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An Improved Optimization Method for iSAM2

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Abstract— **There is an issue called maximum likelihood estimation in SLAM that corresponds to a nonlinear least-square problem. It is expected to earn an accurate solution for large-scale environments with high speed of convergence. Although all the applied optimization methods might be accepted in terms of accuracy and speed of convergence for small datasets, their solutions for large-scale datasets are often far from the ground truth. In this paper, a double Dogleg trust region method is proposed and adjusted with iSAM2 to level up performance and accuracy of the algorithm especially in large-scale datasets. Since the trust region methods are sensitive to their own parameters, Gould parameters are chosen to obtain better performance. Simulations are done on some large-scale datasets and the results indicate that the proposed method is more efficient compared to the conventional iSAM2 algorithm.**

I. Introduction

Optimization-based simultaneous localization and mapping (SLAM) and its demonstration with the aid of graphical models have been studied extensively in past decades. SLAM is utilized greatly in large-scale environments. Hence, the direction of research in recent years is to find an executable real-time solution for large-scale problems. The use of smoothing helps us approach to this objective [1]. In smoothing methods we are dealing with a maximum likelihood estimation corresponding to a nonlinear least-square problem assuming that measurements are Gaussian functions. Because of high complexity of the solution with standard methods, using graphical models in representation of SLAM problem has been noticed in depth. The iSAM2 [2] is a state-of-the-art smoothing method in which the solution is incrementally updated by new observations. It uses graphical models and sparse linear algebra for solving the problem.

We have a limited set of optimization methods that can be adapted to the SLAM. The reason of this limitation is the special structure of existing optimization problem. Furthermore, it is expected to earn an accurate solution for large-scale environments with high convergence speed. Some optimization methods like Gauss-Newton and Levenberg-Marquardt have been used frequently hitherto. Besides, some changes have been made in sparse linear algebra methods for improving the results. In addition, Powell's Dogleg method has been recently offered that is faster than Levenberg-Marquardt and can be readily adapted to incremental smoothing methods like iSAM [3] and iSAM2 [4]. Although all the employed optimization methods might be accepted in terms of accuracy and speed of convergence for small datasets, their solution for large-scale datasets are often far away from the ground truth.

In this paper, we propose and adjust double Dogleg method [5] with Gould parameters for iSAM2 to increase the performance and accuracy of the algorithm especially in large-scale datasets. Double Dogleg method with Gould parameters guarantees global convergence and has more accuracy and speed compared to the previous methods. Double Dogleg method performs in a way that the search point is closer to Gauss-Newton point. Therefore, speed of convergence will increase. On the other hand, since Dogleg and double Dogleg methods are sensitive to trust region parameters, we propose to use Gould parameters [6] instead of Powell parameters [7]. Simulations are done on some large-scale datasets and the results verify that our proposed method is more efficient compared to that of the conventional iSAM2 algorithm in terms of accuracy and speed.

II. Review of iSAM2

A. Graphical Model

The estimation problem in SLAM can be represented by graphical models such as factor graph. Each factor graph can be expressed as a function which is a product of factors of graph: $f(\Theta) = \prod_i f_i(\Theta_i)$. The aim of estimation problem is to obtain the variable assignment Θ^* such that:

$$\Theta^* = \arg\max_{\Theta} f(\Theta). \tag{1}$$

In SLAM, measurements are considered to be Gaussian functions:

$$f_i(\Theta_i) \propto \exp\left(-\frac{1}{2}\|h_i(\Theta_i) - z_i\|^2_{\Sigma_i}\right). \tag{2}$$

By substituting Gaussian factors (2) into objective function $f(\Theta)$ and taking the negative logarithm, the maximization problem (1) converts to a minimization problem:

$$\Theta^* = \arg\min_{\Theta} (-\log f(\Theta)) = \arg\min_{\Theta} \frac{1}{2}\sum_i \|h_i(\Theta_i) - z_i\|^2_{\Sigma_i}. \tag{3}$$

Equation (3) corresponds to the nonlinear least-square problem, which may be solved with a nonlinear optimization method. Linearization is done at each iteration of nonlinear optimizer with the use of current estimate. As a result, a linear least-square problem is obtained as follows.

$$\arg\min_{\eta} (-\log f(\eta)) = \arg\min_{\eta} \|A\eta - b\|^2 \tag{4}$$

Where, $A \in \mathbb{R}^{m\times n}$ is measurement Jacobian matrix, m is the number of factors and n is the number of variables. By computing η at each iteration and adding it to current estimate, the new estimate is obtained [2].

B. Updating Procedure

The iSAM2 is one of the state-of-the-art incremental smoothing methods to solve SLAM problem. It utilizes the features of graphical models and sparse linear algebra concurrently. In iSAM, reordering and relinearization is done

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periodically in batch steps. This issue will usually increase the execution time and reduce the burden of online implementation of the algorithm. In iSAM2, reordering and relinearization is performed at each step and provides an incremental and efficient solution for sparse nonlinear optimization problem. The summarized updating steps of iSAM2 algorithm may be seen in Algorithm 1 [2].

Algorithm 1: Updating procedure of iSAM2

Step 1 Append all new factors.
Step 2 Initialize new variables and append them to variables vector Θ.
Step 3 Fluid relinearization.
Step 4 Update top of the Bayes tree.
- Remove top of the Bayes tree and convert it to factor graph.
- Linearize new factors and relinearize all factors that are marked in previous step in current linearization point Θ.
- Append new factors to new factor graph.
- Reorder variables and eliminate factor graph to Bayes net and convert Bayes net to Bayes tree.
- Append saved unchanged subtrees to bottom of new Bayes tree.

Step 5 Update η.
Step 6 Obtain new estimate by $\Theta \oplus \eta$.

Note that, η (in step 6 of Algorithm 1) may be updated by an optimization method such as a line search method like Gauss-Newton or a trust region method like Dogleg.

III. Optimization Methods Used in iSAM2

The goal of unconstrained optimization problems is to minimize an objective function. Least-square problems are particular type of optimization problems in which the objective function is defined as follows.

$$f(x) = \frac{1}{2}\sum_{j=1}^{m} r_j^2(x) = \frac{1}{2}\|r(x)\|_2^2 \tag{5}$$

$r : \mathbb{R}^n \to \mathbb{R}^m$ is called residual vector which is collection of r_j residuals and $r_j \in \mathbb{R}^{n\times 1}$.

Jacobian $J(x)$ is defined as an $m \times n$ matrix that includes the gradient of residuals:

$$J(x) = \left[\frac{\partial r_j}{\partial x_i}\right]_{\substack{j=1,2,\dots,m\\ i=1,2,\dots,n}} = \begin{bmatrix} \nabla r_1(x)^T \\ \nabla r_2(x)^T \\ \vdots \\ \nabla r_m(x)^T \end{bmatrix}. \tag{6}$$

Therefore, gradient and Hessian of f are obtained as [8]:

$$\nabla f(x) = \sum^{m} r_j(x)\nabla r_j(x) = J(x)^T r(x) \tag{7}$$

and

$$\begin{aligned}\nabla^2 f(x) &= \sum_{j=1}^{m} \nabla r_j(x)\nabla r_j(x)^T + \sum_{j=1}^{m} r_j(x)\nabla^2 r_j(x)\\ &= J(x)^T J(x) + \sum_{j=1}^{m} r_j(x)\nabla^2 r_j(x).\end{aligned} \tag{8}$$

A. Gauss-Newton

In this method, the search direction h_k^{gn} in k'th iteration is earned by solving the following equation:

$$J_k^T J_k h_k = -J_k^T r_k, \tag{9}$$

in which, Hessian of f is approximated with $\nabla^2 f_k \approx J_k^T J_k$ that is close to the Hessian (8) in cases with small residuals. In this occasions, Gauss-Newton method provides high local speed of convergence like Newton method. On the other hand, h_k^{gn} can be expressed as the solution of following linear least-square problem:

$$\min_{h\in\mathbb{R}^n} m_k(h) = \frac{1}{2}\|J_k h_k + r_k\|^2. \tag{10}$$

Equation (10) may be considered as the solution of linearization of vector function $r(x_k + h_k) \approx r_k + J_k h_k$ and replacement of it in $\frac{1}{2}\|\cdot\|^2$ function [8].

In order to use Gauss-Newton method in iSAM2 algorithm, an incremental version of it is created. This procedure in its matrix form is very similar to iSAM which is thoroughly described by Rosen et al. in [4]. However, combination of operations with factor graph, Bayes net and Bayes tree graphical models [9] is the main difference of iSAM2 to iSAM.

Apart from the advantages of Gauss-Newton method, there is an important disadvantage, namely the convergence is not guaranteed. If the norm of $\sum_{j=1}^{m} r_j \nabla^2 r_j$ matrix (second term in (8)) is large enough, the method may divert from real solution. In order to solve this fundamental problem, Levenberg-Marquardt method is proposed. The convergence of this method is guaranteed like that of steepest descent method [8] for the points far from the optimal solution. However, its behavior resembles Gauss-Newton method in the vicinity of optimal point [10]. Although Levenberg-Marquardt has global convergence property, system of equations may be solved several times during an iteration and this issue significantly increases the complexity of computations.

B. Dogleg

Powell has introduced a method named Dogleg [7], which composes the steepest descent and Gauss-Newton methods like Levenberg-Marquardt [11]. Dogleg is a trust region method, and therefore, is inherently suitable for nonlinear least-square problems. In trust region methods, objective function is approximated with a trust model function in a region around the current point. Then, minimum of the model function in this region is considered the solution. At each iteration of Dogleg method, it is desired to find the solution of the following suboptimal problem.

$$\min_{h\in\mathbb{R}^n} f_k + g_k^T h + \frac{1}{2} h^T G_k h \qquad s.t. \|h\| \le \Delta_k \tag{11}$$

Equation (11) is in fact a Taylor expansion of objective function f_k around x_k, in which $\Delta_k > 0$ is the trust region radius, G_k is the Hessian and g_k is the gradient of f_k, while $\|\cdot\|$ denotes the norm. This suboptimal problem is a constrained form of approximate function (10) for nonlinear least-square problem.

Trust region radius Δ_k is chosen at each iteration based on the following gain ratio:

$$\rho_k = \frac{f(x_k) - f(x_k + h_k)}{m_k(0) - m_k(h_k)} \tag{12}$$

in which the numerator and denominator denote actual and predicted reductions, respectively. The amount of ρ_k states the compromise between objective and model functions [8]. Procedure of updating Δ based on ρ is given in Algorithm 2.

Algorithm 2: Update of trust region radius Δ

Inputs: $\rho, h^{dl}, \Delta, 0 < \mu_1 < \mu_2 < 1, 0 < c_1 \leq c_2$
Output: Δ

Step 1 If $\rho > \mu_2$, then put: $\Delta \leftarrow \max\left\{\Delta, c_2\|h^{dl}\|\right\}$
Step 2 Else if $\rho < \mu_1$, then put: $\Delta \leftarrow c_1\Delta$

In trust region methods, if the obtained step makes a reduction at least equal to the reduction of Cauchy step in the model, it has global convergence (sufficient reduction condition) [12], [8]. Cauchy step is specified by the distance of the origin to the minimum point of m in the steepest descent direction (negative of gradient) as, [10]:

$$h^{cd} = -\frac{g^T g}{g^T J^T J g} g \tag{13}$$

Dogleg method is based on selection of one of the two steps h^{cd} and h^{gn} or combination of them. These two steps have the following relation [12]:

$$\|h^{cd}\| \leq \|h^{gn}\|, \qquad m\left(h^{gn}\right) \leq m\left(h^{cd}\right) \tag{14}$$

Suppose that $h^*(\Delta)$ is the exact solution of the suboptimal problem (11). According to (14), two states occur for h^{cd} and h^{gn} in the trust region [8]:

1) Gauss-Newton point is inside the trust region. In this condition, Gauss-Newton step will be an exact solution for (11):

$$h^*(\Delta) = h^{gn}, \qquad when \|h^{gn}\| \leq \Delta \tag{15}$$

2) Gauss-Newton point is outside the trust region. In this condition, as it is shown in Fig. 1, Dogleg method considers a path consisting of two line segments instead of optimal curve $h^*(\Delta)$. The first line segment continues from origin to h^{cd} and the second line segment extends from h^{cd} to h^{gn}. This path is presented by $\tilde{h}(\beta)$, $\beta \in [0, 2]$:

$$\tilde{h}(\beta) = \begin{cases} \beta h^{cd} & 0 \leq \beta < 1 \\ h^{cd} + (\beta - 1)(h^{gn} - h^{cd}) & 1 \leq \beta < 2 \end{cases} \tag{16}$$

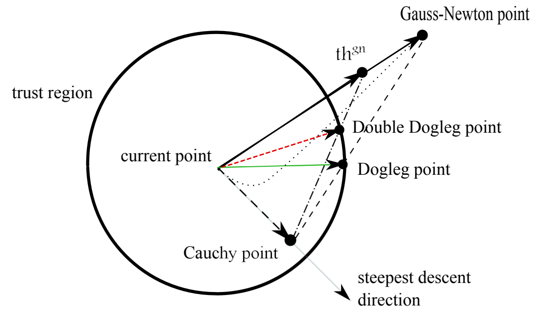


Fig. 1: Gauss-Newton point is outside and Cauchy point is inside the trust region. Dogleg and Double Dogleg steps and optimum path (dotted curve) are specified in the figure.

The value of β is found as the solution of following equation [8].

$$\|h^{cd} + (\beta - 1)(h^{gn} - h^{cd})\|^2 = \Delta^2 \tag{17}$$

The solution of Dogleg method is the minimum point of model function during the approximate path which is obtained with respect to the trust region range (Algorithm 3).

Algorithm 3: Computation of Dogleg step h^{dl}

Inputs: h^{gn}, h^{cd}, Δ
Output: h^{dl}

Step 1 If $\|h^{gn}\| \leq \Delta$, then put: $h^{dl} \leftarrow h^{gn}$
Step 2 Else if $\|h^{cd}\| \geq \Delta$, then put: $h^{dl} \leftarrow \left(\frac{\Delta}{\|h^{cd}\|}\right) h^{cd}$
Step 3 Else put: $h^{dl} \leftarrow h^{cd} + (\beta - 1)(h^{gn} - h^{cd})$, where β is obtained from $\|h^{dl}\| = \Delta$.

IV. The Proposed Optimization Method

In this section, we present an improved optimization method for iSAM2 based on the algorithm introduced by Dennis and Mei [5] named double Dogleg. Dogleg method has suitable convergence properties and is accurate, but it can lessen the effect of the use of Gauss-Newton step, and hence, reduce the speed of convergence of algorithm [13]. Moreover, it seems that a Dogleg step is significantly oriented to gradient direction near the solution [5] and thus, in this case:

- We have approximately used steepest descent method with specific step length, while steepest descent method with optimum step length is very slow near the solution.
- Computations do not depend on Hessian matrix. Hessian matrix was just used for computing step length and it has no role in the determination of the direction.

Therefore, double Dogleg method was introduced by Dennis and Mei for more inclination of Dogleg step toward Gauss-Newton step at each iteration.

A. Double Dogleg

Suppose that h_k^{cd} and h_k^{gn} are respectively Cauchy and Gauss-Newton steps at k'th iteration. Also, $G_k = J_k^T J_k$ denotes the approximate of the Hessian, $g_k = J_k^T r_k$ denotes the gradient of the model and $H_k = G_k^{-1}$. We consider step 3 of Algorithm 3 where $\|h_k^{cd}\| \leq \Delta_k$ and $\|h_k^{gn}\| \geq \Delta_k$.

As was mentioned, essential property of Dogleg step is to guarantee the convergence of the algorithm. If the approximate obtained within this step has a reduction equal to the reduction of Cauchy step in the model, it has a global convergence. Therefore, first, a step that the approximate reduction of which in the model is like Cauchy step is considered in the direction of Gauss-Newton step . In this regard, parameter t is specified as follows:

$$m_k(-tH_k g_k) = m_k\left(h_k^{cd}\right) = m_k\left(-\left(\frac{\|g_k\|^2}{g_k^T G_k g_k}\right)\right), \quad (18)$$

and we have:

$$t^2\left(\frac{1}{2}g_k^T H_k g_k\right) - t\left(g_k^T H_k g_k\right) + \frac{1}{2}\left(\frac{\|g_k\|^4}{g_k^T G_k g_k}\right) = 0. \quad (19)$$

As a result, after solving equation (19), t is obtained as:

$$t = 1 \pm \sqrt{1-c} \quad (20)$$

in which,

$$c = \frac{\|g_k\|^4}{g_k^T G_k g_k g_k^T H_k g_k} \quad (21)$$

that $c \leq 1$. Now, if $\|th_k^{gn}\| > \Delta_k$, Dogleg step is considered between h_k^{cd} and th_k^{gn} [5].

In this method, $\hat{h}(\beta)$ is a new approximate path that is named double Dogleg path. As it is shown in Fig. 1, it consists of two line segments. The first line segment is from the current point to h^{cd} and the second line segment extends from h^{cd} to th^{gn}. In this case, a point named double Dogleg is obtained from

$$\hat{h}(\beta) = h^{cd} + (\beta - 1)(th^{gn} - h^{cd}), \quad 1 \leq \beta \leq 2. \quad (22)$$

With respect to Fig. 1, double Dogleg step (dashed vector) is more oriented toward Gauss-Newton step than Dogleg step, and therefore, it is expected to have more speed of algorithm while maintaining global convergence. It is better to consider parameter t as [5]:

$$t = 1 - \gamma\sqrt{1-c}. \quad (23)$$

Parameter γ is chosen such that m_k is steadily reduced along a line from $\delta = h_k^{cd}$ to $\delta = th_k^{gn}$. Hence,

$$t = 1 - \gamma\sqrt{1-c}, \qquad \delta_k(\theta) = (1-\theta)h_k^{cd} + \theta th_k^{gn}. \quad (24)$$

By this means, the value of γ must be found as:

$$\left(\frac{\partial}{\partial\theta}\right) m_k(\delta(\theta)) \leq 0, \qquad \forall\theta \in [0,1]. \quad (25)$$

By such calculations we may select any γ that satisfies the following relation.

$$0 < \gamma < \sqrt{1-c}. \quad (26)$$

Calculation of double Dogleg point is detailed in Algorithm 4. The only additional cost of double Dogleg method compared to that of Dogleg is the computation of c in step 2 of this algorithm. Since

$$c = \frac{\|g\|^4}{g^T h^{gn}\|J(x)g\|^2} = \frac{\|g\|^2}{g^T h^{gn}}\frac{\|g\|^2}{\|J(x)g\|^2} = \frac{\|g\|^2}{g^T h^{gn}}\alpha \quad (27)$$

Algorithm 4: Computation of Double Dogleg step h^{dl}

Inputs:	h^{gn}, h^{cd}, Δ
Output:	h^{ddl}
Step 1	If $\|h^{gn}\| \leq \Delta$, then put: $h^{ddl} \leftarrow h^{gn}$
Step 2	Else, compute t with using (23). If $\|th^{gn}\| \leq \Delta$, then put: $h^{ddl} \leftarrow \left(\frac{\Delta}{\|h^{gn}\|}\right) h^{gn}$
Step 3	If $\|th^{gn}\| > \Delta$, then compute Cauchy step $h^{cd} = \left(-\frac{\|g\|^2}{g^T G g}\right) g$.
Step 4	Else if $\|h^{cd}\| \geq \Delta$, then put: $h^{ddl} \leftarrow \left(\frac{\Delta}{\|h^{cd}\|}\right) h^{cd}$
Step 5	Else put: $h^{ddl} \leftarrow h^{cd} + (\beta - 1)(th^{gn} - h^{cd})$, where β is obtained from $\|h^{ddl}\| = \Delta$.

and values $g, \|g\|, h^{gn}, \alpha$ are also computed in Dogleg procedure, it is expected that the computation cost of double Dogleg is not much greater than Dogleg.

B. Sensitivity of Trust-region Methods to Parameters

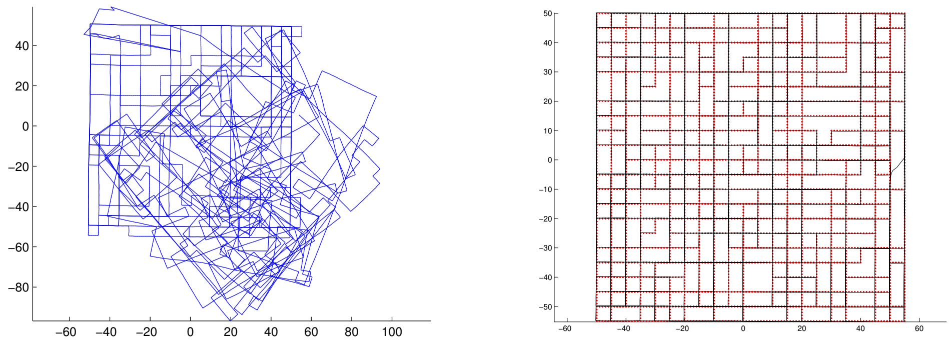
Trust region algorithms are dependent on values of parameters $\Delta_0, \mu_1, \mu_2, c_1, c_2$ (in Algorithm 2). In this section, we present suitable values for these parameters that increase the efficiency of incremental double Dogleg algorithm.

The values $\mu_1 = 0.25, \mu_2 = 0.75, c_1 = 0.5, c_2 = 3$ are proposed by Powell and have been used frequently in practical applications [12]. The question is, what are the most appropriate values for each trust region algorithm. Gould et al. in [6] have studied the sensitivity of the trust region algorithms to the values of parameters in acceptance of step and radius updating. They have chosen a broad spectrum of each parameter in a standard trust region algorithm. Over 20 different choices for parameters μ_1 and μ_2 and over 10 different choices for parameters c_1 and c_2 have been given in intervals $0 \leq \mu_1 \leq 0.4$, $0.5 \leq \mu_2 \leq 0.999$, $0.25 \leq c_1 \leq 0.75$ and $1.5 \leq c_2 \leq 5$.

Numerical considerations and results in several standard optimization problems with selecting broad spectrum of parameters (close to 4000) showed that results of an algorithm may be significantly dependent on the selected values of these parameters. After analysis of the numerical results, they have offered the best choices of parameters in the structure of a standard trust region method as: $c_1 = 0.25$, $c_2 = 3.5$, $\mu_1 = 0.0001$, $\mu_2 = 0.99$. We have studied different choices in our incremental algorithm, and observed that by using this selection, the performance of the optimization solution will increase as well. Therefore, these values have been used for implementation.

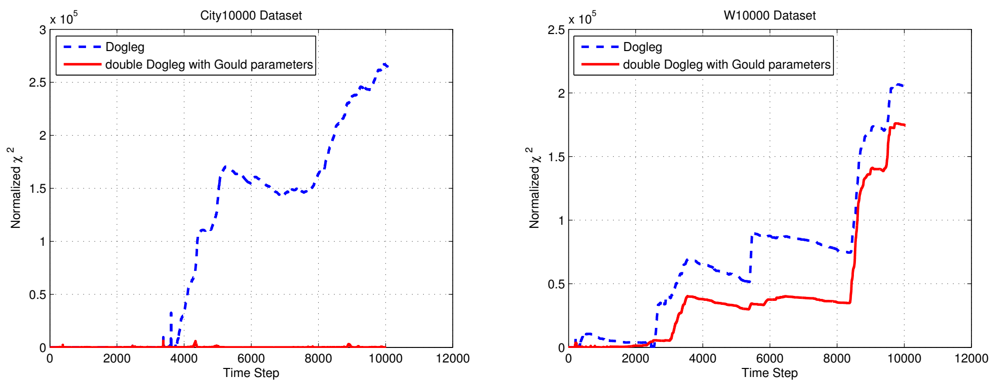
V. Experimental Results

In this section, we compare the performance of our proposed optimization algorithm with conventional optimization methods in iSAM2. Results show significant improvement of double Dogleg with Gould parameters compared to Gauss-Newton and Dogleg methods for large-scale datasets. All tests have been run on a laptop with Intel 2.8 GHz Core i7-2640M processor and 6 GB RAM. GTSAM library has been used



(a) (b)

Fig. 2: City10000 dataset: (a) result of Dogleg. (b) result of double Dogleg with Gould parameters (solid line) and the ground truth (dotted line)



(a) (b)

Fig. 3: Chi-square diagrams of datasets for Dogleg and double Dogleg: (a) City10000 (b) W10000.

TABLE I: Accuracy and time of our proposed method and previous methods in iSAM2 (GN: Gauss-Newton, D: Dogleg, DD: double Dogleg).

			Normalized χ^2			Time(ms)		
Datasets	Poses	Measurements	GN	D	DD with Gould params.	GN	D	DD with Gould params.
City10000	10000	20688	Indeterminate sys.	2.65E+05	2.13	–	137.3	140.5
W10000	10000	64311	Indeterminate sys.	2.05E+05	1.74E+05	–	122	128

from https://collab.cc.gatech.edu/borg/gtsam for iSAM2 algorithm. We have developed the incremental version of double Dogleg with Gould parameters which is compatible with the structure of iSAM2. SLAM are frequently applied in large-scale environments. Thus, the main objective of our proposed method is to improve the results in large-scale datasets. All results are summarized in Table I that consists of three main parts. The first part is about the property of used datasets. The second part, normalized χ^2 column is the result of accuracy for Gauss-Newton (GN), Dogleg (D) and double Dogleg (DD) with Gould parameters. The third part, time column includes the execution time of three mentioned methods. Experiments are performed on City10000 and W10000 large-scale datasets reported in [2]. The number of poses and measurements of datasets can be seen in Table I, which clearly indicates the large size of the optimization problem. Threshold parameters in iSAM2 have been initialized as $\alpha = 0.001$ and $\beta = 0.01$ and relinearization skip has been set to 1. Initial value of the trust region radius has been initialized to 1 in Dogleg and double Dogleg methods. Parameter γ in (23) in double Dogleg algorithm has been set to $\gamma = 0.8\sqrt{(1-c)}$ based on [5], and therefore, $t = 0.2 + 0.8c$ is used. Although we tested other values for γ, the proposed value by Dennis in [5] is more suitable to our problem.

There are many approaches to evaluate the accuracy of solutions. Since there are Gaussian errors in SLAM problem, the least-square objective function is the best choice. Therefore, we choose normalized chi-square (χ^2) criterion that is more compatible to the least-square objective function and also has been used in [2]. Normalized chi-square with $m-n$ degrees of freedom (DOF) is defined as

$$\frac{1}{m-n}\sum_{i}\|h_i(\Theta_i)-z_i\|_{\Sigma_i} \tag{28}$$

in which the numerator is equal to Equation (3). Furthermore, this performance index yields to 1 for a large number of normal distributed observations.

The normalized chi-square values of our proposed method and conventional methods applied to iSAM2 have been shown in Table I. These values have been obtained at the very last step of algorithm after collecting all measurements. As it can be seen, Gauss-Newton could not manage to obtain any solution for both datasets. The reason is that the system becomes indeterminate during the execution of the algorithm. In this case, the Hessian matrix of the system is either not positive semidefinite or the system is ill-conditioned [14]. As a result, Gauss-Newton algorithm diverges and inevitably stops. Unlike Gauss-Newton method, Dogleg method converges to an optimum value for the objective function. Nevertheless, the values of objective function are very large (of order 10^5) and the accuracy of the solution is poor. With respect to the results of double Dogleg with Gould parameters, they have less chi-square values than Dogleg results in both cases, and therefore, we have obtained more accuracy. The improvement of the accuracy is exceptionally well in City10000 dataset in which chi-square is near to 1. This means that the optimal solution is very close to the real value. The ground truth of City10000 dataset and the results of Dogleg and double Dogleg methods can be compared in Fig. 2. Fig. 2(a) shows the result of Dogleg that is very disordered and obviously far from the ground truth. In Fig. 2(b) the result of double Dogleg with Gould parameters has been drawn simultaneously with the ground truth for better comparison. As can be seen, using double Dogleg method with Gould parameters reaches an almost perfect solution. Furthermore, the accuracy increases for W10000 dataset.

Diagrams of chi-square values for both datasets have been presented in Fig. 3. As it is seen in part (a) of this figure, the chi-square for City 10000 dataset is perfectly reduced, while relatively better performance is obtained for W10000 dataset (Fig. 3(b)) using double Dogleg with Gould parameters. The reason of difference between improvements in two datasets is their distinct property. City10000 is a sparse dataset while W10000 is a dense one with a large number of measurements, and almost 6 times of its poses. Thus, our proposed method has a greater impact on sparse datasets than dense ones. Nevertheless, it has slight improvement on dense datasets.

Execution times of Gauss-Newton, Dogleg and double Dogleg in iSAM2 have been compared in Table I as well. These times are related to updating η and computing new estimate at the very last step after collecting all measurements. As previously mentioned, the extra computation cost of double Dogleg is merely for calculation of Equation (27), which is not that significant. Hence, the execution time of double Dogleg method is close to that of Dogleg method, and their difference is less than 0.01 second.

VI. Conclusions

In this paper, double Dogleg trust region optimization method is proposed and adjusted to be used in iSAM2 algorithm. By carefully studying previous optimization methods used in iSAM2, we found that the latest used method, namely Dogleg, is not accurate enough for practical large-scale applications. Double Dogleg method is found with a better performance and accuracy compared to Dogleg method and hence, is implemented on iSAM2. Meanwhile, the proposed method is capable of maintaining the execution time within the acceptable amounts in large-scale datasets. On the other hand, with respect to the influence of values of trust region parameters, we investigated the sensitivity analysis of Gould et al. With the selection of the appropriate values for parameters based on Gould et al. suggestion, we can obtain better performance in the algorithm. Double Dogleg method with Gould parameters can be used in other incremental algorithms as well. It guarantees global convergence and can improve accuracy of the solution especially for large-scale datasets. This method can be utilized in real and practical applications, in accordance to many usages of SLAM in large scale environments.

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