SLAM Based on Intelligent Unscented Kalman Filter

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***Abstract*—The performance of SLAM based on unscented kalman filter (UKF-SLAM) and thus the quality of the estimation depends on the correct a priori knowledge of process and measurement noise. Imprecise knowledge of these statistics can cause significant degradation in performance. In this paper, the adaptive Neuro-Fuzzy has been implemented to adapt the matrix covariance process of UKF-SLAM in order to improve its performance.**

I. INTRODUCTION

The simultaneous localization and mapping (SLAM) is a fundamental problem of navigation of a mobile platform in an environment where both the map of the environment and the localization of the mobile are unknown. There are many solutions to the SLAM problem [1]. The extended kalman filter is the oldest and the most popular approach to solve the SLAM (EKF-SLAM). In this approach, the extended kalman filter (EKF) is employed to estimate the state of vehicle and feature map [2]. The effectiveness of this approach lies on the fact that it holds a fully correlated posterior over robot poses feature map. However, the serious drawbacks of this approach are the linear approximations of non-linear functions and the derivation of Jacobians. As proved in [1-2], the EKF-SALM is inconsistent due to errors introduced during linearization. This introduces inaccurate maps with filter divergence. For the nonlinearity problem, a number of authors have applied the straight unscented kalman filter to SLAM problem [3-6]. The UKF-SLAM avoids linearization by parameterizing the mean and the covariance with a set of sigma points to which the nonlinear transformation is applied. As result, this approach improves the consistency over the EKF-SLAM [7-10]. However, the performance of UKF-SLAM depends largely on the accuracy of knowledge of process covariance matrix and measurement noise covariance matrix [11-16]. In most application of UKF-SLAM these matrixes are unknown. On the other hand it is well known how poor estimates of noise statistics may seriously degrade the Kalman filter performance [11-15]. In this paper, adaptive Neuro-Fuzzy inference system supervise the performance of the UKF-SLAM with adjusts the process covariance matrix. The results show the performance of the proposed method outperforms UKF-SALM. The rest of this paper is organized as follows. In next section we briefly review the required background. The UKF-SLAM is described in section III. The main contribution of this paper is introduced in section IV, which presents the SLAM based on intelligent unscented kalman filter is proposed. The effectiveness of the proposed algorithm is demonstrated using simulation results in sections V.

II. BACKGROUND

A. The SLAM Problem

The goal of SLAM is to simultaneously localize a robot and determine an accurate map of the environment. To describe SLAM, let us denote the map by Θ and the pose of the robot at time t by s_t. The map consists of a collection of features, each of which will be denoted by θ_n and the total number of stationary features will be denoted by N. In this situation, the SLAM problem can be formulized in a Bayesian probabilistic framework by representing each of the robot's position and map location as a probabilistic density function as [17]:

$$p(s_t, \Theta \mid z^t, u^t, n^t) \quad (1)$$

In essence, it is necessary to estimate the posterior density of maps Θ, and poses s_t, given that we know the observation $z^t = \{z_1, ..., z_t\}$, the control input $u^t = \{u_1, ..., u_t\}$ and the data association n^t. Here, data association represents the mapping between map points in Θ and observations in z^t. The SLAM problem is then achieved by applying Bayes filtering as follows [17]:

$$\begin{aligned} & p(s_t, \Theta \mid z^t, u^t, n^t) \propto p(z_t \mid s_t, \Theta, n_t) \\ & p(s_t, \Theta \mid z^{t-1}, u^t, n^t) \end{aligned} \quad (2)$$

with

$$\begin{aligned} & p(s_t, \Theta \mid z^{t-1}, u^t, n^t) = \int p(s_t \mid s_{t-1}, u_t) \\ & p(s_{t-1}, \Theta \mid z^{t-1}, u^t, n^t) ds_{t-1} \end{aligned} \quad (3)$$

Where $p(s_t \mid s_{t-1}, u_t)$ is the dynamics motion model and $p(z_t \mid s_t, \Theta, n^t)$ is the measurement model. The Extended Kalman Filter is a popular choice to approximate of the general Bayes filter. However, as mentioned previous, this approach has two serious drawbacks, namely the linear approximation of nonlinear functions and the calculation of Jacobin matrixes.

B. The Unscented Kalman Filter

Consider the general discrete nonlinear system:

$$\begin{aligned} x_{k+1} &= f(x_k) + w_k \\ y_k &= h(x_k) + v_k \end{aligned} \quad (4)$$

Where x_k and y_k denote the state vector variable and observations at time k, f and h are known possibly nonlinear functions. The w_k and v_k are the Gaussian white process noise and measurement noise, respectively, with covariance:

$$\omega_k \sim \mathrm{N}(0,Q_k) \qquad \upsilon_k \sim \mathrm{N}(0,R_k) \tag{5}$$

The unscented transformation (UT) forms the core of the UKF algorithm. In UKF, the state distribution is represented by the deterministically chosen sigma points which can capture specific mean and covariance of the distribution [18-19]. The nonlinear function is applied to each of these points to yield a transformed sigma point, and then the predicted mean and covariance are calculated form the transformed sigma point. The UKF estimation can be described briefly as follows [18-19]:

1) Initialization

$$\bar{x}_0 = E[x_0]$$
$$P_0 = E[(x_0-\bar{x}_0)(x_0-\bar{x}_0)^T] \tag{6}$$

2) Computing Sigma Points

A set of $2n+1$ weighted samples are chosen as follows:

$$\chi_{k-1} = [\bar{x}_{k-1}, \bar{x}_{k-1} \pm (\sqrt{(N+\lambda)P_{k-1}})_i] \quad i=1...2n \tag{7}$$

3) Time Update

These sigma points pass through the process model and transformed sigma points is calculated as follow:

$$\chi^*_{k|k-1} = f(\chi_{k-1}) \tag{8}$$

Then, the first two moments of the density function of x_k are computed by a weighted linear regression of the transformed sigma points:

$$\bar{\chi}_{k|k-1} = \sum_{i=1}^{2n} \omega_m^{[i]} \chi^*_{i,k|k-1} \tag{9}$$

$$P_{k|k-1} = \sum_{i=0}^{2n} \omega_c^{[i]} (\chi^*_{i,k|k-1} - \bar{\chi}_{k|k-1})(\chi^*_{i,k|k-1} - \bar{\chi}_{k|k-1})^T + Q_k \tag{10}$$

where

$$\gamma_{k|k-1} = h(\chi_{k|k-1}) \tag{11}$$

$$\bar{y}_{k|k-1} = \sum_{i=0}^{2n} \omega_m^{[i]} \gamma_{i,k|k-1} \tag{12}$$

And the weights are given by:

$$\omega_c^{[0]} = \frac{\lambda}{n+\lambda} + (1-\alpha^2+\beta)$$
$$\omega_m^{[0]} = \frac{\lambda}{n+\lambda} \tag{13}$$
$$\omega_m^{[i]} = \omega_c^{[i]} = \frac{\lambda}{2(n+\lambda)} \quad \text{i=1,...,2n}$$
$$\lambda = n(\alpha^2-1)$$

The parameter α determines the speed of the sigma points around $\bar{x}$ and usually set to $1e-4 \le \alpha \le 1$. The constant β is used to incorporate part of the prior knowledge of the distribution of x and for Gaussian distribution $\beta = 2$ is optimal.

4) Measurement Update

$$P_{\bar{y}_k\bar{y}_k} = \sum_{i=0}^{2n} \omega_c^{[i]} (\gamma_{i,k|k-1} - \bar{y}_{k|k-1})(\gamma_{i,k|k-1} - \bar{y}_{k|k-1})^T + R_k$$
$$P_{\bar{x}_k\bar{y}_k} = \sum_{i=0}^{2n} \omega_c^{[i]} (\chi_k^{[i]} - \hat{x}_k)(\gamma_{i,k|k-1} - \bar{y}_{k|k-1})^T \tag{14}$$

The kalman gain matrix and state covariance update derived in the fashion familiar form EKF as follows

$$K_s = P_{\bar{x}_k\bar{y}_k} P^{-1}_{\bar{y}_k\bar{y}_k}$$
$$P_k = P_{k|k-1} - K_s P_{\bar{y}_k\bar{y}_k} K_s^T \tag{15}$$
$$\bar{x}_k = \bar{x}_{k|k-1} + K_s(y_k - \bar{y}_{k|k-1})$$

III. THE UKF-SLAM

In the SLAM problem, the state vector X_a is composed of the vehicle states X_v and the landmark's states X_m. Therefore, the estimates of total the total state vector X_a, maintained in the form of its mean vector $\hat{X}_a$ and the corresponding total error covariance matrix $\hat{P}_a$ is given as follows:

$$\hat{X}_a = [\hat{X}_v^T \ \hat{X}_m^T]^T \tag{16}$$

$$\hat{P}_a = \begin{bmatrix} \hat{P}_v & \hat{P}_{vm} \\ \hat{P}_{vm}^T & \hat{P}_m \end{bmatrix} \tag{17}$$

Where $\hat{X}_v$ is mean estimation of the robot states (robot pose), $\hat{P}_v$ is error covariance matrix associated with $\hat{X}_v$, $\hat{X}_m$ is mean estimate of the feature positions and $\hat{P}_m$ is error covariance matrix associated with $\hat{X}_m$. The map is defined in terms of the position estimates of these statistic features and $\hat{P}_{vm}$ in (17) is cross correlation between vehicle and map. The vehicle pose is initialized assuming that there is no observed feature for now and there is not uncertainty in the starting pose of the vehicle i.e.

$$\hat{X}_a = \hat{X}_v = 0 \qquad \hat{\mathrm{P}}_a = \hat{\mathrm{P}}_v = 0 \tag{18}$$

When vehicle moving, $\hat{X}_v$ and $\hat{P}_v$ become non-zero values. In subsequent iterations, when the first observation is carried out, new features are expected to be initialized and $\hat{X}_m$ and $\hat{P}_m$ appear for the first time. Therefore, the size of $\hat{X}_a$ and $\hat{\mathrm{P}}_a$ increases and this process is continued iteratively. For implementation UKF-SLAM, the first step is prediction. To predict over a time step k to $k+1$, the zeroth sigma point, $\chi_{k-1}^{a[0]}$, is defined equal to the current state and a father $2n$ sigma points are determined by adding and subtracting in the turn the transpose of the ith row of the square root of $\hat{P}^a_{k-1}$ to current mean. Therefore, a symmetric set of

$2n+1$ sigma points $\chi_{k-1}^{a[i]}$ for augmented state as follows [8-9]:

$$\chi_{k-1}^{a[0]} = \hat{X}_{k-1}^{a} \quad (19)$$

$$\chi_{k-1}^{a[i]} = \hat{X}_{k-1}^{a} + \sqrt{(n+\lambda)\hat{P}_{k-1}^{a}}\ , \qquad i=1,...n \quad (20)$$

$$\chi_{k-1}^{a[i]} = \hat{X}_{k-1}^{a} - \sqrt{(n+\lambda)\hat{P}_{k-1}^{a}}\ , \qquad i=n+1,...2n \quad (21)$$

Each sigma point $\chi_{k-1}^{a[i]}$ contains the state and control noise components given by:

$$\chi_{k-1}^{a[i]} = \left[\chi_{v,k-1}^{[i]} \quad \chi_{m,k-1}^{[i]} \quad \chi_{k}^{u[i]}\right]^{T} \quad (22)$$

The set of sigma points $\chi_{v,k-1}^{[i]}$ is transformed by the motion model using the current control u_k with the added model control noise component $\chi_{k-1}^{u[i]}$ of each sigma point

$$\bar{\chi}_{v,k}^{[i]} = f_v(\chi_{v,k-1}^{[i]}, u_k + \chi_{k}^{u[i]}) \quad (23)$$

Also, we have for features equation as following:

$$\bar{\chi}_{m,k}^{[i]} = \chi_{m,k-1}^{[i]} \quad (24)$$

The first two moments of the predicted state are computed by a linear weighted regression of the transformed sigma points $\bar{\chi}_{k-1}^{a[i]}$ as following:

$$\hat{X}_{k|k-1}^{a} = \sum_{i=0}^{2n} \omega_g^{[i]} \bar{\chi}_{k-1}^{a[i]}$$

$$\hat{P}_{k|k-1}^{a} = \sum_{i=0}^{2n} \omega_c^{[i]} (\bar{\chi}_{k-1}^{a[i]} - \hat{X}_{k|k-1}^{a})(\bar{\chi}_{k-1}^{a[i]} - \hat{X}_{k|k-1}^{a})^T + Q_k \quad (25)$$

$$\omega_g^{[0]} = \frac{\lambda}{(n+\lambda)} \qquad \omega_c^{[0]} = \frac{\lambda}{(n+\lambda)} + (1-\alpha^2+\beta)$$

$$\omega_g^{[i]} = \omega_c^{[i]} = \frac{\lambda}{2(n+\lambda)} \qquad \text{(i=1,...,2n)}$$

Here, the parameter α determines the speed of the sigma points and usually set to $1e-4 \le \alpha \le 1$. Also, the parameter β is used to incorporate the knowledge of the higher order moments of the posterior distribution. For a Gaussian prior, the optimal choice is $\beta = 2$. Data association provides the observation z_k statistically compatible and related to the augmented state vector by a non-linear function h_k as follows:

$$z_k = h_k(\hat{X}_{g\,k|k-1}^{a}) \quad (26)$$

Hence, the updated state estimate and its corresponding covariance matrix can be computed by following equations:

$$\hat{X}_{k}^{a} = \hat{X}_{k|k-1}^{a} + P_{\chi\upsilon}(S_k)^{-1}(z_k - \bar{z}_k) \quad (27)$$

$$\hat{P}_{k}^{a} = \hat{P}_{k|k-1}^{a} - K_k S_k (K_k)^T \quad (28)$$

where

$$\bar{z}_k = \sum_{i=0}^{2n} w_g^{[i]} \gamma_k^{[i]} \quad (29)$$

$$K_k = P_{\chi\upsilon}(S_k)^{-1}$$

with

$$\gamma_k^{[i]} = h(\bar{\chi}_{k-1}^{a[i]}) \quad (30)$$

$$S_k = \sum_{i=0}^{2n} w_c^{[i]} (\gamma_k^{[i]} - \bar{z}_k)(\gamma_k^{[i]} - \bar{z}_k)^T + R_k \quad (31)$$

$$P_{\chi\upsilon} = \sum_{i=0}^{2n} \omega_c^{[i]} (\bar{\chi}_{k-1}^{a[i]} - \hat{X}_{k|k-1}^{a})(\gamma_k^{[i]} - \bar{z}_k)^T \quad (32)$$

IV. SLAM BASED ON INTELLIGENT UNSCENTED KALMAN FILTER

The UKF-SLAM, assumes complete a priori knowledge of the process and measurement noise statistics (matrices Q_k and R_k respectively). However, in a real-life UKF-SLAM, these matrixes are unknown. An incorrect a prior knowledge of Q_k and R_k may lead to performance degradation and it can even lead to practical divergence [11-15]. One of the efficient ways to overcome the above weakness is to use an adaptive algorithm. We assume that the noise covariance R_k is completely known. Hence, the algorithm to estimate the process noise covariance Q_k can be derived. The adaptation is adaptively adjusting the process noise covariance matrix Q_k by using adaptive neuro-fuzzy inference System (ANFIS). In this case, an innovation based adaptive estimation (IAE) algorithm to adapt the process noise covariance matrix Q_k is derived. The technique known as covariance-matching is used [15], [16]. The basic idea behind this technique is to make the actual value of the covariance of the residual to be consistent with its theoretical value [15-16]. The innovation sequence $r_k = (z_k - \hat{z}_k)$ has a theoretical covariance that is obtained from the UKF algorithm:

$$S_k = \sum_{i=0}^{2N} w_c^{[i]} (\gamma_k^{[i]} - \bar{z}_k)(\gamma_k^{[i]} - \bar{z}_k)^T + R_k \quad (33)$$

The actual residual covariance $\hat{C}_k$ can be approximated through averaging inside a moving window of size N_m as following:

$$\hat{C}_k = \frac{1}{N} \sum_{i=k-N_m+1}^{k} (r_i^T r_i) \quad (34)$$

If the actual value of covariance $\hat{C}_k$ has discrepancies with its theoretical value, then the diagonal elements of Q_k based on the size of this discrepancy can be adjusted. The size of discrepancy is given by a variable called the degree of mismatch (DOM_k), defined as:

$$DOM_k = S_k - \hat{C}_k \quad (35)$$

It may be deduced from equation (33) that a variation in Q_k will affect the value of S_k Thus, if a mismatch between S_k and $\hat{C}_k$ is observed then a correction can be made through augmenting or diminishing the value of Q_k .The tree general adaptation rules are defined as following:

1. If $DOM_k(1,1)$ is Low and $DOM_k(2,2)$ is Low then Q_k is High

2. If $DOM_k(1,1)$ is Zero and $DOM_k(2,2)$ is Zero then Q_k is Zero

3. If $DOM_k(1,1)$ is High and $DOM_k(2,2)$ is High then Q_k is Low

Then Q_k is adapted in this way

$$Q_k = Q_k \Delta Q_k \tag{36}$$

Where ΔQ_k is the ANFIS output and $DOM_k(1,1)$ and $DOM_k(2,2)$ are ANFIS input. The ANFIS model has been considered as a two-input-single-output system as Fig.1. The parameters of conclusion part of ANFIS are trained using the steepest gradient descent. Fig.2 shows the robot trajectory and landmark location. The star points (*) depict the location of the landmarks that are known and stationary in the environment. The kinematics equations for the mobile robot are as follows:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} (V + v_v)\cos(\phi + [\gamma + v_\gamma]) \\ (V + v_v)\sin(\phi + [\gamma + v_\gamma]) \\ \dfrac{(V + v_v)}{B}\sin(\gamma + v_\gamma) \end{bmatrix} \tag{37}$$

Where (x, y) is the Cartesian coordinates, ϕ is the orientation respective to the global environment, B is the base line of the vehicle and $u = [V \quad \gamma]^T$ is the control input at time t consist of a velocity input V and a steer input γ. The process noise $v = [v_v \quad v_\gamma]^T$ is assumed to be Gaussian. The vehicle is assumed to be equipped with a range-bearing sensor that provides a measurement of range r_i and bearing θ_i to an observed feature ρ_i relative to the vehicle. The observation z of feature ρ_i in the map can be expressed as:

$$\begin{bmatrix} r_i \\ \theta_i \end{bmatrix} = \begin{bmatrix} \sqrt{(x - x_i)^2 + (y - y_i)^2} + \omega_r \\ \tan^{-1}\dfrac{y - y_i}{x - x_i} - \phi + \omega_\theta \end{bmatrix} \tag{38}$$

Where (x_i, y_i) is the landmark position in map and $W = [\omega_r \quad \omega_\theta]^T$ relates to observation noise. The robot moves at a speed 3m/s and with a maximum steering angle 30 deg. Also, the robot has 4 meters wheel base and is equipped with a range-bearing sensor with a maximum range of 20 meters and a 180 degrees frontal field-of-view. The control noise is $\sigma_v = 0.3$ m/s and $\sigma_\gamma = 3^o$. A control frequency is 40 HZ and observation scans are obtained at 5 HZ. The measurement noise is 0.1 m in range and 1^o in bearing. For evaluate proposed method the performance of it is compared with UKF-SLAM, we consider the situation where process noise is wrongly considered. The performance of the proposed method is compared with classical UKF-SLAM where its process covariance matrix Q_k is kept static throughout the experiment in two environments: sparse environment with 35 landmarks and dense environment with 75 landmarks.

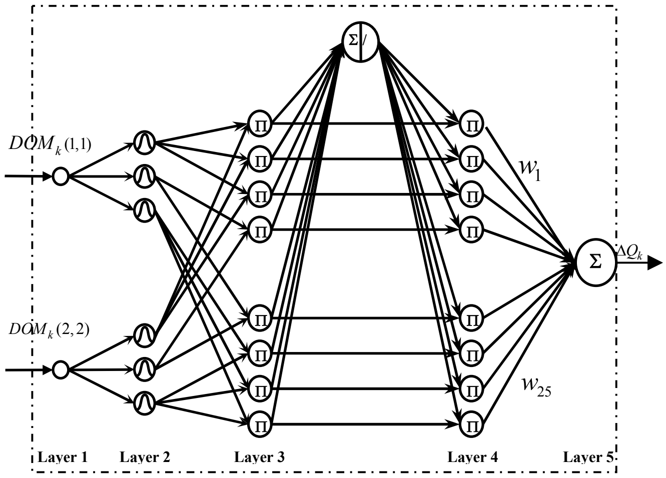


Fig. 1.The adaptive Neuro-fuzzy for tuning Q_k

V. SIMULATION RESULTS

Simulation experiments have been carried out to evaluate the performance of the proposed approach in comparison with EKF-SLAM. The proposed method has been tested for the benchmark environment, with varied number and position of the landmarks, available in [20].

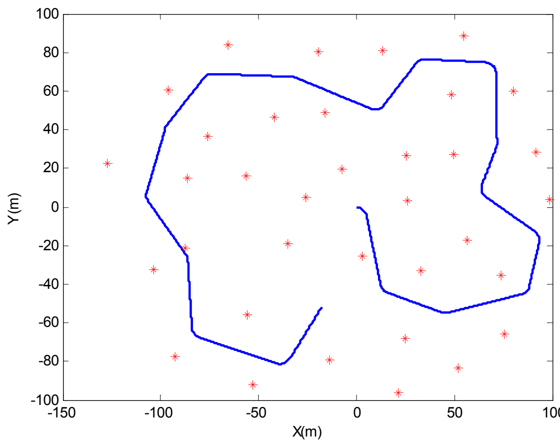
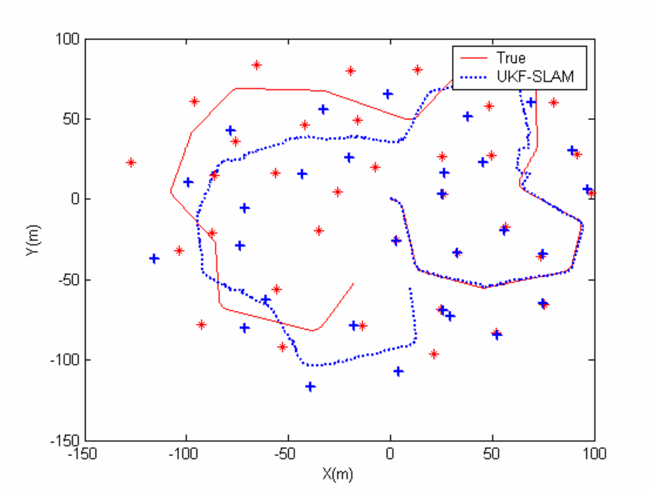
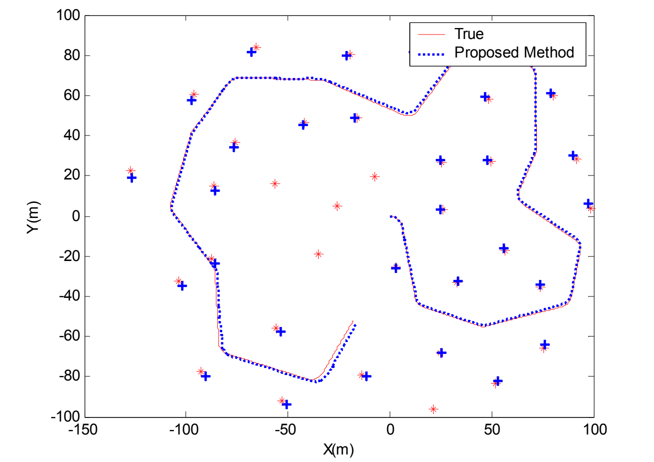


Fig. 2.The experiment environment: The landmark positions and path of robot.

Fig.3 and Fig.4 show the performance of the two algorithms. As observed, SLAM based on Adaptive Neuro-Fuzzy UKF is more accurate than EKF-SLAM. This is because proposed method adaptively tuned the process covariance matrix Q_k and converges to the actual covariance matrix Q_k while covariance matrix measurement Q_k in UKF-SLAM is kept fixed over time.

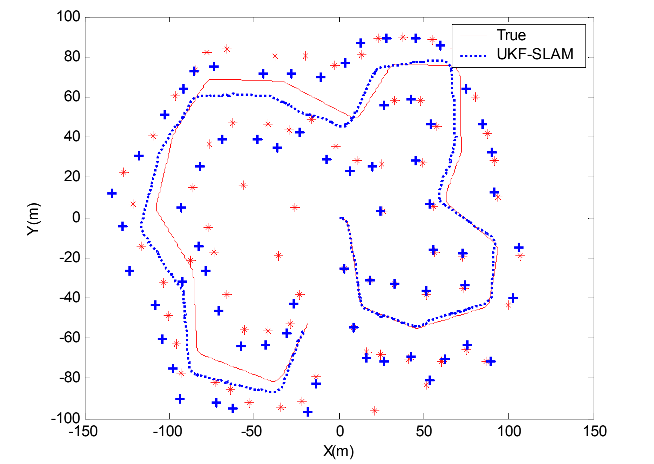


(a)

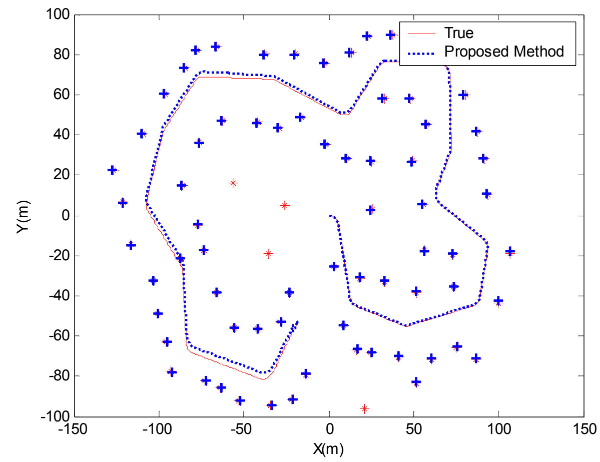


(b)

Fig. 3. Estimated robot path and estimated landmark with true robot path and true landmarks (with 35 landmarks):(a) UKF-SLAM (b) proposed method



(a)



(b)

Fig. 4. Estimated robot path and estimated landmark with true robot path and true landmarks (with 75 landmarks):(a) UKF-SLAM (b) proposed method

VI. CONCLUSION

The preset paper has proposed SLAM based on intelligent unscented kalman filter. The simulation results are shown that while the UKF-SLAM showed unreliable performance, in proposed method, proposed method can provide robust and accurate performance in each sample situation in each case study.

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