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Kinematic Performance Indices Analyzed on Four Planar Cable Robots via Interval Analysis

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***Abstract*—In this paper, some new kinematic performance indices are proposed and examined on four planar cable driven parallel manipulators. The main kinematic indices are based on kinematic sensitivity and controllable workspace of the robot. Interval analysis is adopted as a mathematical framework to compute feasible kinematic sensitivity and worst kinematic sensitivity indices. For determining the feasible kinematic sensitivity, the controllable workspace is combined with the desired kinematic sensitivity property. The area of the foregoing region and the worst kinematic sensitivity corresponding to it are introduced as practical design indices. Then four typical design of planar cable robot are examined by the following performance measures, and one of such designs are selected and implemented in practice.**

I. Introduction

Cable Driven Redundant Parallel Manipulators (CDRPMs) consist of a moving platform which is connected by the means of actuated cables to the base. Redundancy is an inherent requirement for CDRPMs due to the fact that cables can only pull but cannot push the moving platform. Thus, in a non-singular posture, the moving platform of CDRPM can perform n Degree-of-freedom (DOF) by considering at least $n+1$ cables. CDRPMs are special design of parallel manipulators (PMs) that heritage the advantages of PMs such as high acceleration and high load carrying capability and at the same time, have alleviated some of their shortcomings, such as restricted workspace.

The workspace analysis of CDRPM is investigated upon different perspectives and different types of workspace are proposed in the literature. In short, four different types of workspace have been introduced: (1) Wrench feasible workspace [3], (2) Dynamic workspace [1], (3) Static workspace [4] and (4) Controllable workspace (CWS) [13]. In this paper, more emphasis is placed on the controllable workspace which represents the most general feasible workspace of CDRPMs. Controllable workspace pertains at finding the set of poses (position and orientation) of the moving platform in which any wrench can be generated by the moving platform while cables are all in tension.

In design of PMs, usually kinematics performance indices are used to reduce the singularities and to improve the performance of the mechanism under study. Most popular indices are Yoshikawa manipulability [14] and the dexterity indices [12], which entail some limits and as stated in [2], seems to have not drawn a consensus among the robotics community [2]. Recently, two different indices referred to as point-displacement and rotational kinematic sensitivities are proposed which their meaning is thought to be clear and definite to the designer of a robotic manipulator [7]. These indices provide tight upper bounds on the magnitudes of the moving platform rotations and point-displacements, respectively under a unit-magnitude array of actuated-joint displacement [2], [11].

The mathematical framework of this paper is based on interval analysis [8], using the INTLAB package. Our interest toward applying interval analysis to the kinematic analysis can be summarized as follows: (1) In contrast to many other intelligent mathematical tools which would result in a lengthy computational process and may converge to a local optimum, interval analysis is not a *black box*, since it requires to combine heuristics and numerical concepts to make it more effective, and (2) For the problem in which infinity norm are involved, interval analysis may solve the problem more efficiently rather than other methods since infinity norm is a non-analytical function and consequently mathematical operations are not tractable.

This paper aims at using several kinematic indices for surveying the performance of different configurations of planar CDRPMs. To this end, four different designs with specific features are studied by using indices such as controllable workspace area, worst kinematic sensitivity and feasible kinematic sensitivity (FKS). FKS workspace is a part of controllable workspace in which the kinematic sensitivity is less than a desired value and can be also regarded as a performance index [5].

The remainder of this paper is organized as follows. First, the general concept of interval analysis is broadly reviewed. Then based on the work presented in [11], the general idea of kinematic sensitivity is recalled. The paper follows by introducing the set of indices used for evaluating the CDRPMs under study in this paper, namely, the feasible kinematic sensitivity workspace and the worst kinematic sensitivity within the workspace of the robot. Next, these indices are calculated for several designs of CDRPMs and different designs are surveyed with the viewpoint of feasible kinematic sensitivity and more practical indices such as feasibility fabrication, mechanical interference and valume of moving platform.

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II. Background Materials

A. Interval Analysis

Interval analysis is amongst the numerical methods proposed in the literature that allows to safely solve the problem, and to obtain a guaranteed result. The basic principles of interval analysis are simple, where efficient implementation requires a high expertise level. In interval analysis, one deals with intervals of numbers instead of the numbers themselves:

$$[x] = [\underline{x}, \overline{x}] = \{x | x \in R, \underline{x} \leq x \leq \overline{x}\} \tag{1}$$

where $\underline{x}$ is the left endpoint and $\overline{x}$ is the right endpoint of the interval. In this paper, interval analysis is not introduced in detail, since it is beyond the scope of this study and reader are referred to [9] for a more comprehensive detail. It should be noted that all the interval algorithms proposed in this paper are implemented in Matlab which uses INTLAB, a package supporting interval calculations.

B. Kinematic Sensitivity Indices

Kinematic sensitivity is defined as the maximum error that occurs in the Cartesian workspace as a result of bounded errors in the joint space ($\|\rho\| \leq 1$). In order to obtain consistent unit indices, two indices have been defined in [2]:

$$\sigma_{r_c,f} \equiv \max_{\|\rho\|_c=1} \|\phi\|_f \qquad \text{and} \qquad \sigma_{p_c,f} \equiv \max_{\|\rho\|_c=1} \|\mathbf{p}\|_f \tag{2}$$

in which, $\rho \in \mathbb{R}^n$ represents small actuator displacements and $x = [\mathbf{p}, \phi]$ stands for the pose of the moving platform. Moreover, $c = \{2, \infty\}$ and $f = \{2, \infty\}$ are respectively the types of norm for which the constraint and the objective are expressed. From the results obtained from [11], it can be inferred that two situations may correctly represent the kinematic sensitivity: (1) The constraint and objective functions are both expressed using ∞-norms ($c = f = \infty$) which will be used for the purposes of this paper and (2) The constraint and objective functions are expressed respectively with ∞ and 2-norms ($c = \infty$, $f = 2$).

III. Kinematic Performance Indices

This section is devoted entirely to an overview on the computation of the kinematic sensitivity of CDRPMs, based on the results obtained by authors in detail in [5]. More specifically, the main objective of this section is to investigate the proposed interval-based algorithm in [5] which leads to obtain a region within the workspace of the mechanism, referred to as *feasible kinematic sensitivity workspace*, where the kinematic sensitivity is less than a desired value, σ_d. In extension to the latter work and as the main contribution of this paper, the worst kinematic sensitivity is proposed which provides an insight into the performance of the mechanism under study.

A. Feasible Kinematic Sensitivity

As aforementioned, the controllable workspace is considered in this paper [13]. For controllable workspace analysis, the analytic method proposed in [15] is used. In this method a set of external wrenches is introduced and called *fundamental wrenches* in order to provide a physical interpretation of controllable workspace. Moreover, an analytical method is developed to determine the controllable workspace of redundant CDRPMs based on fundamental wrenches. The proposed method is generally applicable to any cable manipulators with any redundant cables as long as its Jacobian matrix is of full rank. The set of fundamental wrench for cable manipulator with one degree of redundancy refers to a set of $n+1$ vectors; each of them is equal to an opposite direction of column vector of Jacobian transpose as [15]:

$$\mathbf{A} = -\mathbf{J}^T = [\mathbf{A}_1 | \mathbf{A}_2 | ... | \mathbf{A}_{n+1}]_{n \times (n+1)}, \ \mathbf{w}_f = -\mathbf{A}_i, \ i = 1, ..., n+1 \tag{3}$$

In which $\mathbf{A}$ and $\mathbf{J}$ denote the structure and Jacobian matrix, respectively, and $\mathbf{w}_f$ is the fundamental wrench vector. According to the proposed theorem in [15], the controllable workspace can be obtained when all the determinant of the following matrix are positive.

$$\Delta_{ij} = \det[\mathbf{A}_1 ... \mathbf{A}_{j-1} \quad -\mathbf{w}_i \quad \mathbf{A}_{j+1} ... \mathbf{A}_{i-1} \mathbf{A}_{i+1} ... \mathbf{A}_{n+1}], i \neq j \tag{4}$$

The method to obtain the FKS workspace is introduced previously by the authors in [5]. The volume of feasible kinematic sensitivity can be used as a suitable measure for optimal design of such manipulators. In fact, this region is produced from blending controllable workspace and the area having the desired kinematic sensitivity.

B. Worst Kinematic Sensitivity

The main concern of this part is to provide an interval-analysis-based determination for the maximum kinematic sensitivity (point-displacement or rotational) within the controllable workspace of a planar CDRPM. The method used for finding the maximum value of a function is based on the simplest and most accurate global optimization algorithm which is described in [9] with great details and for what follows a briefly overview is provided. The objective consists in obtaining the maximum magnitude of a given function, $f(x)$ called f^*, using interval analysis. The main idea to solve this problem lies in the following relation:

$$\tau \equiv \max(\inf((C_j))) \leq f^* \leq \max(\sup(F(C_j))), 1 \leq j \leq n \tag{5}$$

where C_j are constitutive sub-intervals of interval $[X]$ and n is the number of constitutive sub-intervals and $\inf(F(C_j))$ and $\sup(F(C_j))$ are the lower and upper bounds of interval $F(C_j)$. Based on the above relation, if the upper bound of $F(C_j)$ is smaller than the lower bound of $F(C_i)$, $i \neq j$, certainly the inner points of C_j interval will have smaller fitness with respect to C_i and do not include the optimum point and should be excluded from the rest of the computation process. The pseudo-code presented in Table I shows the calculation algorithm for obtaining the worst kinematic sensitivity in the robot workspace. In this pseudo-code, D is an interval vector which encompasses the robot workspace (controllable space) and is the input function argument. Also KS($\cdot$) is an interval function which calculates the point-displacement or rotational kinematic sensitivity of its interval argument. At the end, the

TABLE I
THE PSEUDO-CODE FOR THE CALCULATION OF THE WORST KINEMATIC SENSITIVITY WITHIN THE WORKSPACE.

```
Function: Compute-Max-Kinematic-Sensitivity(D)
S = D    j = 1    Results = ∅
τ = max(inf(KS(S)))
for i = 1 : size(S)
   if τ ≤ (sup(KS(S(i))))
      R(j) = S(i)
      j = j + 1
   end
end
while R ≠ ∅
   k = argmax_j(inf(KS(R(j))))
   X = R(k)
   split X into X_(1) and X_(2)
   τ̂ = max(inf(KS(X_(1))), inf(KS(X_(2))))
   if τ < τ̂
      τ̂ = τ
   end
   for n = 1 : 2
      if τ ≤ sup(KS(X_(n)))
         if width(KS(X_(n)) ≤ δ
            Results = Results ∪ X_(n)
         else
            R = R ∪ X_(n)
         end
      end
   end
end
```

output of the pseudo-code are sub-intervals, called *Results*, that determine the worst kinematic sensitivity according to the arbitrary accuracy specified by δ.

IV. ANALYSIS OF FOUR DESIGN OF CDRPMS BY KINEMATIC PERFORMANCE INDICES

Four designs of planar CDRPM's are the central subject of this paper to be analyzed by the introduced kinematic performance indices. These designs are called V-inverted V; X; upper-lower V and U-inverted U and are depicted schematically in Fig. 1. These CDRPMs perform two translational DOFs along the x- and y- axes in the x-y plane which is represented by P_{xy} and one orientational DOF, ϕ, around z-axis.

A. V-Inverted V Design

Figure 1(a) illustrates a design for which four cables are connecting two connection points at the moving platform to two pair of adjacent points at the fixed frame. As it can be observed from Fig.1(a), the design parameters of this mechanism are as follows: $R_a = 10$, $R_b = 10$. Figure 2(a) represents the results obtained for the V-inverted V design for the controllable workspace, while Figure 2(b) depicts its FKS workspace.

In Fig. 2(a), the internal region (green area) determines the controllable workspace. In contrast, the external region (red area) specifies the uncontrollable workspace. The middle boundary is the region that separates the controllable workspace from the uncontrollable workspace. By using the

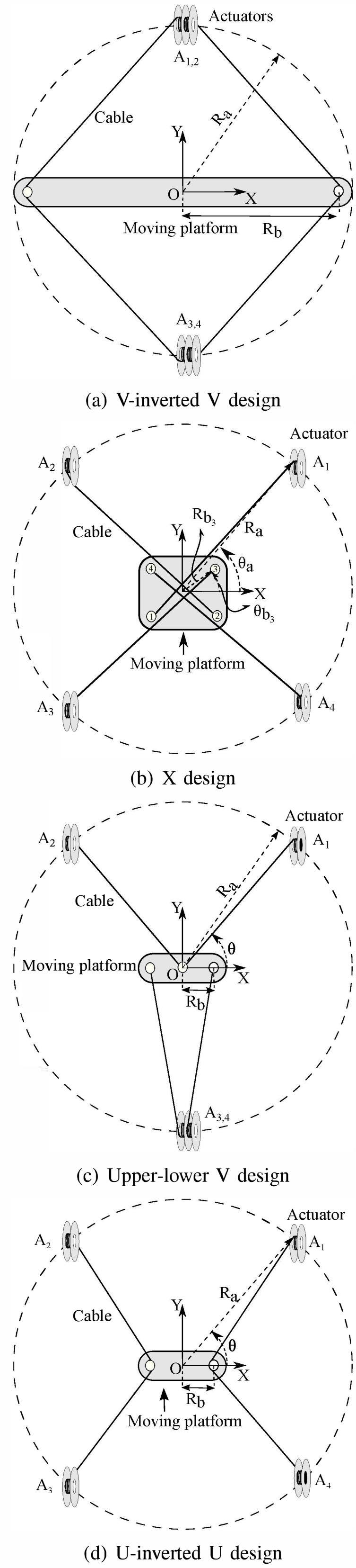
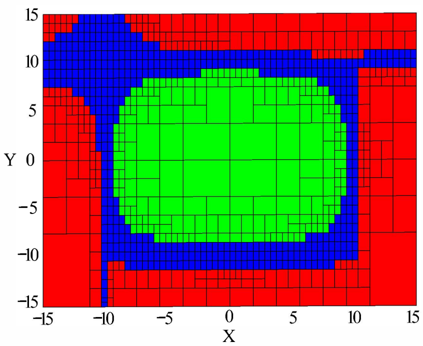
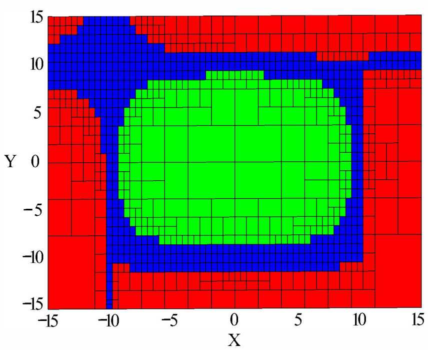


Fig. 1. The schematics of the four proposed CDRPMs.



(a) The controllable workspace



(b) The FKS workspace

Fig. 2. The interval illustration for the results of the V-inverted V design.

pseudo-code of Table I and according to the specified accuracy, δ, the worst point-displacement or rotational kinematic sensitivity in the controllable workspace, $\max(\sigma_p)$ and $\max(\sigma_r)$, are obtained and their value are shown in Table II. In a situation that the worst rotational or point-displacement kinematic sensitivity would be greater than the desired value, FKS workspace will have smaller area than the controllable workspace, since there is no area of controllable workspace with undesired kinematic sensitivity in the FKS workspace. As it is shown in Table II, the area of the controllable workspace of this robot for $\phi = 0$, is approximately equal to the FKS workspace area, since the worst point-displacement and rotational kinematic sensitivity ($\max(\sigma_p)$, $\max(\sigma_r)$) is less than the desired kinematic sensitivity (σ_{p_d}, σ_{r_d}). According to Table II, a the prominent characteristic of this design is its large controllable workspace, so as its controllable workspace area is about twice the area of that in other designs.

In summary, one can express illustrious features of this design as follows: (1) very large workspace that is provided by large moving platform dimensions. (2) proper mechanism accuracy (desired point displacement and rotational kinematic sensitivity) and (3) the collision-free design due to the placement of all cables in $\mathscr{P}_{xy}$. The following issues can be noted as the shortcomings of this design: (1) due the dimension of the moving platform, mechanical interferences are possible with outer objects close to the base which should be taken into account for the installation of the robot. (2) large platform increases the inertia of the mechanism and as a consequence reduces the agility of the mechanism. (3) due to shape and the relatively large dimensions of the platform, by a small force along the z- axis or a perturbation torque around x- or y- axis it may perturb the moving platform position from the $\mathscr{P}_{xy}$ plane which results in the loss of control of the robot.

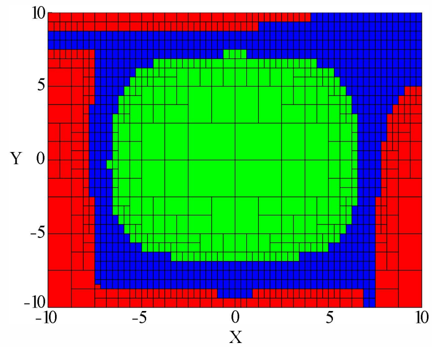
B. X Design

Figure 1(b) depicts schematically the second design. In this design, some parameters such as the moving platform could have regular or non-regular shapes. However, according to the manufacturing considerations, it is more convenient that the fixed frame is rectangular. Thus, it can be shown that this mechanism admits up to 10 design parameters and in order to have a compact design, with a large the area of the controllable workspace these parameters should be prescribed. By using differential evolutionary algorithm [10], and with regard to the design constrains $2 \leq R_a \leq 10$ and $0.1 \leq R_{b_i} \leq 1$, these parameters are optimized to have maximum area of controllable workspace. The results of this optimization are as follow: $R_{b_1} = 0.74$, $\theta_{b_1} = 270°$, $R_{b_2} = 0.77$, $\theta_{b_2} = 284°$, $R_{b_3} = 0.72$, $\theta_{b_3} = 75°$, $R_{b_4} = 0.70$, $\theta_{b_4} = 88°$, $R_a = 10$, $\theta_a = 34°$. Then, the controllable workspace of this design for $\phi = 0$ is computed by interval analysis and is demonstrated in Fig. 3(a). The periphery solid line ellipsoids shown in the Fig. 3(b), and throughout this paper, depict the areas with maximum (worst) rotational kinematic sensitivity. Furthermore, the periphery dash line ellipsoids indicate the parts of the controllable workspace that have the worst point-displacement kinematic sensitivity.

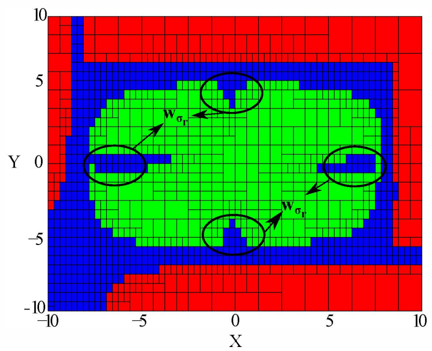
As it is detailed in Table II, some of the advantages of this design can be listed as follows: (1) leading to a maximum controllable workspace among the proposed mechanisms with small platforms (2) Entailing the desired point-displacement and rotational kinematic sensitivity (3) having small moving platform which makes possible higher speed and acceleration. On the other hand, the drawback of this design is its complex and expensive manufacturing process. In order to avoid collision among the crossed cables, they should be placed in various parallel planes parallel to $\mathscr{P}_{xy}$ plane, which is more complex and expensive to be manufactured. Moreover, the moving platform in such a design may perform some undesirable rotation around x- or y-axis and keeping the motion in a plane is a prohibitive task.

TABLE II
THE COMPARISON TABLE OF THE VARIOUS DESIGNS OF CDRPMs
($\delta = 0.01$, $\sigma_{d_p} = 0.3$ AND $\sigma_{d_r} = 0.2$).

Design	CWS area (m^2)	W_{σ_p}	W_{σ_r}	FKS area (m^2)	Feasibility fabrication	Mechanical interference
V-inverted V	297	0.10	0.02	297	×	×
X	158	0.17	0.19	148	×	✓
Upper-lower V	105	0.15	0.91	95.11	✓	×
U-inverted U	151	0.14	0.14	149.6	✓	×

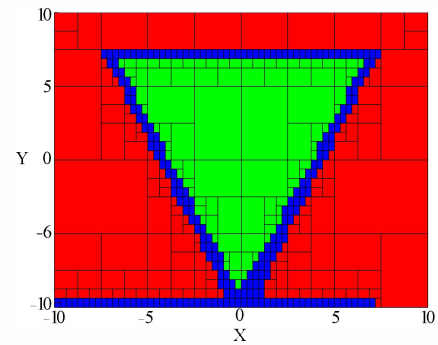


(a) The controllable workspace

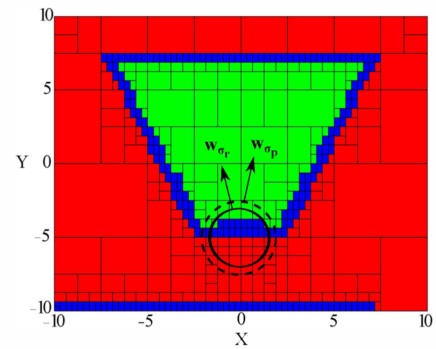


(b) The FKS workspace

Fig. 3. The interval illustration for the results of the X design.



(a) The controllable workspace



(b) The FKS workspace

Fig. 4. The interval illustration for the results of the upper-lower V design.

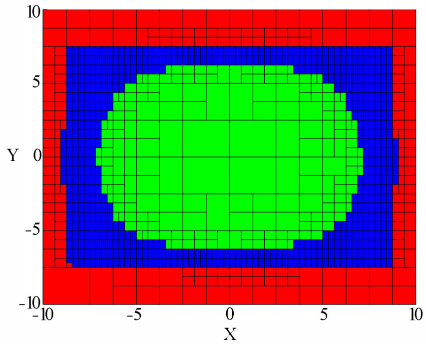
C. Upper-Lower V Design

The third mechanism is illustrated in Fig. 1(c). An interesting feature of this robot can be regarded as the independence of the shape and the area of the controllable workspace from orientation of the moving platform, i.e., ϕ. In other words, as it can be observed from Fig. 4(a), the controllable workspace of this mechanism is always an isosceles triangle the base of which is parallel to the x- axis and for non-singular posture this area remains constant for different orientation of the moving platform. According to Fig. 1(c), the design parameters are as follows: $R_a = 10$, $R_b = 1$ and $\theta = \frac{\pi}{6}$. Another interesting issue is that no change occurs in the shape and area of this controllable workspace with respect to changes in R_b so that this figure is only sensitive to the parameters R_a and θ. However for having a desirable rotational kinematic sensitivity, the lower bound of R_b parameter should be obtained which satisfies the latter index. Figure 4(b) shows the FSK workspace of this mechanism. It can be observed that the bottom triangle vertex of the controllable workspace has been removed from the controllable workspace, owing to the unacceptable rotational kinematic sensitivity.

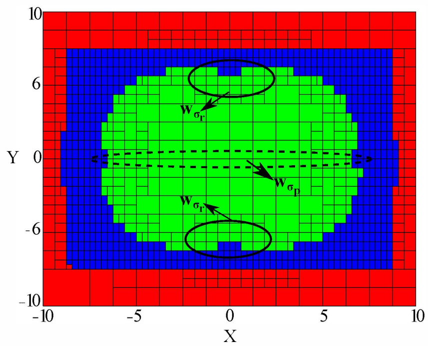
The positive characteristics of this design may be listed as: (1) the shape of the controllable workspace is invariant to the rotation angles (2) performing the desired point-displacement kinematic sensitivity (3) having a triangular shape for the controllable workspace (4) having small platform. Regarding the first item, it should be noted that stability in the workspace shape is an important issue to guarantee the robot specifications for an external user. In the configurations which do not have a constant controllable workspace, the region that the manufacturer presents to a user, will be the intersection area of the controllable workspaces for different orientations of the moving platform. It is obvious that this space is less than or equal to the controllable workspace in a specific orientation. However, weaknesses of this design are as follows: (1) low extent controllable workspace, and (2) undesirable rotational kinematic sensitivity. In order to alleviate this problem R_b must be chosen as large as possible.

D. U-Inverted U Design

Figure 1(d) depicts the schematic of the fourth design. It can be observed that four attachment points at the base are connected symmetrically to two adjacent points at the moving platform. The four attachment points at the base, A_i, $i = 1,..,4$ are distributed symmetrically on the perimeter of a circle with R_a as radius. Having R_a and θ, all of the attachment points of the cables to the fixed frame can be computed. The moving platform can be regarded as straight line for which the mobile frame O_{xy} is placed at the mid point of the former line. Design parameters of this mechanism, as indicated in Fig. 1(d), are $R_a = 10$, $R_b = 1$ and $\theta = \frac{\pi}{4}$. By using differential evolutionary algorithm, these above parameters are determined so as the controllable workspace area for $\phi = 0$ is maximized. Fig. 5(a) and Fig. 5(b) are illustrated respectively the controllable and



(a) The controllable workspace



(b) The FKS workspace

Fig. 5. The interval illustration for the results of the U-inverted U design.

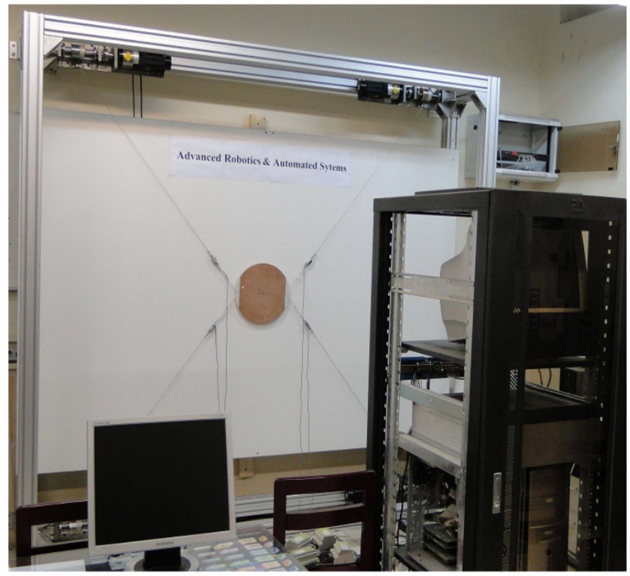


Fig. 6. The planar cable robot built by Advanced Robotics and Automated Systems (ARAS) at K.N Toosi University.

FKS workspace of the U-inverted U design.

Some features of this design are as follows: (1) easy fabrication (2) having desired accuracy (3) small platform. Also, the negative points can be summarized as follows: (1) small controllable workspace, and (2) different controllable workspace for different rotational DOF. According to Table II, fabrication feasibility, reasonable cost, and at the same time, appropriate accuracy and sufficient area of the workspace of the U-inverted U design, convinced us to implement this design in fabrication of the cable robot illustrated in Fig. 6. The experimental verification of positioning performance of this robot has been presented in [6].

V. Conclusion

In this paper, several kinematic indices were proposed to survey the performance of four designs of planar cable robots. Feasible kinematic sensitivity index has been used to determine the efficiency of the workspace, and the worst kinematic sensitivity is proposed as the mechanism accuracy index. The framework for computing the these indices in practice is based on interval analysis. Four generic designs of planar cable robots have been studied with the aid of these to have a better insight of kinematics performance of such designs in practice. The fabrication feasibility, mechanical interference and the moving platform volume are also considered and then an acceptable mechanism was selected for further fabrication by our group.

References

[1] G. Barrette and C. Gosselin, "Determination of the Dynamic Workspace of Cable-driven Planar Parallel Mechanisms," *Journal of Mechanical Design*, vol. 127, p. 242, 2005.

[2] P. Cardou, S. Bouchard, and C. Gosselin, "Kinematic-sensitivity Indices for Dimensionally Nonhomogeneous Jacobian Matrices," *IEEE Transactions on Robotics*, vol. 26, no. 1, pp. 166–173, 2010.

[3] I. Ebert-Uphoff and P. Voglewede, "On the Connections Between Cable-driven Robots, Parallel Manipulators and Grasping," in *Proceedings of the IEEE International Conference on Robotics and Automation, ICRA'04.*, vol. 5, 2004, pp. 4521–4526.

[4] A. Fattah and S. Agrawal, "On the Design of Cable-suspended Planar Parallel Robots," *Journal of Mechanical Design*, vol. 127, p. 1021, 2005.

[5] S. Khalilpour, A. Zarif Loloei, H. Taghirad, and M. Tale Masouleh, "Feasible Kinematic Sensitivity in Cable Robots Based on Interval Analysis," *Cable-driven Parallel Robots*, pp. 233–249, 2012.

[6] M. Khosravi and H. Taghirad, *Cable-driven Parallel Robots*. Springer, 2012, ch. Experimental Performance of Robust PID Controller on a Planar Cable Robot, pp. 337–352.

[7] M. Saadatzi, M. Masouleh, H. Taghirad, C. Gosselin, and P. Cardou, "On the Optimum Design of 3-RPS Parallel Mechanisms," in *proceedings of the 19th Iranian Conference on Electrical Engineering, ICEE'11*, 2011, pp. 1–6.

[8] R. E. Moore, *Interval Analysis*. Englewood Cliffs, N.J.: Prentice-Hall, 1966.

[9] R. Moore, R. Kearfott, and M. Cloud, *Introduction to Interval Analysis*. Society for Industrial Mathematics, 2009.

[10] K. Price, R. Storn, and J. Lampinen, *Differential Evolution: A Practical Approach to Global Optimization*. Springer-Verlag New York Inc, 2005.

[11] M. Saadatzi, M. Masouleh, H. Taghirad, C. Gosselin, and P. Cardou, "Geometric Analysis of the Kinematic Sensitivity of Planar Parallel Mechanisms," *Transactions of the Canadian Society for Mechanical Engineering*, vol. 35, no. 4, p. 477, 2011.

[12] J. Salisbury and J. Craig, "Articulated Hands," *The International Journal of Robotics Research*, vol. 1, no. 1, pp. 4–17, 1982.

[13] R. Verhoeven and M. Hiller, "Estimating the Controllable Workspace of Tendon-based Stewart Platforms," *Advances in Robot Kinematics*, pp. 277–284, 2000.

[14] T. Yoshikawa, "Analysis and Control of Robot Manipulators with Redundancy," in *proceedings of the First International Symposium Robotics Research*. Mit Press Cambridge, MA, 1984, pp. 735–747.

[15] A. Zarif Loloei and H. Taghirad, "Controllable Workspace of Cable Driven Redundant Parallel Manipulators by Fundamental Wrench Analysis," to be appear in, March 2013.