delay τ satisfying $0<\tau\le\bar{\tau}$, if there exist scalars $\alpha_1\in\Re$, $\alpha_2<0$, positive definite symmetric matrices L, P, T, Z_1, Z_2, R_1, $R_2\in\Re^{n\times n}$, matrices M_1, $M_2\in\Re^{n\times n}$, K_p, $K_d\in\Re^{l\times n}$ satisfying matrix inequalities (6) ~ (9). Moreover, H_∞ PD control law is given by $u=K_py+K_d\dot{y}$.

$$\begin{bmatrix} M_1 & \alpha_1 I \\ \alpha_1 I & Z_1\end{bmatrix}>0 \tag{7}$$

$$\begin{bmatrix} 2M_2 & \bar{\tau}\alpha_2 I \\ \bar{\tau}\alpha_2 I & 2Z_2\end{bmatrix}>0 \tag{8}$$

$$\begin{bmatrix} 2M_2 & \bar{\tau}\alpha_2 I \\ \bar{\tau}\alpha_2 I & 2Z_2\end{bmatrix}>0 \tag{9}$$

in which,

$$\Omega_1 = LA^T+AL+2\alpha_1 L+\bar{\tau}(M_1+M_2)+\alpha_2(2-\bar{\tau})L+T$$

$$\Omega_2 = BK_pC_y-\alpha_1 I-\alpha_2 I$$

$$\Omega_3 = BK_dC_y-\bar{\tau}\alpha_2 I$$

$$\Omega_4 = \bar{\tau}\left(BK_pC_y+ABK_dC_y\right)^T, \qquad \Omega_5=\Omega_4/\bar{\tau}$$

Proof: In this case a Lyapunov-Krasovskii functional candidate for the system (3) has the form [25]

$$V=V_1+V_2+V_3+V_4$$

$$
\begin{bmatrix}
\Omega_1 & \Omega_2 & \Omega_3 & 0 & LF & 0 & \bar{\tau}LA^T & LA^T & \bar{\tau}L(AA)^T & L(AA)^T & LC_z^T \\
* & -Q & 0 & 0 & 0 & 0 & \bar{\tau}(BK_pC_y)^T & (BK_pC_y)^T & \bar{\tau}(ABK_pC_y)^T & (ABK_pC_y)^T & 0 \\
* & * & -R_1-\bar{\tau}\alpha_2 P & 0 & 0 & 0 & \bar{\tau}(BK_dC_y)^T & (BK_dC_y)^T & \Omega_4 & \Omega_5 & 0 \\
* & * & * & -R_2 & 0 & 0 & 0 & 0 & \bar{\tau}(BK_dC_y)^T & (BK_dC_y)^T & 0 \\
* & * & * & * & -m^2\gamma^2 I & 0 & \bar{\tau}F^T & F^T & \bar{\tau}(AF)^T & (AF)^T & D_z^T \\
* & * & * & * & * & -(1-m)^2\gamma^2 I & 0 & 0 & \bar{\tau}F^T & F^T & 0 \\
* & * & * & * & * & * & -\bar{\tau}Z_1^{-1} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -R_1^{-1} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & -\bar{\tau}Z_2^{-1} & 0 & 0 \\
* & * & * & * & * & * & * & * & * & -R_2^{-1} & 0 \\
* & * & * & * & * & * & * & * & * & * & -I
\end{bmatrix} < 0 \tag{6}
$$

where

$$V_1 = x(t)^T Px(t) \tag{10}$$

$$V_2 = \int_{-\tau}^{0}\int_{t+\beta}^{t} \dot{x}^T(\alpha) Z_1 \dot{x}(\alpha)\, d\alpha\, d\beta \tag{11}$$

$$V_3 = \int_{t-\tau}^{t} x^T(\alpha) Qx(\alpha)\, d\alpha + \int_{t-\tau}^{t} \dot{x}^T(\alpha) R_1 \dot{x}(\alpha)\, d\alpha \tag{12}$$

$$V_4 = \tau^{-1}\int_{-\tau}^{0}\int_{-\tau}^{\beta}\int_{t+\eta}^{t} \ddot{x}^T(\alpha) Z'\ddot{x}(\alpha)\, d\alpha\, d\eta\, d\beta + (1/2)\int_{t-\tau}^{t} \ddot{x}^T(\alpha) R'\ddot{x}(\alpha)\, d\alpha \tag{13}$$

where $P=P^T>0$, $Q=Q^T>0$, $R_1=R_1^T>0$, $R'=R'^T>0$, $Z_1=Z_1^T>0$ and $Z'=Z'^T>0$. Differentiating V_1 with respect to t gives us

$$
\begin{aligned}
\dot{V}_1 &= 2x^T(t)P\dot{x}(t) \\
&= 2x^T(t)P\{Ax(t) + BK_pC_y x(t-\tau) \\
&\quad + BK_dC_y \dot{x}(t-\tau) + Ew(t)\}
\end{aligned}
$$

It is possible to write

$$x(t-\tau) = x(t) - \int_{t-\tau}^{t} \dot{x}(\alpha)\, d\alpha \tag{14}$$

Let us introduce the following relation for the delayed derivative of the state:

$$
\begin{aligned}
&\dot{x}(t-\tau) \\
&= \tau^{-1}\left[x(t) - x(t-\tau) - \int_{-\tau}^{0}\int_{t-\tau}^{t+\beta} \ddot{x}(\alpha)\, d\alpha\, d\beta\right]
\end{aligned} \tag{15}
$$

Therefore,

$$
\begin{aligned}
\dot{V}_1 &= 2x^T P(A + BK_pC_y + \tau^{-1}BK_dC_y)x(t) \\
&\quad - 2\tau^{-1}x^T PBK_dC_y x(t-\tau) \\
&\quad - 2x^T PBK_pC_y \int_{t-\tau}^{t} \dot{x}(\alpha)\, d\alpha \\
&\quad - 2\tau^{-1}x^T PBK_dC_y \int_{-\tau}^{0}\int_{t-\tau}^{t+\beta} \ddot{x}(\alpha)\, d\alpha\, d\beta \\
&\quad + 2x^T(t)PFw(t)
\end{aligned}
$$

Applying Lemma 1 and Remark 1, the following upper bound for $\dot{V}_1$ is obtained:

$$
\begin{aligned}
&\dot{V}_1 \le \\
&x^T\{A^T P + PA + \tau(X_1 + X'/2) + Y_1 + Y_1^T + \tau^{-1}(Y' + Y'^T)\}x \\
&+ x^T(t)(PBK_pC_y - Y_1)x(t-\tau) \\
&+ x^T(t-\tau)(PBK_pC_y - Y_1)^T x(t) \\
&+ x^T(t)(PBK_dC_y - Y')\dot{x}(t-\tau) \\
&+ \dot{x}^T(t-\tau)(PBK_dC_y - Y')^T x(t) \\
&- \tau^{-1}x^T(t)Y'x(t-\tau) - \tau^{-1}x^T(t-\tau)Y'^T x(t) + 2x^T(t)PFw(t) \\
&+ \tau^{-1}\int_{-\tau}^{0}\int_{t-\tau}^{t+\beta} \ddot{x}^T(\alpha)Z'\ddot{x}(\alpha)\, d\alpha\, d\beta + \int_{t-\tau}^{t} \dot{x}^T(\alpha)Z_1\dot{x}(\alpha)\, d\alpha
\end{aligned} \tag{16}
$$

where

$$\begin{bmatrix} X_1 & Y_1 \\ Y_1^T & Z_1 \end{bmatrix} > 0 \tag{17}$$

and,

$$\begin{bmatrix} X' & Y' \\ Y'^T & Z' \end{bmatrix} > 0 \tag{18}$$

Also, the time derivative of V_4 can be represented as follows:

$$
\begin{aligned}
\dot{V}_4 &= -\tau^{-1}\int_{-\tau}^{0}\int_{t-\tau}^{t+\beta} \ddot{x}^T(\alpha)Z'\ddot{x}(\alpha)\, d\alpha\, d\beta \\
&\quad + (\tau/2)\ddot{x}^T(t)Z'\ddot{x}(t) + (1/2)\ddot{x}^T(t)R'\ddot{x}(t) \\
&\quad - (1/2)\ddot{x}^T(t-\tau)R'\ddot{x}(t-\tau)
\end{aligned} \tag{19}
$$

It can be shown that the time derivative of V_2 and V_3 are

$$\dot{V}_2 = \dot{x}^T(t)(\tau Z_1)\dot{x}(t) - \int_{t-\tau}^{t} \dot{x}^T(\alpha)Z_1\dot{x}(\alpha)\, d\alpha \tag{20}$$

$$\dot{V}_3 = x^T(t)Qx(t) - x^T(t-\tau)Qx(t-\tau) + \dot{x}^T(t)R_1\dot{x}(t) - \dot{x}^T(t-\tau)R_1\dot{x}(t-\tau) \quad (21)$$

Therefore, we have

$$\dot{V}(t) = \sum_{i=1}^{4}\dot{V}_i \quad (22)$$

Now consider (16), (19) ~ (22) and define $\tau^{-1}Y' = Y_2$. By the assumption of $Y' = Y'^T < 0$ and then adding and subtracting the terms $x^T(t)(\tau Y_2)x(t)$ and $\dot{x}^T(t-\tau)(\tau Y_2^T)\dot{x}(t-\tau)$ in (22), an upper bound for $\dot{V}$ is obtained as

$$\begin{aligned}\dot{V}(t) \le\ & x^T\{A^TP + PA + \tau(X_1 + (1/2)X') + Y_1 + Y_1^T + 2Y_2\}x \\ & + 2x^T(t)(PBK_pC_y - Y_1)x(t-\tau) \\ & + 2x^T(t)PBK_dC_y\dot{x}(t-\tau) \\ & + (x^T(t) + \dot{x}(t-\tau))^T(-\tau Y_2)(x^T(t) + \dot{x}(t-\tau)) \\ & + 2x^T(t)PFw(t) - x^T(t)Y_2x(t-\tau) \\ & - x^T(t-\tau)Y_2x(t) + (\tau/2)\ddot{x}^T(t)Z'\ddot{x}(t) \\ & + (1/2)\ddot{x}^T(t)R'\ddot{x}(t) \\ & - (1/2)\ddot{x}^T(t-\tau)R'\ddot{x}(t-\tau) \\ & + (Ax + BK_pC_yx(t-\tau) + BK_dC_y\dot{x}(t-\tau) + Fw(t))^T. \\ & \tau Z_1(Ax + BK_pC_yx(t-\tau) + BK_dC_y\dot{x}(t-\tau) + Fw(t)) \\ & + x^T(t)Qx(t) - x^T(t-\tau)Qx(t-\tau) \\ & + \dot{x}^T(t)R_1\dot{x}(t) - \dot{x}^T(t-\tau)R_1\dot{x}(t-\tau) \\ & + 2(Ax(t) + BK_pC_yx(t-\tau) + BK_dC_y\dot{x}(t-\tau) + Fw(t))^T. \\ & (R_1)Fw(t) + x^T(t)(\tau Y_2)x(t) \\ & + \dot{x}^T(t-\tau)(\tau(t)Y_2)^T\dot{x}(t-\tau)\end{aligned} \quad (23)$$

Assume zero initial condition, i.e. $\phi(t)$=0, $\forall t\in[-\tau,0]$ we have $V(q(t))|_{t=0}$=0. For a prescribed $\gamma > 0$, consider the following performance index:

$$J_{zd}(w) = \int_0^{\infty}(z^Tz - \gamma^2d^Td)d\tau \quad (24)$$

where $d(t) = \begin{bmatrix} w(t) \\ \dot{w}(t)\end{bmatrix}$,

Therefore, the performance index (24) can be rewritten as

$$J_{zd}(w) = \int_0^{\infty}(z^Tz - \gamma^2w^Tw - \gamma^2\dot{w}^T\dot{w})d\tau$$

Since $V(t)|_{t=0}$=0 and $V(t)|_{t\to\infty}\ge 0$, we obtain

$$\begin{aligned}J_{zd}(w) &= \int_0^{\infty}(z^Tz - \gamma^2w^Tw - \gamma^2\dot{w}^T\dot{w} + \dot{V}(t))d\tau \\ &\quad + V(t)|_{t=0} - V(t)|_{t\to\infty} \\ &\le \int_0^{\infty}(z^Tz - \gamma^2w^Tw - \gamma^2\dot{w}^T\dot{w} + \dot{V}(t))d\tau\end{aligned}$$

Hence, the following inequality is obtained:

$$\begin{aligned}J_{zd}(w) \le \int_0^{\infty}\{&x^TC^TCx + 2x^TC^TD_zw + w^TD_z^TD_zw \\ &- m^2\gamma^2w^Tw - (1-m)^2\gamma^2\dot{w}^T\dot{w} + \dot{V}(t)\}d\tau\end{aligned} \quad (25)$$

Considering (23) and $0 < \tau \le \bar{\tau}$ a new upper bound for (25) is obtained as

$$J_{zd} \le \int_0^{\infty}\{\zeta^T\Pi\zeta + x^T(t)(\tau Y_2)x(t) + \dot{x}^T(t-\tau)(\tau Y_2)^T\dot{x}(t-\tau)\}d\tau \quad (26)$$

with defined

$\zeta = [x(t) \;\; x(t-\tau) \;\; \dot{x}(t-\tau) \;\; \ddot{x}(t-\tau) \;\; w(t) \;\; \dot{w}(t)]$

and $\Pi = [\Sigma_{ij}]$ where $\Sigma_{ij} = \Sigma_{ij}^T$ and i,j= 1,2,...6.

in which,

$$\Sigma_{11} = A^TP + PA + Y_1 + Y_1^T + \bar{\tau}(X_1 + X'/2) + (2-\bar{\tau})Y_2 + A\Upsilon_1A + A^TA^T\Upsilon_2AA + Q + C_z^TC_z$$

$$\Sigma_{12} = PBK_pC_y - Y_1 - Y_2 + A^T\Upsilon_1BK_pC_y + A^TA^T\Upsilon_2ABK_pC_y$$

$$\Sigma_{13} = PBK_dC_y - \bar{\tau}Y_2 + A^T\Upsilon_1BK_dC_y + A^TA^T\Upsilon_2(BK_pC_y + ABK_dC_y)$$

$$\Sigma_{14} = A^TA^T\Upsilon_2BK_dC_y$$

$$\Sigma_{15} = PF + A^T\Upsilon_1F + A^TA^T\Upsilon_2AF + C_z^TD_z$$

$$\Sigma_{16} = A^TA^T\Upsilon_2F$$

$$\Sigma_{22} = -Q + (BK_pC_y)^T\Upsilon_1BK_pC_y + (ABK_pC_y)^T\Upsilon_2ABK_pC_y$$

$$\Sigma_{23} = (BK_pC_y)^T\Upsilon_1BK_dC_y + (BK_pC_y)^TA^T\Upsilon_2(BK_pC_y + ABK_dC_y)$$

$$\Sigma_{24} = (BK_pC_y)^TA^T\Upsilon_2BK_dC_y$$

$$\Sigma_{25} = (BK_pC_y)^T\Upsilon_1F + (ABK_pC_y)^T\Upsilon_2AF$$

$$\Sigma_{26} = (ABK_pC_y)^T\Upsilon_2F$$

$$\Sigma_{33} = -R_1 - \bar{\tau}Y_2 + (BK_dC_y)^T\Upsilon_1BK_dC_y + (BK_pC_y + ABK_dC_y)^T\Upsilon_2(BK_pC_y + ABK_dC_y)$$

$$\Sigma_{34} = (BK_pC_y + ABK_dC_y)^T\Upsilon_2BK_dC_y$$

$$\Sigma_{35} = (BK_dC_y)^T\Upsilon_1F + (ABK_dC_y + BK_pC_y)^T\Upsilon_2AF$$

$$\Sigma_{36} = (BK_pC_y + ABK_dC_y)^T\Upsilon_2F$$

$$\Sigma_{44} = -R'/2 + (BK_dC_y)^T\Upsilon_2BK_dC_y$$

$$\Sigma_{45} = (BK_dC_y)^T\Upsilon_2AF \qquad \Sigma_{46} = (BK_dC_y)^T\Upsilon_2F$$

$$\Sigma_{55} = F^T\Upsilon_1F + F^TA^T\Upsilon_2AF + D_z^TD_z - m^2\gamma^2I$$

$$\Sigma_{56} = F^TA^T\Upsilon_2F \qquad \Sigma_{66} = F^T\Upsilon_2F - (1-m)^2\gamma^2I$$

where $\Upsilon_1 = R_1 + \bar{\tau}Z_1$ and $\Upsilon_2 = R'/2 + (\bar{\tau}/2)Z'$.

Consider the constraint $Y_2 = Y_2^T < 0$, if $\zeta^T\Pi\zeta < 0$, then the negative semi definiteness of J_{zd} in (26) is guaranteed for any constant time-delay τ satisfying $0 < \tau \le \bar{\tau}$. Hence, by assuming $w\in\mathcal{W}^{1,2}(0,\infty,\Re^p)\cap\mathcal{L}^2(0,\infty,\Re^p)$ and Π<0 then implies that J_{zd}<0 and therefore $\|T_{zd}\|_\infty<\gamma$. By Lemma 2, the inequality $\|T_{zd}\|_\infty<\gamma$ guarantees $\|T_{zw}\|_\infty<\gamma$

$$\begin{bmatrix} \Omega_1 & BK_pC_y-\alpha_1 I-\alpha_2 I & BK_dC_y-\bar{\tau}\alpha_2 I & 0 & \bar{\tau}LA^T & LA^T & \bar{\tau}L(AA)^T & L(AA)^T \\ * & -Q & 0 & 0 & \bar{\tau}(BK_pC_y)^T & (BK_pC_y)^T & \bar{\tau}(ABK_pC_y)^T & (ABK_pC_y)^T \\ * & * & -R_1-\bar{\tau}\alpha_2 P & 0 & \bar{\tau}(BK_dC_y)^T & (BK_dC_y)^T & \bar{\tau}(BK_pC_y+ABK_dC_y)^T & (BK_pC_y+ABK_dC_y)^T \\ * & * & * & -R_2 & 0 & 0 & \bar{\tau}(BK_dC_y)^T & (BK_dC_y)^T \\ * & * & * & * & -\bar{\tau}Z_1^{-1} & 0 & 0 & 0 \\ * & * & * & * & * & -R_1^{-1} & 0 & 0 \\ * & * & * & * & * & * & -\bar{\tau}Z_2^{-1} & 0 \\ * & * & * & * & * & * & * & -R_2^{-1} \end{bmatrix} < 0 \tag{32}$$

to be satisfied. Use Schur complement, one may show that condition $\Pi<0$ is equivalent to the following matrix inequality:

$$\left[\begin{array}{c|c} \Xi_{11} & \Xi_{12} \\ \hline \Xi_{12}^T & \Xi_{22} \end{array}\right] < 0 \tag{27}$$

with LMI (17) and

$$\begin{bmatrix} X' & \tau Y_2 \\ \tau Y_2 & Z' \end{bmatrix} > 0 \tag{28}$$

where

$$\Xi_{11} = \begin{bmatrix} \Omega_1 & \Omega_2 & \Omega_3 & 0 & PF & 0 \\ * & -Q & 0 & 0 & 0 & 0 \\ * & * & -R_1-\bar{\tau}Y_2 & 0 & 0 & 0 \\ * & * & * & -R'/2 & 0 & 0 \\ * & * & * & * & -m^2\gamma^2 I & 0 \\ * & * & * & * & * & -(1-m)^2\gamma^2 I \end{bmatrix}$$

$$\Xi_{12} = \begin{bmatrix} \bar{\tau}\Delta_1 Z_1 & \Delta_1 R_1 & \bar{\tau}\Delta_2 Z'/2 & \Delta_2 R'/2 & \Delta_3 \end{bmatrix}$$

where

$$\Omega_1 = A^T P + PA + Y_1 + Y_1^T + \bar{\tau}\left(X_1 + (1/2)X'\right) + (2-\bar{\tau})Y_2 + Q$$

$$\Omega_2 = PBK_pC_y - Y_1 - Y_2$$

$$\Omega_3 = PBK_dC_y - \bar{\tau}Y_2$$

$$\Omega_4 = BK_pC_y + ABK_dC_y$$

$$\Delta_1 = \begin{bmatrix} A & BK_pC_y & BK_dC_y & 0 & F & 0 \end{bmatrix}^T$$

$$\Delta_2 = \begin{bmatrix} AA & ABK_pC_y & \Omega_4 & BK_dC_y & AF & F \end{bmatrix}^T$$

$$\Delta_3 = \begin{bmatrix} C_z & 0 & 0 & 0 & D_z & 0 \end{bmatrix}^T$$

and

$$\Xi_{22} = diag\left(-\bar{\tau}Z_1 \quad -R_1 \quad -\bar{\tau}Z'/2 \quad -R'/2 \quad -I\right)$$

Furthermore, we may define $Y_1=\alpha_1 P$, $Y_2=\alpha_2 P$, $L=P^{-1}$ with arbitrary scalar values α_1 and $\alpha_2<0$. Denote P^{-1}, Z_1^{-1}, R_1^{-1} as L, F_1, and H_1 respectively, by performing a congruence transformation to (27) by diag $diag\left(L, I, I, I, I, I, F_1, H_1, 2Z'^{-1}, 2R'^{-1}, I\right)$ together with introducing the change of variables $M_1=LX_1L$, $M_2=L(X'/2)L$, $R_2=R'/2$, $Z_2=Z'/2$, the matrix inequality (6) is derived. Similarly, Pre and post multiplying the LMI (17) by diag (L , I), the LMI (7) is provided. Moreover, by performing a congruence transformation to (18) by diag (L , I), we can obtain

$$\begin{bmatrix} LX'L & \tau\alpha_2 I \\ \tau\alpha_2 I & Z' \end{bmatrix} > 0$$

Using Schur complement, we have

$$LX'L - (\tau\alpha_2 I)(Z')^{-1}(\tau\alpha_2 I) > 0$$

Substitute $M_2 = L(X'/2)L$ and $2Z_2=Z'$, the following matrix inequality is derived.

$$2M_2 - (\tau\alpha_2 I)(2Z_2)^{-1}(\tau\alpha_2 I) > 0 \tag{29}$$

On the other hand we have

$$2M_2 - (\tau\alpha_2 I)(2Z_2)^{-1}(\tau\alpha_2 I)^T \geq 2M_2 - (\bar{\tau}\alpha_2 I)(2Z_2)^{-1}(\bar{\tau}\alpha_2 I)^T \tag{30}$$

Therefore, satisfying the following inequality guarantees the inequality (29) to be satisfied.

$$2M_2 - (\bar{\tau}\alpha_2 I)(2Z_2)^{-1}(\bar{\tau}\alpha_2 I)^T > 0 \tag{31}$$

Applying Schur complement, the matrix inequality (8) is obtained. To guarantee asymptotic stability of the difference operator

$$\mathcal{D}(x_t) = x(t) - BK_dC_y x(t-\tau)$$

it suffices to guarantee $\bar{\sigma}(BK_dC_y)<1$ or $(BK_dC_y)^T(BK_dC_y)<I$. Using Schur complement and performing a congruence transformation by diag (L, I), the matrix inequality (9) is provided. This completes the proof. ■

Remark 2: It should be noted that the resulting matrix inequality (6) is not an LMI condition due to the terms LQL, Z_i^{-1} and R_i^{-1}. A good remedy to deal with this problem is to apply iterative method presented by Moon et al. [24]. Exploiting this method enables us to replace the existing nonconvex optimization problem with a nonlinear minimization problem with LMI conditions. This iteration algorithm works efficiently for many examples. Moreover, it helps us to find an initial guess in the feasibility region and improve the solution iteratively by applying BMI conditions obtained during the proof of Theorem 1.

Remark 3: Theorem 1 can be further modified to cope with stabilization problem, leading to the following corollary.

Corollary 1: For given scalar $\bar{\tau}>0$, the closed-loop system (3) is asymptotically stable for any constant time-delay τ satisfying $0<\tau\leq\bar{\tau}$, if there exist scalars $\alpha_1\in\Re$, $\alpha_2<0$, positive definite symmetric matrices L, P, T, Z_1, Z_2, R_1, R_2 $\in\Re^{n\times n}$, matrices M_1, M_2 $\in\Re^{n\times n}$, K_p, K_d $\in\Re^{l\times n}$ satisfying matrix inequalities (32) ~ (35). Moreover, H$_\infty$ PD control law is given by $u=K_p y+K_d\dot{y}$.

$$\begin{bmatrix} M_1 & \alpha_1 I \\ \alpha_1 I & Z_1 \end{bmatrix}>0 \tag{33}$$

$$\begin{bmatrix} 2M_2 & \bar{\tau}\alpha_2 I \\ \bar{\tau}\alpha_2 I & 2Z_2 \end{bmatrix}>0 \tag{34}$$

and,

$$\begin{bmatrix} I & (BK_dC_y)^T \\ BK_dC_y & I \end{bmatrix}>0 \tag{35}$$

in which,

$$\Omega_1=LA^T+AL+2\alpha_1 L+\bar{\tau}(M_1+M_2)+\alpha_2(2-\bar{\tau})L+T$$

4. CASE STUDY: CONCENTRATION IN AN UNSTABLE REACTOR

In this example we consider a chemical reactor with unstable dynamical behaviour. For control purposes, $C(t)$, the output concentration, is the output variable and $C_i(t)$, the input concentration, is the manipulated variable. Furthermore, the concentration transducer needs a dead time of 20 seconds to give the output variable. The linearized transfer function from $C_i(t)$ to $C(t)$ at the operating point is obtained as follows [22]:

$$P(s)=\frac{3.433e^{-20s}}{103.1s-1}$$

with an unstable pole at s=1/103.1. Normey-Rico et.al [22] presented a PI controller in smith predictor (SP) structure to stabilize the above system as

$$C_1(s)=3.29\left(1+\frac{1}{43.87s}\right)$$

. Two additional filters $F(s)$ and $F_r(s)$ are proposed in SP to improve the set-point response and the predictive properties.

To evaluate the method presented in this paper, we consider a cascade controller as shown in figure 1. Applying our method proposed in Theorem 1 and considering remark 2, the PD controller in the inner loop is designed to stabilize the process. For tracking purposes a PI controller is applied using Fertik method [23].

Let us consider $\bar{\tau}=20$ and apply Theorem 1 to find an H$_\infty$ PD controller for the above system. By solving the matrix inequalities (6) ~ (9), the deigned PD controller for the inner loop is obtained as

$$C_2(s)=0.707(1+.1425s)$$

For a set point tracking problem we apply Fertik method and design a PI controller for the outer loop as

$$C_3(s)=0.647\left(1+\frac{1}{51.2s}\right)$$

We also use an additional filter to improve the set point response of the closed-loop system as

$$F(s)=\frac{1}{30s+1}$$

To see the closed-loop response of our proposed structure and the one presented in [22], we consider $\bar{\tau}=20$, a set point change to 5 mol/l at t=50s and a flow step disturbance is introduced at t=700 s. Simulation results are shown in figure 2. As it is shown in Figure 2, despite the results reported in [22], the closed-loop system shows an unstable behaviour in a large simulation time applying controller $C_1(s)$ whereas our designed controller provides stable response with good disturbance rejection in presence of time-delay. In order to evaluate the robustness of the closed-loop system with respect to time-delay variations or dead-time estimation error, consider the closed-loop system for the ±25% time-delay variation, that is τ=25s and τ=15s. Figures 3 & 4 show the simulation results of the closed-loop for time-delay variations. As it is shown in figures 3-4, our designed controller provides stable and well-damped response even in presence of considerable variations in time-delay. Moreover, it is observed in the simulations that the closed-loop system with controller $C_2(s)$ remains in the stable region for a maximum delay of τ_{max}=29.6 s.

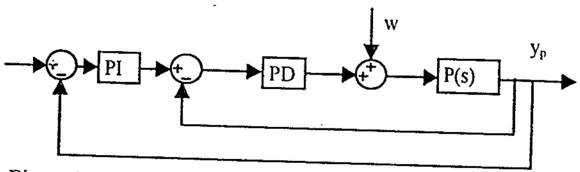

Figure 1. Cascade Control Structure

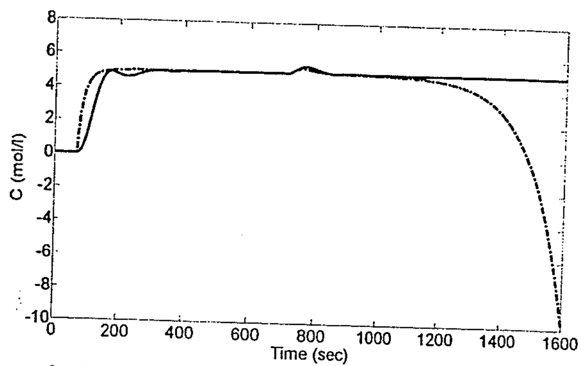

Figure 2: Step response of the plant output with τ =20 s. Proposed controller (solid) and Normey et al. 's controller (dashed)

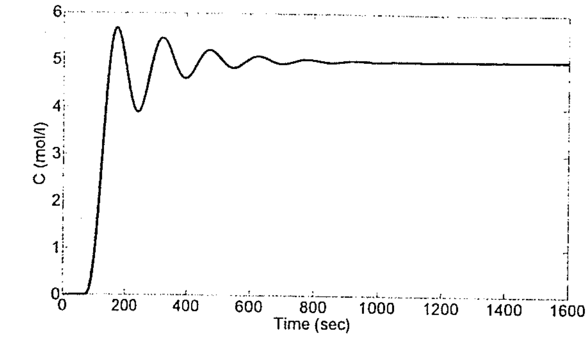

Figure 3. Step response of the plant output with τ=25 s.

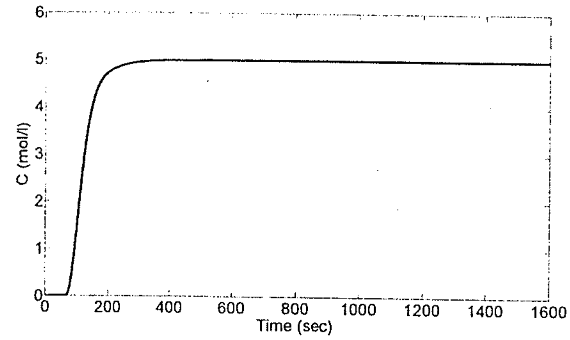

Figure 4. Step response of the plant output with τ=15 s.

5. CONCLUSIONS

H_∞ control of a time-delay system with input delay for uncertain time-delay is elaborated in this paper. The resulting closed-loop system with the PD control law is a particular system of neutral type. In this system, the coefficients of delayed terms depend on the control law parameters. The Lyapunov theory is used to derive a set of delay-dependent sufficient conditions in presence of uncertain time-delay. A sufficient condition is derived for the existence of an H_∞ PD controller for the closed loop system in terms of matrix inequalities. Moreover, a practical example is presented in this paper to illustrate the effectiveness of our method. Simulations show good disturbance rejection as well as robustness in presence of time-delay variations.

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