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Robust PID Control of Cable–Driven Robots with Elastic Cables

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***Abstract*—In this paper robust PID control of fully-constrained cable-driven robots with elastic cables is studied in detail. To develop the idea, a robust PID control for cable-driven robots with ideal rigid cables is firstly designed and then, this controller is extended for the robots with elastic cables. To overcome vibrations caused by inevitable elasticity of cables, a composite control law is proposed based on singular perturbation theory. The proposed control algorithm includes robust PID control for corresponding rigid model and a corrective term. Using the proposed control algorithm the dynamics of the cable-driven robot is divided into slow and fast subsystems. Then, based on the results of singular perturbation theory, stability analysis of the total system is performed. Finally, the effectiveness of the proposed control law is investigated through several simulations on a planar cable-driven robot.**

I. Introduction

Cable driven parallel manipulators (CDPMs) are a special class of parallel robots in which the rigid extensible links are replaced by actuated cables. In a CDPM the end–effector is connected to the base by a number of active cables. While the cables length is changing, the end-effector is forced toward the desired position and orientation. Replacing rigid links by cables inaugurates many potential applications such as very large workspace robots [1], high speed manipulation [2], handling of heavy materials [3], and cleanup of disaster areas [4].

Using cables instead of rigid links, however, introduces new challenges in the study of CDPMs. Cables have the limitation that they can only apply tensile forces and no compression. Due to this fact, well-known results in the theory of control for robotics can not be used directly for CDPMs. Dynamic behavior of cables is another challenge in the control study of CDPMs. In comparison to the large amount of researches performed on the control of conventional robots, relatively few has been reported on the control of CDPMs. With assumption of massless rigid string model for the cables, many of control schemes which are applicable for series and parallel robots, may be adapted for CDPMs [2], [5], [6]. Inclusion of dynamic behavior of the cables into dynamic analysis of CDPMs leads in complicated control laws, and research on this topic is infancy. In [7], using elastic massless spring model for the cables dynamics of the CDPM is derived and a new control algorithm in the length space is proposed.

The main goal of this paper is to present a new approach in control of CDPMs with elastic cables, by using popular PID controller and singular perturbation theory [8]. In this paper, first dynamics of cable robot with ideal rigid cables is introduced and robust PID control algorithm is proposed for this model. Then, UUB stability of the rigid system with proposed controller is analyzed through Lyapunov theory. Next, dynamic model is extended for elastic cables, and a control strategy is developed using singular perturbation theory. By using Tikhonov's theorem slow and fast variables are separated and incorporated in the stability analysis of total closed-loop system. Finally, simulation results on a planar cable robot are given to demonstrate the effectiveness of the proposed control algorithm in practice.

II. Robust PID Control of Rigid Cable Robot

In this section we assume that the elasticity of cables can be ignored and cables behave as massless rigid strings. Based on this assumption, the standard model for the dynamics of n-cable parallel robot is given as [7]:

$$\boldsymbol{M}_{eq}(\boldsymbol{x})\ddot{\boldsymbol{x}} + \boldsymbol{N}_{eq}(\boldsymbol{x},\dot{\boldsymbol{x}}) = \boldsymbol{J}^T\boldsymbol{u}_r \tag{1}$$

in which,

$$\begin{aligned}
\boldsymbol{M}_{eq}(\boldsymbol{x}) &= r\boldsymbol{M}(\boldsymbol{x}) + r^{-1}\boldsymbol{J}^T\boldsymbol{I}_m\boldsymbol{J}\\
\boldsymbol{C}_{eq}(\boldsymbol{x},\dot{\boldsymbol{x}}) &= r\boldsymbol{C}(\boldsymbol{x},\dot{\boldsymbol{x}}) + r^{-1}\boldsymbol{J}^T\boldsymbol{I}_m\dot{\boldsymbol{J}}\\
\boldsymbol{N}_{eq}(\boldsymbol{x},\dot{\boldsymbol{x}}) &= r\boldsymbol{N}(\boldsymbol{x},\dot{\boldsymbol{x}}) + r^{-1}\boldsymbol{J}^T\boldsymbol{I}_m\dot{\boldsymbol{J}}\dot{\boldsymbol{x}} + r^{-1}\boldsymbol{J}^T\boldsymbol{D}\boldsymbol{J}\dot{\boldsymbol{x}}\\
\boldsymbol{N}(\boldsymbol{x},\dot{\boldsymbol{x}}) &= \boldsymbol{C}(\boldsymbol{x},\dot{\boldsymbol{x}})\dot{\boldsymbol{x}} + \boldsymbol{G}(\boldsymbol{x}) + \boldsymbol{F}_d\dot{\boldsymbol{x}} + \boldsymbol{F}_s(\dot{\boldsymbol{x}}) + \boldsymbol{T}_d
\end{aligned}$$

Where $\boldsymbol{x} \in \boldsymbol{R}^6$ is the vector of generalized coordinates, $\boldsymbol{M}(\boldsymbol{x})$ is the inertia matrix, $\boldsymbol{I}_m$ is diagonal matrix of actuator inertias reflected to the cable side of the gears, $\boldsymbol{C}(\boldsymbol{x},\dot{\boldsymbol{x}})$ represents the Coriolis and centrifugal terms, $\boldsymbol{G}(\boldsymbol{x})$ is the gravitational terms, r is radius of pulleys and $\boldsymbol{u}_r$ represents the input torque. $\boldsymbol{J}$ represents the Jacobian matrix of the system and relates $\dot{\boldsymbol{x}}$ to derivative of the cable length vector by: $\dot{\boldsymbol{L}} = \boldsymbol{J}\dot{\boldsymbol{x}}$. $\boldsymbol{F}_d$ denotes coefficient matrix of viscous friction and $\boldsymbol{F}_s$ is a Coulomb friction term, $\boldsymbol{T}_d$ denotes disturbances which could represent any inaccuracy in dynamic model.

In design of robust PID controller it is assumed that all dynamical terms such as $\boldsymbol{M}_{eq}(\boldsymbol{x})$, and $\boldsymbol{C}_{eq}(\boldsymbol{x},\dot{\boldsymbol{x}})$ are uncertain and only some information about their bounds is available. The control law is designed based on these bounds and assumptions to satisfy robust stability conditions. Recall dynamic model of system (1), and choose a PID controller for the system as follows:

$$\boldsymbol{u}_1 = \boldsymbol{J}^T \boldsymbol{u}_r = \boldsymbol{K}_V \dot{\boldsymbol{e}} + \boldsymbol{K}_P \boldsymbol{e} + \boldsymbol{K}_I \int_0^t \boldsymbol{e}(s) ds \tag{2}$$

or

$$\boldsymbol{u}_r = \boldsymbol{J}^\dagger \left[\boldsymbol{K}_V \dot{\boldsymbol{e}} + \boldsymbol{K}_P \boldsymbol{e} + \boldsymbol{K}_I \int_0^t \boldsymbol{e}(s) ds \right] + r\boldsymbol{Q} \tag{3}$$

in which, $\boldsymbol{e} = \boldsymbol{x}_d - \boldsymbol{x}$ and $\boldsymbol{y} = \left[\int_0^t \boldsymbol{e}^T(s) ds \quad \boldsymbol{e}^T \quad \dot{\boldsymbol{e}}^T \right]^T$. $\boldsymbol{J}^\dagger$ is pseudo-inverse of $\boldsymbol{J}^T$, which achieves minimum norm response and $\boldsymbol{Q}$, which is called the internal forces, spans null space of $\boldsymbol{J}^T$ and must satisfy

$$\boldsymbol{J}^T \boldsymbol{Q} = \boldsymbol{0} \tag{4}$$

It means that this term ($\boldsymbol{Q}$) does not contribute into motion of the end-effector and only provides positive tension in the cables. In this paper we assume that the system always satisfies the vector closure conditions and at all times, positive internal forces can be produced such that the cables are in tension. As it is demonstrated in [9], in spite of uncertainties in all parameters, following relations hold for dynamic terms of the cable robot.

$$\begin{cases} \underline{m} \leq \|\boldsymbol{M}_{eq}\| \leq \overline{m} \\ \|\boldsymbol{C}_{eq}(\boldsymbol{x}, \dot{\boldsymbol{x}})\| \leq \beta_3 + \beta_4 \|\boldsymbol{y}\| \\ \|\boldsymbol{G}_{eq}(\boldsymbol{x})\| \leq \xi_{g_{e_q}} \\ \|\boldsymbol{N}_{eq}(\boldsymbol{x}, \dot{\boldsymbol{x}})\| \leq \beta_0 + \beta_1 \|\boldsymbol{y}\| + \beta_2 \|\boldsymbol{y}\|^2 \end{cases} \tag{5}$$

A. Stability Analysis

Implement the control law $\boldsymbol{u}_r$ in (1) to get:

$$\dot{\boldsymbol{y}} = \boldsymbol{A}\boldsymbol{y} + \boldsymbol{B}\Delta\boldsymbol{A} \tag{6}$$

where,

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I}_6 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}_6 \\ -\boldsymbol{M}_{eq}^{-1}\boldsymbol{K}_I & -\boldsymbol{M}_{eq}^{-1}\boldsymbol{K}_P & -\boldsymbol{M}_{eq}^{-1}\boldsymbol{K}_V \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{M}_{eq}^{-1} \end{bmatrix}, \quad \Delta\boldsymbol{A} = \boldsymbol{N}_{eq}(\boldsymbol{x}, \dot{\boldsymbol{x}}) + \boldsymbol{M}_{eq}\ddot{\boldsymbol{x}}_d$$

To analyze robust stability of the system, consider the following Lyapunov function [10].

$$V_R(\boldsymbol{y}) = \boldsymbol{y}^T \boldsymbol{P} \boldsymbol{y}$$

in which, $\boldsymbol{P}$ is equal to:

$$\frac{1}{2} \begin{bmatrix} \mu\boldsymbol{K}_P + \mu\boldsymbol{K}_I + \mu^2\boldsymbol{M}_{eq} & \mu\boldsymbol{K}_V + \boldsymbol{K}_I + \mu^2\boldsymbol{M}_{eq} & \mu\boldsymbol{M}_{eq} \\ \mu\boldsymbol{K}_V + \boldsymbol{K}_I + \mu^2\boldsymbol{M}_{eq} & \mu\boldsymbol{K}_V + \boldsymbol{K}_P + \mu^2\boldsymbol{M}_{eq} & \mu\boldsymbol{M}_{eq} \\ \mu\boldsymbol{M}_{eq} & \mu\boldsymbol{M}_{eq} & \boldsymbol{M}_{eq} \end{bmatrix}.$$

Now choose, $\boldsymbol{K}_P = k_P\boldsymbol{I}$, $\boldsymbol{K}_V = k_V\boldsymbol{I}$, and $\boldsymbol{K}_I = k_I\boldsymbol{I}$. Then we may prove the following results.

Lemma 1: Assume the following inequalities hold:

$$\mu > 0 \quad , \quad 2\mu < 1$$
$$s_1 = \mu(k_P - k_V) - (1-\mu)k_I - \mu\overline{m} > 0$$
$$s_2 = k_P - k_I - \mu\overline{m} > 0$$

Then $\boldsymbol{P}$ is positive definite and satisfies the following condition:

$$\underline{\lambda}(\boldsymbol{P})\|\boldsymbol{y}\|^2 \leq V_R(\boldsymbol{y}) \leq \overline{\lambda}(\boldsymbol{P})\|\boldsymbol{y}\|^2 \tag{7}$$

in which,

$$\underline{\lambda}(\boldsymbol{P}) = min\{\frac{1-2\mu}{2}\underline{m}, \frac{s_1}{2}, \frac{s_2}{2}\}$$
$$\overline{\lambda}(\boldsymbol{P}) = max\{\frac{1+2\mu}{2}\overline{m}, \frac{s_3}{2}, \frac{s_4}{2}\}$$

and

$$s_3 = \mu(k_P + k_v) + (1+\mu)k_I + (1+2\mu)\mu\overline{m}$$
$$s_4 = \mu\overline{m}(1+2\mu) + 2\mu k_V + k_P + k_I$$

$\underline{m}$ and $\overline{m}$ are defined as before in (5).

Proof is based on Gershgorin theorem and is similar to that in [11]. Now since P is positive definite, by using skew-symmetry of $\dot{\boldsymbol{M}}_{eq} - 2\boldsymbol{C}_{eq}$ and some manipulations, one may write.

$$\dot{V}_R(\boldsymbol{y}) = -\boldsymbol{y}^T\boldsymbol{H}\boldsymbol{y} + \tfrac{1}{2}\boldsymbol{y}^T \begin{bmatrix} \mu\boldsymbol{I} \\ \mu\boldsymbol{I} \\ \boldsymbol{I} \end{bmatrix} (\boldsymbol{C}_{eq} + \boldsymbol{C}_{eq}^T)$$
$$\begin{bmatrix} \mu\boldsymbol{I} & \mu\boldsymbol{I} & \boldsymbol{I} \end{bmatrix} \boldsymbol{y} + \boldsymbol{y}^T \begin{bmatrix} \mu\boldsymbol{I} \\ \mu\boldsymbol{I} \\ \boldsymbol{I} \end{bmatrix} \Delta\boldsymbol{A} +$$
$$\tfrac{1}{2}\boldsymbol{y}^T \begin{bmatrix} \boldsymbol{0} & \mu^2\boldsymbol{I} & \mu^2\boldsymbol{I} \\ \mu\boldsymbol{I} & 2\mu^2\boldsymbol{I} & (\mu^2+\mu)\boldsymbol{I} \\ \mu^2\boldsymbol{I} & (\mu^2+\mu)\boldsymbol{I} & \mu\boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{M}_{eq} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{M}_{eq} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{M}_{eq} \end{bmatrix} \boldsymbol{y}$$

where,

$$\boldsymbol{H} = \begin{bmatrix} \mu k_I\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & (\mu k_P - \mu k_V - k_I)\boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & k_V\boldsymbol{I} \end{bmatrix}$$

Hence, we have

$$\dot{V}_R(\boldsymbol{y}) \leq -\rho\|\boldsymbol{y}\|^2 + \lambda_1\|\boldsymbol{C}_{eq}\|\|\boldsymbol{y}\|^2 +$$
$$\mu^{-1}\lambda_1\|\boldsymbol{y}\|\|\Delta\boldsymbol{A}\| + \lambda_2\overline{m}\|\boldsymbol{y}\|^2$$

in which,

$$\rho = min\{\mu k_I\, , \, \mu(k_P - k_V) - k_I\, , \, k_V\}$$

Using inequalities (5), one may write $\dot{V}_R(\boldsymbol{y})$ as

$$\dot{V}_R(\boldsymbol{y}) \leq \|\boldsymbol{y}\|(\xi_0 - \xi_1\|\boldsymbol{y}\| + \xi_2\|\boldsymbol{y}\|^2) \tag{8}$$

in which,

$$\begin{cases} \xi_0 = \mu^{-1}\lambda_1\beta_0 + \mu^{-1}\lambda_1\lambda_3\overline{m} \\ \xi_1 = \rho - \lambda_1\beta_3 - \frac{1}{2}\lambda_2\overline{m} - \mu^{-1}\lambda_1\beta_1 \\ \xi_2 = \lambda_1\beta_4 + \mu^{-1}\lambda_1\beta_2 \end{cases} \tag{9}$$

where, $\lambda_1 = \lambda_{max}(\boldsymbol{R}_1)$, $\lambda_2 = \lambda_{max}(\boldsymbol{R}_2)$, and $\lambda_3 = sup\|\ddot{\boldsymbol{x}}_d\|$, and λ_{max} denotes the largest eigenvalue of the corresponding matrix, and

$$\boldsymbol{R}_1 = \begin{bmatrix} \mu^2\boldsymbol{I} & \mu^2\boldsymbol{I} & \mu\boldsymbol{I} \\ \mu^2\boldsymbol{I} & \mu^2\boldsymbol{I} & \mu\boldsymbol{I} \\ \mu\boldsymbol{I} & \mu\boldsymbol{I} & \mu\boldsymbol{I} \end{bmatrix}, \boldsymbol{R}_2 = \begin{bmatrix} \boldsymbol{0} & \mu^2\boldsymbol{I} & \mu^2\boldsymbol{I} \\ \mu\boldsymbol{I} & 2\mu^2\boldsymbol{I} & (\mu^2+\mu)\boldsymbol{I} \\ \mu^2\boldsymbol{I} & (\mu^2+\mu)\boldsymbol{I} & \mu\boldsymbol{I} \end{bmatrix}$$

According to the result obtained so far, we may state the stability conditions for the error system based on the following theorem.

Theorem 1: The error system (6) is uniformly ultimately bounded (UUB), if ξ_1 is chosen large enough.

Proof: According to Equations (7) and (8) and lemma 3.5 from [10], if the following conditions hold, the system is UUB stable with respect to $B(0, d)$, where

$$d = \frac{2\xi_0}{\xi_1 + \sqrt{\xi_1^2 - 4\xi_0\xi_2}}\sqrt{\frac{\overline{\lambda}(\boldsymbol{P})}{\underline{\lambda}(\boldsymbol{P})}}$$

The conditions are:

$$\begin{aligned} \xi_1 &> 2\sqrt{\xi_0\xi_2} \\ \xi_1^2 + \xi_1\sqrt{\xi_1^2 - 4\xi_0\xi_2} &> 2\xi_0\xi_2(1 + \sqrt{\frac{\overline{\lambda}(\boldsymbol{P})}{\underline{\lambda}(\boldsymbol{P})}}) \\ \xi_1 + \sqrt{\xi_1^2 - 4\xi_0\xi_2} &> 2\xi_2\|\boldsymbol{y}_0\|\sqrt{\frac{\overline{\lambda}(\boldsymbol{P})}{\underline{\lambda}(\boldsymbol{P})}} \end{aligned}$$

These conditions can be simply met by choosing large enough ξ_1. According to (9), this can be met by choosing appropriate large control gains $\boldsymbol{K}_P, \boldsymbol{K}_V$, and $\boldsymbol{K}_I$. ■

III. Robot with Elastic Cables

A. Dynamic Model

In a CDPM vibration caused by inevitable elasticity in cables, may be a major concern for some applications which require high accuracy or high bandwidth. New research results have shown that in fully-constrained cable robots, dominant dynamics of cables are corresponding to longitudinal vibration [12], therefore, axial spring model can suitably describe the effects of dominant dynamics of cable.

In order to model a general cable driven robot with n cables assume that: $L_{1i} : i = 1, 2, ..., n$ denotes the length of i^{th} cable with tension which can be measured by a string pot. $L_{2i} : i = 1, 2, ..., n$ denotes the cable length corresponding to the i^{th} actuator and may be measured by the motor shaft encoder. If the system is rigid, then $L_{1i} = L_{2i}, \forall i$. Let us denote:

$$\boldsymbol{L} = (L_{11}, L_{12}, \cdots, L_{1n}, L_{21}, L_{22}, \cdots, L_{2n}) = (\boldsymbol{L}_1^T, \boldsymbol{L}_2^T)$$

With this notation, final equations of motion are derived in [7], which may be written as following:

$$\boldsymbol{M}(\boldsymbol{x})\ddot{\boldsymbol{x}} + \boldsymbol{N}(\boldsymbol{x}, \dot{\boldsymbol{x}}) = \boldsymbol{J}^T\boldsymbol{K}(\boldsymbol{L}_2 - \boldsymbol{L}_1) \tag{10}$$

$$\boldsymbol{I}_m\ddot{\boldsymbol{q}} + \boldsymbol{D}\dot{\boldsymbol{q}} + r\boldsymbol{K}(\boldsymbol{L}_2 - \boldsymbol{L}_1) = \boldsymbol{u} \tag{11}$$

in which,

$$\begin{aligned} &\boldsymbol{N}(\boldsymbol{x}, \dot{\boldsymbol{x}}) = \boldsymbol{C}(\boldsymbol{x}, \dot{\boldsymbol{x}})\dot{\boldsymbol{x}} + \boldsymbol{G}(\boldsymbol{x}) + \boldsymbol{F}_d\dot{\boldsymbol{x}} + \boldsymbol{F}_s(\dot{\boldsymbol{x}}) + \boldsymbol{T}_d \\ &\boldsymbol{L}_2 - \boldsymbol{L}_0 = r\boldsymbol{q} \quad , \quad \dot{\boldsymbol{L}}_1 = \boldsymbol{J}\dot{\boldsymbol{x}} \end{aligned}$$

In these equations $\boldsymbol{L}_0$ denotes the initial cables length vector at $\boldsymbol{x} = 0$ and other parameters are defined as before. For notational simplicity we assumed that all cable stiffness coefficients are equal[1]. Furthermore, assume that the stiffness values of the cables are in order of magnitude larger than other system parameters. To idealize this assumption, assume that $\boldsymbol{K} = O(1/\epsilon^2)$ where ϵ is a small parameter.

Equations (10) and (11) represent CDPM as a nonlinear and coupled system. This representation includes both rigid and flexible subsystems and their interactions. It can be shown that the model of cable driven parallel robot with elastic cables is reduced to (1), if the cable stiffness $\boldsymbol{K}$ tends to infinity. Furthermore, this model has inherited the properties of rigid dynamics (1), such as positive definiteness of inertia matrix and skew symmetricity of $\dot{\boldsymbol{M}}_{eq} - 2\boldsymbol{C}_{eq}$.

B. Control

In this section we show that the control law (3) developed for rigid robot, can be modified for the robot with elastic cables. First, consider a composite control law by adding a corrective term to the control law (3) in the form of

$$\boldsymbol{u} = \boldsymbol{u}_r + \boldsymbol{K}_d(\dot{\boldsymbol{L}}_1 - \dot{\boldsymbol{L}}_2) \tag{12}$$

where $\boldsymbol{u}_r$ is given by (3) in terms of $\boldsymbol{x}$ and $\boldsymbol{K}_d$ is a constant and positive diagonal matrix whose diagonal elements are in order of $O(1/\epsilon)$. Notice that

$$\boldsymbol{L}_2 = r\boldsymbol{q} + \boldsymbol{L}_0 \Longrightarrow \dot{\boldsymbol{L}}_2 = r\dot{\boldsymbol{q}} \ , \quad \ddot{\boldsymbol{L}}_2 = r\ddot{\boldsymbol{q}} \tag{13}$$

Substitute control law (12) in (11) and define variable $\boldsymbol{z}$ as

$$\boldsymbol{z} = \boldsymbol{K}(\boldsymbol{L}_2 - \boldsymbol{L}_1) \tag{14}$$

The closed loop dynamics reduces to

$$r^{-1}\boldsymbol{I}_m\ddot{\boldsymbol{z}} + \boldsymbol{K}_d\dot{\boldsymbol{z}} + r\boldsymbol{K}\boldsymbol{z} = \boldsymbol{K}(\boldsymbol{u}_r - r^{-1}\boldsymbol{I}_m\ddot{\boldsymbol{L}}_1) \tag{15}$$

By the assumption on $\boldsymbol{K}$ and our choice for $\boldsymbol{K}_d$ we may write

$$\boldsymbol{K} = \frac{\boldsymbol{K}_1}{\epsilon^2} \quad ; \quad \boldsymbol{K}_d = \frac{\boldsymbol{K}_2}{\epsilon} \tag{16}$$

where $\boldsymbol{K}_1$, $\boldsymbol{K}_2$ are of $O(1)$. Therefore (15) can be written as

$$\epsilon^2 r^{-1}\boldsymbol{I}_m\ddot{\boldsymbol{z}} + \epsilon\boldsymbol{K}_2\dot{\boldsymbol{z}} + r\boldsymbol{K}_1\boldsymbol{z} = \boldsymbol{K}_1(\boldsymbol{u}_r - r^{-1}\boldsymbol{I}_m\ddot{\boldsymbol{L}}_1) \tag{17}$$

Now equations (10) and (17) can be written together as

$$\boldsymbol{M}(\boldsymbol{x})\ddot{\boldsymbol{x}} + \boldsymbol{N}(\boldsymbol{x}, \dot{\boldsymbol{x}}) = \boldsymbol{J}^T\boldsymbol{z} \tag{18}$$

$$\epsilon^2 r^{-1}\boldsymbol{I}_m\ddot{\boldsymbol{z}} + \epsilon\boldsymbol{K}_2\dot{\boldsymbol{z}} + r\boldsymbol{K}_1\boldsymbol{z} = \boldsymbol{K}_1(\boldsymbol{u}_r - r^{-1}\boldsymbol{I}_m\ddot{\boldsymbol{L}}_1) \tag{19}$$

The variable $\boldsymbol{z}$ and its time derivative $\dot{\boldsymbol{z}}$ may be considered as the fast variables while the end-effector position variable $\boldsymbol{x}$ or $\boldsymbol{L}_1$ and its time derivative $\dot{\boldsymbol{x}}$ are considered as the slow variables. Using the results of singular perturbation theory, elastic system (18) and (19) can be approximated by the quasi–steady state system or slow subsystem and the boundary layer system or fast subsystem as follows. With $\epsilon = 0$, equation (19) becomes

$$\bar{\boldsymbol{z}} = r^{-1}(\bar{\boldsymbol{u}}_r - r^{-1}\boldsymbol{I}_m\ddot{\bar{\boldsymbol{L}}}_1) \tag{20}$$

[1] This assumption does not reduce the generality of the problem, and for the general case this can be easily reached by variable scaling.

in which, the overbar variables represent the values of variables when $\epsilon = 0$. Substitute (20) into (18)

$$\boldsymbol{M}(\bar{\boldsymbol{x}})\ddot{\bar{\boldsymbol{x}}} + \boldsymbol{N}(\bar{\boldsymbol{x}}, \dot{\bar{\boldsymbol{x}}}) = r^{-1}\boldsymbol{J}^T(\bar{\boldsymbol{u}}_r - r^{-1}\boldsymbol{I}_m\ddot{\bar{\boldsymbol{L}}}_1) \qquad (21)$$

Furthermore, substitute $\ddot{\bar{\boldsymbol{L}}}_1 = \boldsymbol{J}\ddot{\bar{\boldsymbol{x}}} + \dot{\boldsymbol{J}}\dot{\bar{\boldsymbol{x}}}$ in the above equation as:

$$\boldsymbol{M}_{eq}(\bar{\boldsymbol{x}})\ddot{\bar{\boldsymbol{x}}} + \boldsymbol{N}_{eq}(\bar{\boldsymbol{x}}, \dot{\bar{\boldsymbol{x}}}) = \boldsymbol{J}^T\bar{\boldsymbol{u}}_r \qquad (22)$$

Equation (22) is called quasi-steady state system. Note that (22) is the rigid model (1) in terms of $\bar{\boldsymbol{x}}$. Using Tikhonov's theorem [8], for $t > 0$ the elastic force $\boldsymbol{z}(t)$ and the end-effector position $\boldsymbol{x}(t)$ satisfy

$$\boldsymbol{z}(t) = \bar{\boldsymbol{z}}(t) + \boldsymbol{\eta}(\tau) + O(\epsilon)$$
$$\boldsymbol{x}(t) = \bar{\boldsymbol{x}}(t) + O(\epsilon) \;\;,\;\; \boldsymbol{L}_1(t) = \bar{\boldsymbol{L}}_1(t) + O(\epsilon)$$

where, $\tau = t/\epsilon$ is the fast time scale and $\boldsymbol{\eta}$ is the fast state variable that satisfies the following boundary layer equation,

$$r^{-1}\boldsymbol{I}_m\frac{d^2\boldsymbol{\eta}}{d\tau^2} + \boldsymbol{K}_2\frac{d\boldsymbol{\eta}}{d\tau} + r\boldsymbol{K}_1\boldsymbol{\eta} = \boldsymbol{0} \qquad (23)$$

Considering these results, elastic system (18) and (19) according to (20), can be approximated up to $O(\epsilon)$ by

$$\boldsymbol{M}_{eq}(\boldsymbol{x})\ddot{\boldsymbol{x}} + \boldsymbol{N}_{eq}(\bar{\boldsymbol{x}}, \dot{\bar{\boldsymbol{x}}}) = \boldsymbol{J}^T(\boldsymbol{u}_r + r\boldsymbol{\eta}(\tau)) \qquad (24)$$

$$r^{-1}\boldsymbol{I}_m\frac{d^2\boldsymbol{\eta}}{dt^2} + \boldsymbol{K}_d\frac{d\boldsymbol{\eta}}{dt} + r\boldsymbol{K}\boldsymbol{\eta} = \boldsymbol{0} \qquad (25)$$

Notice that the controller gain $\boldsymbol{K}_d$ can be suitably chosen such that the boundary layer system (23) becomes asymptotically stable. By this means, with sufficiently small values of ϵ, the response of the elastic system (10) and (11) with the composite control (12) consisting of the rigid control $\boldsymbol{u}_r$ given by (3) and the corrective term $\boldsymbol{K}_d(\dot{\boldsymbol{L}}_1 - \dot{\boldsymbol{L}}_2)$, will be nearly the same as the response of rigid system (1) with the rigid control $\boldsymbol{u}_r$ alone. This will happen after some initially damped transient oscillation of the fast variables $\boldsymbol{\eta}(t/\epsilon)$.

C. Stability Analysis of Total System

Control of rigid model and its stability analysis were discussed in previous section. Furthermore, It is demonstrated that the boundary layer or the fast subsystem (23) is asymptotically stable, if the corrective term is used in the control law. However, in general, individual stability of the boundary layer and that of quasi-steady state subsystems does not guarantee the stability of total closed-loop system. In this section, the stability of the total system is analyzed in details. Recall the dynamic equations of elastic system (24) and (25), and apply the control law (3) from previous section. Consider $\boldsymbol{y} = \begin{bmatrix} \int_0^t \boldsymbol{e}^T ds & \boldsymbol{e}^T & \dot{\boldsymbol{e}}^T \end{bmatrix}^T$ and $\boldsymbol{h} = \begin{bmatrix} \boldsymbol{\eta}^T & \dot{\boldsymbol{\eta}}^T \end{bmatrix}^T$, in which, $\boldsymbol{e} = \boldsymbol{x}_d - \boldsymbol{x}$. Then the dynamics equations can be rewritten as,

$$\dot{\boldsymbol{y}} = \boldsymbol{A}\boldsymbol{y} + \boldsymbol{B}\Delta\boldsymbol{A} + \boldsymbol{C}\left[\boldsymbol{I} \;\; \boldsymbol{0}\right]\boldsymbol{h} \qquad (26)$$
$$\dot{\boldsymbol{h}} = \tilde{\boldsymbol{A}}\boldsymbol{h} \qquad (27)$$

in which,

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I}_6 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}_6 \\ -\boldsymbol{M}_{eq}^{-1}\boldsymbol{K}_I & -\boldsymbol{M}_{eq}^{-1}\boldsymbol{K}_P & -\boldsymbol{M}_{eq}^{-1}\boldsymbol{K}_V \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{M}_{eq}^{-1} \end{bmatrix} \;,\; \boldsymbol{C} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ -r\boldsymbol{M}_{eq}^{-1}\boldsymbol{J}^T \end{bmatrix}$$

and

$$\Delta\boldsymbol{A} = \boldsymbol{N}_{eq}(\boldsymbol{x}, \dot{\boldsymbol{x}}) + \boldsymbol{M}_{eq}\ddot{\boldsymbol{x}}_d$$
$$\tilde{\boldsymbol{A}} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -r^2\boldsymbol{I}_m^{-1}\boldsymbol{K} & -r\boldsymbol{I}_m^{-1}\boldsymbol{K}_d \end{bmatrix}$$

The stability of this system may be analyzed by the following Lemma and Theorem.

Lemma 2: There is a positive definite matrix $\boldsymbol{K}_d$ such that the closed-loop system (27) is asymptotically stable.

Proof: Consider the following Lyapunov function candidate:

$$V_F = \boldsymbol{h}^T\boldsymbol{W}\boldsymbol{h} \;\;,\;\; \boldsymbol{W} = \frac{1}{2}\begin{bmatrix} r^2(\boldsymbol{K}_d + \boldsymbol{K}) & r\boldsymbol{I}_m \\ r\boldsymbol{I}_m & \boldsymbol{I}_m \end{bmatrix} \qquad (28)$$

According to Shur complement, in order to have positive definite $\boldsymbol{W}$, it is sufficient to have $\boldsymbol{K}_d > \boldsymbol{I}_m$. Differentiate V_F along trajectories of (27):

$$\dot{V}_F = -\boldsymbol{h}^T\boldsymbol{S}\boldsymbol{h} \;\;,\;\; \boldsymbol{S} = \begin{bmatrix} r^3\boldsymbol{K} & \boldsymbol{0} \\ \boldsymbol{0} & r(\boldsymbol{K}_d - \boldsymbol{I}_m) \end{bmatrix} \qquad (29)$$

Since, $\boldsymbol{K}$, $\boldsymbol{K}_d$ and $\boldsymbol{I}_m$ are diagonal positive definite matrices, $\dot{V}_F$ becomes negative definite if $\boldsymbol{K}_d > \boldsymbol{I}_m$. If this condition holds the closed-loop system (27) is asymptotically stable. ■

Theorem 3: The closed-loop system (26) and (27) is UUB stable if ξ_1 and $\boldsymbol{K}_d$ are chosen suitably large.

Proof: Consider the following composite Lyapunov function candidate

$$V(\boldsymbol{y}, \boldsymbol{h}) = V_R + V_F = \boldsymbol{y}^T\boldsymbol{P}\boldsymbol{y} + \boldsymbol{h}^T\boldsymbol{W}\boldsymbol{h} \qquad (30)$$

in which, $\boldsymbol{y}^T\boldsymbol{P}\boldsymbol{y}$ denotes the Lyapunov function candidate for the rigid subsystem, and $\boldsymbol{h}^T\boldsymbol{W}\boldsymbol{h}$ denotes that for the fast subsystem (23). According to Rayleigh-Ritz inequality:

$$\begin{cases} \underline{\lambda}(\boldsymbol{P})\|\boldsymbol{y}\|^2 \le \boldsymbol{y}^T\boldsymbol{P}\boldsymbol{y} \le \overline{\lambda}(\boldsymbol{P})\|\boldsymbol{y}\|^2 \\ \underline{\lambda}(\boldsymbol{W})\|\boldsymbol{h}\|^2 \le \boldsymbol{h}^T\boldsymbol{W}\boldsymbol{h} \le \overline{\lambda}(\boldsymbol{W})\|\boldsymbol{h}\|^2 \end{cases}$$

in which $\underline{\lambda}$, $\overline{\lambda}$ are the smallest and largest eigenvalues of the matrices, respectively. By adding these inequalities one can write,

$$\begin{aligned} \begin{bmatrix} \|\boldsymbol{y}\| & \|\boldsymbol{h}\| \end{bmatrix} \begin{bmatrix} \underline{\lambda}(\boldsymbol{P}) & 0 \\ 0 & \underline{\lambda}(\boldsymbol{W}) \end{bmatrix} \begin{bmatrix} \|\boldsymbol{y}\| \\ \|\boldsymbol{h}\| \end{bmatrix} &\le V(\boldsymbol{y}, \boldsymbol{h}) \\ \le \begin{bmatrix} \|\boldsymbol{y}\| & \|\boldsymbol{h}\| \end{bmatrix} \begin{bmatrix} \overline{\lambda}(\boldsymbol{P}) & 0 \\ 0 & \overline{\lambda}(\boldsymbol{W}) \end{bmatrix} & \begin{bmatrix} \|\boldsymbol{y}\| \\ \|\boldsymbol{h}\| \end{bmatrix} \end{aligned} \qquad (31)$$

Define $\boldsymbol{Z}_t = \begin{bmatrix} \|\boldsymbol{y}\| \\ \|\boldsymbol{h}\| \end{bmatrix}$ and apply Rayleigh-Ritz inequality, it can be written as,

$$\underline{\lambda}\|\boldsymbol{Z}_t\|^2 \le V(\boldsymbol{y}, \boldsymbol{h}) \le \overline{\lambda}\|\boldsymbol{Z}_t\|^2 \qquad (32)$$

in which,

$$\underline{\lambda} = min(\underline{\lambda}(\boldsymbol{P}), \underline{\lambda}(\boldsymbol{W}))$$
$$\overline{\lambda} = max(\overline{\lambda}(\boldsymbol{P}), \overline{\lambda}(\boldsymbol{W}))$$

Differentiate $V(\boldsymbol{y}, \boldsymbol{h})$ along trajectories of (26) and (27). Hence,

$$\dot{V}(\boldsymbol{y}, \boldsymbol{h}) = 2\boldsymbol{y}^T\boldsymbol{P}\dot{\boldsymbol{y}} + \boldsymbol{y}^T\dot{\boldsymbol{P}}\boldsymbol{y} + 2\boldsymbol{h}\boldsymbol{W}\dot{\boldsymbol{h}} = -\boldsymbol{h}^T\boldsymbol{S}\boldsymbol{h} + \left[2\boldsymbol{y}^T\boldsymbol{P}(\boldsymbol{A}\boldsymbol{y} + \boldsymbol{B}\Delta\boldsymbol{A}) + \boldsymbol{y}^T\dot{\boldsymbol{P}}\boldsymbol{y}\right] + 2\boldsymbol{y}^T\boldsymbol{P}\boldsymbol{C}\begin{bmatrix}\boldsymbol{I} & \boldsymbol{0}\end{bmatrix}\boldsymbol{h}$$

Using Rayleigh-Ritz inequality,

$$-\boldsymbol{h}^T\boldsymbol{S}\boldsymbol{h} \leq -\lambda_{min}(\boldsymbol{S})\|\boldsymbol{h}\|^2 \tag{33}$$

According to (8) it can be concluded that,

$$2\boldsymbol{y}^T\boldsymbol{P}(\boldsymbol{A}\boldsymbol{y} + \boldsymbol{B}\Delta\boldsymbol{A}) + \boldsymbol{y}^T\dot{\boldsymbol{P}}\boldsymbol{y} \leq \|\boldsymbol{y}\|(\xi_0 - \xi_1\|\boldsymbol{y}\| + \xi_2\|\boldsymbol{y}\|^2)$$

Furthermore,

$$2\boldsymbol{y}^T\boldsymbol{P}\boldsymbol{C}\begin{bmatrix}\boldsymbol{I} & \boldsymbol{0}\end{bmatrix}\boldsymbol{h} \leq 2r\overline{m}\overline{\lambda}(\boldsymbol{P})\sigma_{max}(\boldsymbol{J}^T)\|\boldsymbol{y}\|\|\boldsymbol{h}\|$$

In which, λ_{min}, and σ_{max} denote the smallest eigenvalue and largest singular value of the corresponding matries, respectively. By using the above inequalities, one may write

$$\dot{V}(\boldsymbol{y}, \boldsymbol{h}) \leq -\boldsymbol{Z}_t^T\boldsymbol{R}\boldsymbol{Z}_t + \xi_0\|\boldsymbol{Z}_t\| + \xi_2\|\boldsymbol{Z}_t\|^3 \tag{34}$$

in which,

$$\boldsymbol{R} = \begin{bmatrix} \xi_1 & -r\overline{m}\overline{\lambda}(\boldsymbol{P})\sigma_{max}(\boldsymbol{J}^T) \\ -r\overline{m}\overline{\lambda}(\boldsymbol{P})\sigma_{max}(\boldsymbol{J}^T) & \lambda_{min}(\boldsymbol{S}) \end{bmatrix} \tag{35}$$

$\boldsymbol{R}$ is positive definite if

$$\lambda_{min}(\boldsymbol{S}) > \frac{r^2\overline{m}^2\overline{\lambda}^2(\boldsymbol{P})\sigma^2_{max}(\boldsymbol{J}^T)}{\xi_1} \tag{36}$$

This condition is met by a suitable choice of $\boldsymbol{K}_d$ for the fast subsystem. Therefore, one can write,

$$\dot{V}(\boldsymbol{y}, \boldsymbol{h}) \leq \|\boldsymbol{Z}_t\|(\xi_0 - \lambda_{min}(\boldsymbol{R})\|\boldsymbol{Z}_t\| + \xi_2\|\boldsymbol{Z}_t\|^2) \tag{37}$$

Now, according to (37) and (32), and Lemma 3.5 of [10], if these conditions are met then the closed-loop system (26) and (27) is UUB stable with respect to $Y(0, d')$ where:

$$d' = \frac{2\xi_0}{\lambda_{min}(\boldsymbol{R}) + \sqrt{\lambda^2_{min}(\boldsymbol{R}) - 4\xi_0\xi_2}}\sqrt{\frac{\overline{\lambda}(\boldsymbol{P})}{\underline{\lambda}(\boldsymbol{P})}}$$

and the stability conditions are:

$$\lambda_{min}(\boldsymbol{R}) > 2\sqrt{\xi_0\xi_2}$$

$$\lambda^2_{min}(\boldsymbol{R}) + \lambda_{min}(\boldsymbol{R})\sqrt{\lambda^2_{min}(\boldsymbol{R}) - 4\xi_0\xi_2} > 2\xi_0\xi_2(1 + \sqrt{\frac{\overline{\lambda}(\boldsymbol{P})}{\underline{\lambda}(\boldsymbol{P})}})$$

$$\lambda_{min}(\boldsymbol{R}) + \sqrt{\lambda^2_{min}(\boldsymbol{R}) - 4\xi_0\xi_2} > 2\xi_2\|\boldsymbol{Z}_{t0}\|\sqrt{\frac{\overline{\lambda}(\boldsymbol{P})}{\underline{\lambda}(\boldsymbol{P})}}$$

These conditions are satisfied by increasing $\lambda_{min}(\boldsymbol{R})$, through appropriate choice of large ξ_1, and $\lambda_{min}(\boldsymbol{S})$. Note that, ξ_1 is a function of the robust PID control gains $\boldsymbol{K}_I$, $\boldsymbol{K}_P$ and $\boldsymbol{K}_V$ and $\lambda_{min}(\boldsymbol{S})$ is affected by the control gain $\boldsymbol{K}_d$ for the fast subsystem. Therefore, robust stability of the closed-loop system is guaranteed by suitable choice of the controller gains such that the above conditions are met. ■

IV. SIMULATIONS

To show effectiveness of the proposed control algorithm a simulation study has been performed on a planar cable robot. Our model of a planar cable robot [13], consists of a moving platform that is connected by four cables to the base platform, as shown in Fig. (1). As it is shown in Fig. (1), A_i denote the fixed base points of the cables and B_i denote points of connection of the cables on the moving platform. The position of the center of the mass of the moving platform $\boldsymbol{P}$, is denoted by $\boldsymbol{P} = [x_P, y_P]$, and the orientation of the moving platform is denoted by ϕ with respect to the fixed coordinate frame. Hence, the manipulator posses three degrees of freedom $\boldsymbol{x} = [x_P, y_P, \phi]$, with one degree of actuator redundancy.

The equations of motion can be written in the following form,

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{G} + \boldsymbol{F}_d\dot{\boldsymbol{x}} + \boldsymbol{F}_s(\dot{\boldsymbol{x}}) + \boldsymbol{T}_d = \boldsymbol{J}^T\boldsymbol{K}(\boldsymbol{L}_2 - \boldsymbol{L}_1)$$
$$\boldsymbol{I}_m\ddot{\boldsymbol{q}} + \boldsymbol{D}\dot{\boldsymbol{q}} + r\boldsymbol{K}(\boldsymbol{L}_2 - \boldsymbol{L}_1) = \boldsymbol{u}$$

in which, $\boldsymbol{x} = [x_P, y_P, \phi]$, and

$$\boldsymbol{M} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_z \end{bmatrix} \quad \text{and} \quad \boldsymbol{G} = \begin{bmatrix} 0 \\ mg \\ 0 \end{bmatrix}$$

The following parametric values in SI units are used in the simulations; $\boldsymbol{I}_m = 0.6\ \boldsymbol{I}_{4\times4}$, $r = 0.035$, $\boldsymbol{K} = 1000\boldsymbol{I}$, $m = 2.5$, and $I_z = 0.03$. In order to demonstrate a highly flexible system $\boldsymbol{K}$ is intentionally chosen very low. To show the effectiveness of the proposed composite control algorithm suppose that the system is at the origin and have to track the following smooth reference trajectories in x, y, and ϕ coordinates,

$$\begin{aligned} x_d &= 0 \\ y_d &= 0.4 + 2e^{-t} - 2.4e^{-t/1.2} \\ \phi_d &= 0.2\sin(0.1\pi t) \end{aligned}$$

The controller is based on (12) and consists of rigid control $\boldsymbol{u}_r$ given by (3) and the corrective term. Controller gain matrices are chosen as $\boldsymbol{K}_P = 250\ \boldsymbol{I}_{3\times3}$, $\boldsymbol{K}_V = 40\ \boldsymbol{I}_{3\times3}$, $\boldsymbol{K}_I = 20\ \boldsymbol{I}_{3\times3}$ and $\boldsymbol{K}_d = 350\ \boldsymbol{I}_{4\times4}$ to satisfy the stability conditions. In the first step, only rigid control law $\boldsymbol{u}_r$, is applied to the manipulator. As is illustrated in Fig. (2), the

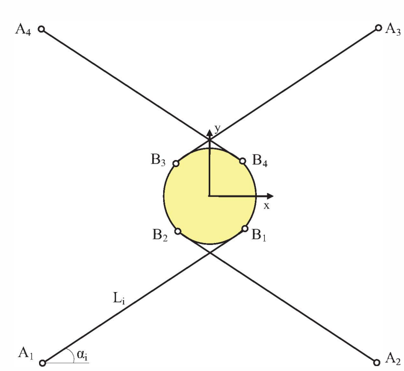

Fig. 1. The schematics of planar cable mechanism

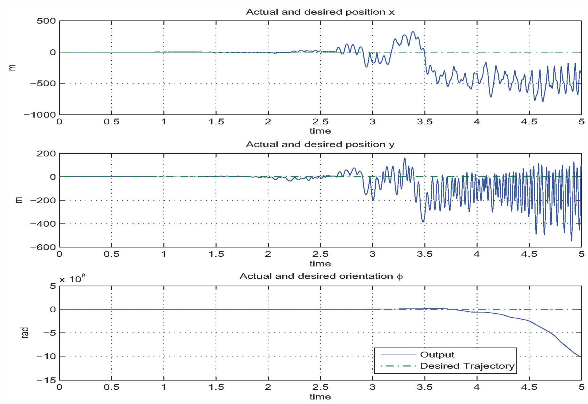

Fig. 2. The closed-loop system experiences instability, if only rigid controller $\boldsymbol{u}_r$ is applied.

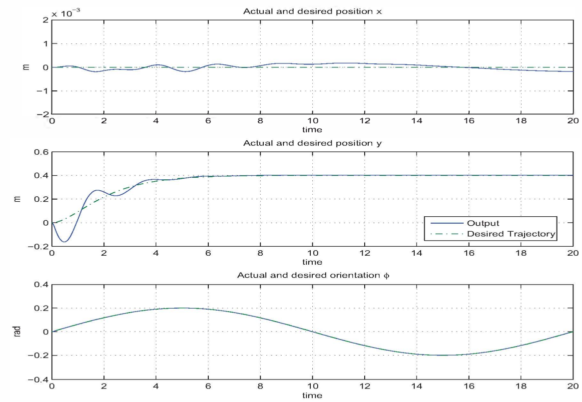

Fig. 3. Suitable tracking performance of the closed-loop system to smooth reference trajectories; Proposed control algorithm.

manipulator experiences instability if the rigid control $\boldsymbol{u}_r$ is solely applied to the system. The main reason for instability is divergence of its fast variables.

Figure (3) illustrates dynamic behavior of the closed-loop system with the proposed control algorithm. Internal force $\boldsymbol{Q}$ is used whenever at least one cable becomes slack (or $L_{1i} < L_{2i}$, $i = 1, \cdots, 4$), in order to ensure that the cables remain in tension. Although, the system is very flexible the proposed control algorithm can suitably stabilize the system. As it is seen in this figure, position and orientation outputs track the desired values pretty well, and the steady state errors are very small, while as it is shown in Fig. (4), all cables are in tension for the whole maneuver.

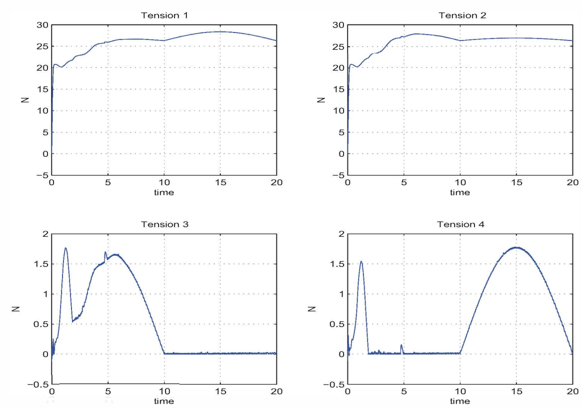

Fig. 4. Simulation results showing the cables tension for smooth reference trajectories.

V. Conclusions

In this paper robust PID control of CDPMs with elastic cables is examined in detail. Initially, Robust PID control of CDPMs with ideal cables is studied and it is proved that the proposed controller stabilize the system in presence of dynamic uncertainties. Then, by using singular perturbation theory this algorithm is modified to be applicable in the case of CDPMs with elastic cables. The proposed composite control algorithm consists of a robust PID control according to the corresponding rigid model and a corrective term to stabilize the fast subsystem. The stability of the closed-loop system is analyzed through Lyapunov second method, and it is shown that the proposed composite controller is capable to stabilize the system in presence of flexible cables. Finally, the performance of the proposed composite controller is examined through simulations.

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