Feedback Error Learning Control of Trajectory Tracking of Nonholonomic Mobile Robot

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***Abstract*— In this paper a new controller for nonholonomic system is introduced. This feedback error learning controller benefits from both nonlinear and adaptive controller properties. The nonlinear controller is used to stabilize the nonholonomic behavior of the systems. This controller is a sliding mode controller which is designed based on backstepping method. The adaptive controller tries to face with uncertainty and unknown dynamic of the mobile robot. This part uses neural network controller for adaptation. The experimental results show the effectiveness of proposed controller and suitable and robust tracking performance of a mobile robot, which is significantly better than traditional controllers.**

I. INTRODUCTION

Stabilizing a nonholonomic system is a challenging issue. As in a nonholonomic robot controllable degrees of freedom are less than total degrees of freedom , one must try to find suitable control inputs to guarantee stability of all states. A good survey on nonholonomic system is given in [1] and the references within. Different approaches have been proposed for stabilizing and tracking purpose of a nonholonomic systems. These methods can be divided into two main categories: open loop strategies, and (nonlinear) close loop methods. Due to uncertainty of these systems, neural network controllers are used in both cases. Some examples of open loop strategies are given in [2], [3]. Nonlinear theories are wildly used as closed loop strategies. [4], [5], [6], [7], [8] are some examples of this class of strategies, where, [4] uses feedback linearization for stabilizing a nonholonomic system, while, [5] proposes an algorithm for exponentially stabilizing of an uncertain system. In [6], [7] adaptive stabilization of an uncertain system is investigated.

According to Brockett's theorem, nonholonomic systems with restricted mobility can not be stabilized to desired configuration via linear control methods [9]. Therefore, nonlinear control theories are needed to be implemented for such systems. However, most of these strategies need an exact state space model. In some cases, control inputs are based on linear and angular velocities, which means that the system dynamics are neglected in the process of controller design. This may lead us to use neural network controller due to ability for adaptation in model uncertainties. On the other hand, Backstepping is widely used for control of mobile robot. The classical back stepping controller is a starting point for development of such controllers in practice. For example [10] used the basic idea of backstepping in defining sliding surfaces.

A combination of sliding and neural network controller is proposed in this paper. This controller benefits from both classical nonlinear and neural network controllers. Due to uncertainty in such systems, a learning control may be considered. A simple and powerful scheme for adaptive learning is considered as the feedback error learning (FEL) scheme, while [11] is a good survey on FEL. The proposed algorithm has shown brilliant performance in terms of tracking. This paper is organized as follows: in section 2, kinematic model of mobile robot is introduced and error equations are derived. Section 3 contains description about sliding mode controller, FEL and proposed controller. In section 4 experimental results obtained by mobile robot in real world situations are presented and finally in section 5 conclusion and comparison between proposed controller and sliding mode controller are analyzed.

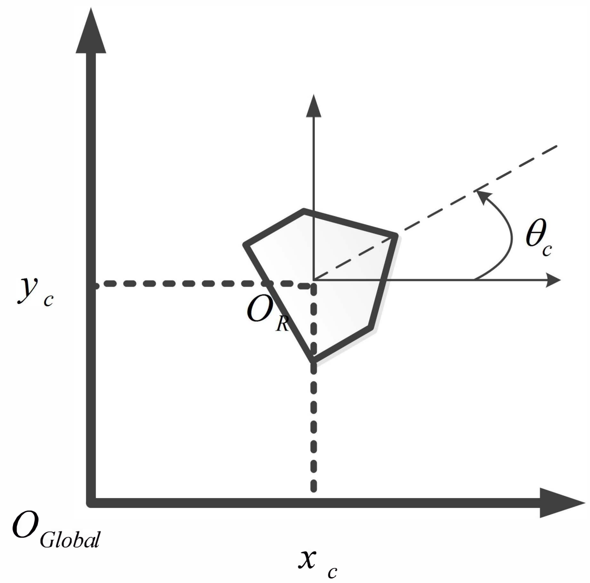

Fig. 1: Robot position and orientation in global frame

II. KINEMATIC MODEL

For designing a classical controller, a kinematic model is needed to describe the system behavior. A mobile robot moving in plane can be expressed by 3 degree of freedom in global frame. Lets $p_c = [x_c, y_c, \theta_c]^T$ shows robot current position and orientation in global frame. As it can be seen in Fig. 1 x_c and y_c show robot position in x and y directions and θ represents robot orientation with respect to x direction. The mobile robot motion is controlled by linear and angular velocity. The mobile robot kinematic can be defined as follows [12]:

$$\dot{p}=\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix}=\begin{bmatrix}\cos\theta & -d\sin\theta\\ \sin\theta & d\cos\theta\\ 0 & 1\end{bmatrix}\begin{bmatrix}v\\ \omega\end{bmatrix} \tag{1}$$

In Eq. 1, d denotes the distance between robot centroid and robot frame. The control objective is to steer the mobile robot such that follows the desired trajectory. Suppose $p_r=[x_r,y_r,\theta r]^T$ and $q_r=[v_r,\omega_r]^T$ represent desire position and velocity, respectively. By considering tracking problem, error equations described in robot frame are more useful for the controller design and it can been easily derived by coordinate transformation from global frame as follows [13]:

$$p_e=\begin{bmatrix}x_e\\ y_e\\ \theta_e\end{bmatrix}=\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_r-x_c\\ y_r-y_c\\ \theta_r-\theta_c\end{bmatrix} \tag{2}$$

Using Eq. 1 and Eq. 2, time derivative of tracking error is determined as follows:

$$\dot{p_e}=\begin{bmatrix}y_e\omega-v+v_r\cos\theta_e-d\omega_r\sin\theta_e\\ -x_e\omega-d\omega+v_r\sin\theta_e+d\omega_r\cos\theta_e\\ \omega_r-\omega\end{bmatrix} \tag{3}$$

Trajectory tracking of a mobile robot is to design controller inputs which makes the posteriori error bounded for any initial error:

$$\lim_{t\to\infty}\|(x_e,y_e,\theta_e)\|^T=0 \tag{4}$$

III. Controller Design

A. Sliding Mode Controller

The mobile robot which is used in this paper is a tracked mobile robots. Control inputs used for this system are linear and angular velocity. However, robot moves in plane and has 3 degrees of freedom. Such underactuated systems are called *nonholonomic systems* if they constitute velocity type constraint(s) which are called nonholonomic constraint. For a mobile robot, the nonholonomic constraint can be written as bellow:

$$\dot{x}\sin\theta-\dot{y}\cos\theta=0 \tag{5}$$

The most popular controller for nonholonomic mobile robot is controller based on backstepping method. The backstepping controller which is suggested by [14], is widely used for controlling nonholonomic mobile robots. It has been proved in this reference that tracking error is uniformly bounded and converges to zero. Backstepping method suggests changing in controller structure such that the stability of the system is guaranteed through a Lyapunov analysis. This transformation can be used to define sliding surfaces. Considering $x_e=0$ and $\theta_e=-\arctan v_r y_e$ andy_e becomes stable. Based on[14] a change of variable in backstepping controller is as follows

$$\theta_e=-\arctan v_r y_e \tag{6}$$

Eq. 6 can be used as a sliding surface. Sliding surfaces are defined as follows:

$$S=\begin{bmatrix}S_1\\ S_2\end{bmatrix}=\begin{bmatrix}x_e\\ \theta_e+\arctan v_r y_e\end{bmatrix} \tag{7}$$

If these sliding surfaces converge to zero, all states will become stable. The time derivative of sliding surfaces are defined as following:

$$\dot{S}=\begin{bmatrix}\dot{S_1}\\ \dot{S_2}\end{bmatrix}=\begin{bmatrix}-k_1\frac{S_1}{|S_1|+\delta_1}-\exp^d S_1\\ -k_2\frac{S_2}{|S_2|+\delta_2}-\exp^d S_2\end{bmatrix} \tag{8}$$

Time derivative of 7 results in:

$$\begin{aligned}\dot{S} &= \begin{bmatrix}\dot{x}_e\\ \dot{\theta}_e+\frac{\partial a}{\partial v_r}\dot{v_r}+\frac{\partial a}{\partial y_e}\dot{y_e}\end{bmatrix}\\ &= \begin{bmatrix}y_e\omega-v+v_r\cos\theta_e-d\omega_r\sin\theta_e\\ \omega_r-\omega+\frac{\partial a}{\partial v_r}\dot{v_r}+\frac{\partial a}{\partial y_e}\dot{y_e}\end{bmatrix}\end{aligned} \tag{9}$$

In this equation $\frac{\partial a}{\partial v_r}$ and $\frac{\partial a}{\partial y_e}$ are written for abbreviations and can be calculated as follows:

$$\frac{\partial a}{\partial v_r}=\frac{y_e}{1+(v_r y_e)^2} \tag{10}$$

$$\frac{\partial a}{\partial y_e}=\frac{v_r}{1+(v_r y_e)^2} \tag{11}$$

The equation 9 can be used for derivation of the control command:

$$\begin{bmatrix}v\\ \omega\end{bmatrix}=\begin{bmatrix}y_e\omega+v_r\cos\theta_e-d\omega_r\sin\theta_e-\dot{S_1}\\ \frac{\omega_r+\frac{\partial a}{\partial v_r}\dot{v_r}+\frac{\partial a}{\partial y_e}(\dot{y_e}v_r\sin\theta_e+d\omega_r\cos\theta_e)-\dot{S_2}}{1+\frac{\partial a}{\partial y_e}(x_e+d)}\end{bmatrix} \tag{12}$$

Some mobile robots need two commands to be prompted, namely, the velocity of right and left wheel. These velocities are simply computed by linear transformation of angular and linear velocity as follows:

$$\begin{bmatrix}\omega_{right}\\ \omega_{left}\end{bmatrix}=\frac{1}{r}\begin{bmatrix}1 & \frac{b}{2}\\ 1 & \frac{-b}{2}\end{bmatrix}\begin{bmatrix}V\\ \omega\end{bmatrix} \tag{13}$$

where, r and b denote the radius of wheel and wheel base, respectively.

B. Feedback Error Learning Scheme

The closed loop system which uses a feedback error learning (FEL) scheme may be seen as a hybrid system. FEL is a feed-forward neural network structure that is parallel to the usual feedback controller [15]. The artificial neural network (ANN) learns the inverse dynamics of the controlled object, which is made by feedback controller using training signal while the process of training artificial neural network is online. The total control effort applied to the plant is the sum of the feedback control output and network control output[16], [17]. The stability of this structure has been analyzed by many researchers[18], [19], [20]. In [20] stability for general plants is explained. Stability of FEL is proved by use of strict positiveness of the closed loop system. In [18] other assumption is considered that the plant is stable and stably invertible. In this paper we have used of desired states instead of usually used real states. If the inverse of system is obtained after a period of time, the feedback controller will be omitted from the loop. If the system is affected by any noise or disturbance the feedback controller will be re-entered to the loop and generates the new inverse dynamics.

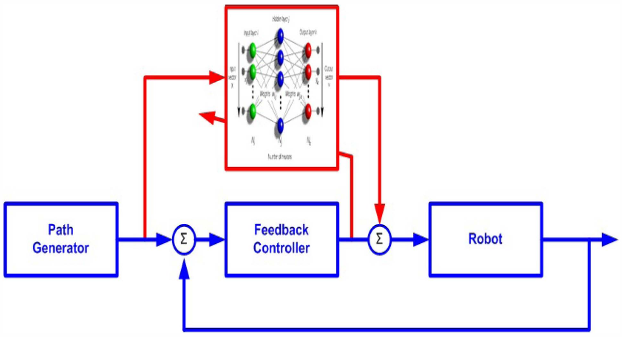

Fig. 2: Feedback error learning (FEL) structure

The advantage of this structure is that the network controller is trained without needing any jacobian of the system, and therefore this method is model free.

C. Proposed Controller

Presented method is a combination of two controllers in parallel form. This combination uses of classic controller that performs in parallel with an artificial neural network trained online [15]. As mentioned before, this kind of architecture for neural controllers is known as feedback error learning. The output signal that is generated by classic controller is used for training the net and it is back propagated for learning purposes [11]. For classic controller we used sliding mode controller that is described in previous section. For neural network part we have used a multi-layer perceptron net (MLP). When we have to deal with nonlinearity and uncertainty, one of the best choice for controller is artificial neural network [21]. It could be useful in some cases that we do not have mathematical model of system or our model is too poor and inadequate to represent the system with sufficient accuracy. Neural network can provide a nonlinear map only by input and output data of system [24]. Some of the neural models are used in control field such as radial base function (RBF) and MLP [22], [23].

In this paper for better efficiency we have used two MLP networks, one for computing angular velocity and another for linear velocity. Each network has two hidden layers and three inputs contain position and angle of robot respect to base. Training algorithm is descending gradient with learning rate equal to 0.1. Output of each network will be added to the output of Sliding Mode controller and applied to the robot. The structure of controller can be seen in Fig.2.

IV. Experimental Results

A. System Setup

In this section some experiments have been executed to validate feedback error learning scheme. The proposed controller is implemented on KNTU Mellon mobile robot. This robot is equipped with a laser range finder and two encoders. The maximum range reading of laser range finder is 8 meter and the ego-motion estimation is done by well known ICP algorithm.

To evaluate proposed controller several steps are considered: First, we consider that the control input is produced by sliding mode controller. In this case the Neural Network is omitted from the control loop. Besides Neural Network is added to control structure to make the feedback error learning scheme. Finally we compare the results of these steps.

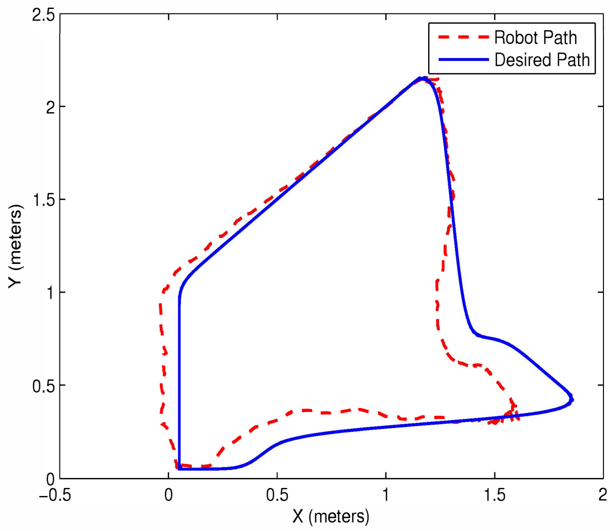

Fig. 3: Desired and robot path by Sliding Mode controller

B. Sliding Mode Controller

In this section a desired path is considered and the purpose is to track this path by minimum error. Sliding Mode controller is used for producing control input. Two parameters K1 and K2 in equation 8 should be design. Choosing small values for them terminates instability in system because the effect of uncertainty could not be compensated. By do some experiments these parameters are selected as K1=5 and K2=6. Maximum linear velocity is 0.08(m/s) and maximum angular velocity is considered as 0.15(rad/s). Fig.3 shows the desired path and also the path that is covered by robot. Because of high uncertainty disturbance in system the performance of tracking is not optimum. In Fig.4 the errors of robot position and orientation are shown. It is also extracted that the errors of tracking are higher than desired values. The results of quantitative evaluation is given in the future section.

C. Feedback Error Learning

As mentioned in the previous part feedback error learning is a close loop controller that uses of classic control and neural network approximator. In this part similar to the sliding mode controller some experiments are considered. MLP model is used for Neural Network (NN) with 2 layers. We have used two NN for better efficiency, one for computing angular velocity and another for linear velocity. Fig.5 shows the desired path and also the path that is covered by robot. This figure demonstrates that tracking is more satisfaction by means of FEL controller. The Errors of tracking is shown in Fig.6. FEL decreases the error of robot position and orientation.

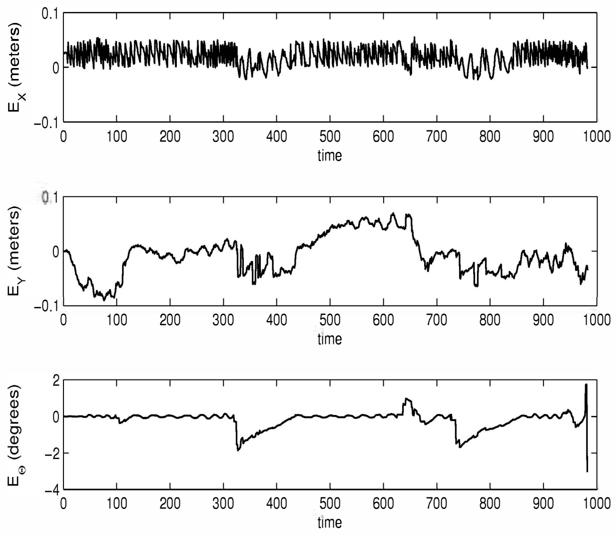

Fig. 4: Robot position and orientation error by Sliding Mode controller

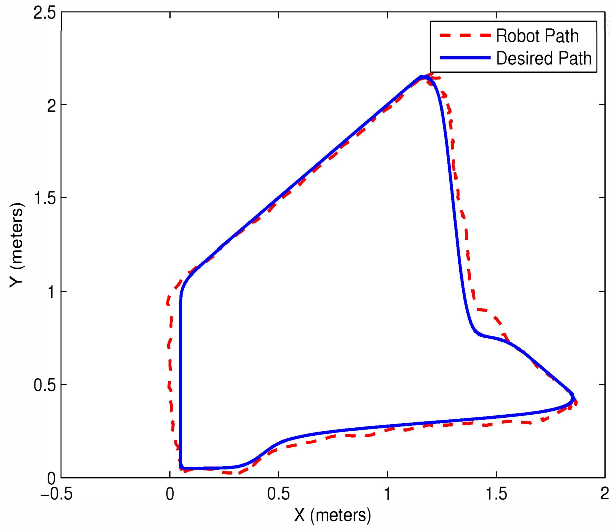

Fig. 5: Desired and robot path by FEL

TABLE I: Mean Square Error of different proposed methods - Sliding mode controller & Feedback Error Learning

MSE	X(m)	Y(m)	θ(d)
SM	9.269e-04	0.0059	0.4704
FEL	7.653e-04	9.495e-04	0.232

For quantitative evaluation and comparison robot position and orientation is considered. In table I Mean Square Error(MSE) of robot position and orientation with a desired path is shown. Results illustrates that adding neural network to the control loop removes the effects of uncertainty and disturbance and tracking is done by higher performance and lower errors.

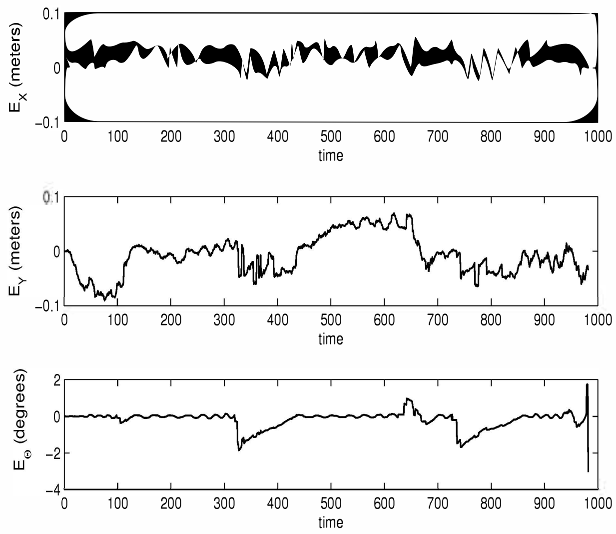

Fig. 6: Robot position and orientation error by FEL

V. Conclusions

In this paper a feedback error learning scheme is presented which contains one sliding mode controller and one neural network controller. The sliding mode controller is suitable for nonholonomic systems which can not be stabilized by state feedback. There are different sliding surfaces that can be used for nonholonomic systems. In this paper sliding surfaces are defined based on backstepping controller. Due to dynamic of mobile robot, robot can not track exact angular and linear velocity that produced by sliding mode controller. It causes the total performance not to be satisfactory. Using a neural network controller in FEL scheme, the total performance will improve. The experimental results on mobile robots confirm better performance of FEL than that of a single sliding mode controller.

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