Region of Convergence Expansion of A Robust Model Predictive Controller

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***Abstract*—In this paper a gain scheduling method is proposed for robust model predictive control of a useful class of nonlinear discrete-time systems. The system is composed of a linear model perturbed by an additive state-dependent nonlinear term. Robust model predictive controllers are designed in the literature to compensate for the uncertainty of the system. In order to enlarge the region of convergence it is assumed that system has several equilibrium points and multiple robust controllers are designed. By switching between the controllers it is verified that the region of convergence shall be enlarged, while the overall stability of the system is preserved. In the proposed method, the stability analysis based on Lyapunov functions for each level set is performed, while state feedback control law is designed by minimization of a desired cost function formed in linear matrix inequalities. The simulation results show the applicability of the proposed method.**

Keywords—gain scheduling; robust model predictive control; region of convergence; switching; Lyapunov function

I. Introduction

Nonlinear model predictive control (NMPC) is a powerful technique that can control most physical processes which are inherently nonlinear. Although in most cases NMPC is based on linear models of systems, there are cases where nonlinear effects are significantly important, such that the linear models assumption is not valid. Using a nonlinear model changes the control problem from a convex quadratic program to a non-convex nonlinear problem, which is much more difficult to solve. Complexity of on-line computation is a major problem in nonlinear MPC. This is a critical issue in real time applications, control of high dimensional systems and applications requiring large prediction horizons. Suboptimal techniques are presented in most papers for the NMPC problem. The NMPC problem can be effectively solved by a convex optimization algorithm. This method is implemented either through linear dynamic approximation in [1,2] or by using a linear differential equation [3,4]. But there is a need for effective techniques which control the wide class of nonlinear models. One approach that has come into favor in recent years is Robust Model Predictive Control (RMPC). This algorithm is applied to systems composed of a linear constant part perturbed by an additive state-dependent nonlinear term. In [5] an RMPC algorithm designs a state feedback control law that minimizes an infinite horizon cost function within the framework of linear matrix inequalities. In this algorithm the stability domain is limited to an ellipsoidal invariant set about origin. Although this estimation maybe conservative, some techniques have been proposed to enlarge the estimated region of convergence. Region of convergence of an equilibrium point is the set of initial conditions from which the states converge to equilibrium point. In [6,7] an efficient RMPC algorithm with a time varying terminal constraint set is developed to enlarge the region of convergence. On this topic gain scheduling is the most familiar technique in recent years. In [8] this algorithm is used to enlarge the region of convergence for continuous-time systems but it isn't applied to constrained systems. Unfortunately gain scheduling does not provide guarantees on the stability and performance of the closed loop system at operating conditions [8].

In this paper, for nonlinear discrete-time systems several equilibrium points are considered. Then RMPC technique designs a feedback control law about each equilibrium point and by switching between the controllers the system reaches to the desired equilibrium point, while the global stability of closed-loop system is guaranteed. Indeed, the overall stability of system preserved by using control Lyapunov function theory.

This paper is organized as follows. In Section II some preliminaries and problem statement is presented. Section III describes the extension of the proposed MPC problem in [5] to the system with various equilibrium points. State constraint is incorporated to problem in form of linear matrix inequalities. Section IV explains the controller synthesis. In Section V, a two-tank system is exampled to illustrate the procedure. Finally, the benefits of the proposed algorithm are concluded in Section VI.

II. Preliminaries And Problem Statement

In this section the algorithm presented in [5] is extended to systems with various equilibrium points. Consider the nonlinear discrete-time dynamic system

$$x(k+1) = f\big(x(k), u(k)\big) \qquad (1)$$

where k is the discrete time index, $x(k) \in R^n$ the state, $u(k) \in R^m$ the input, $f(\cdot\,,\,\cdot) \in C^2$, and $f(0,0) = 0$.

Desired equilibrium point is considered origin. Then by using following equality other equilibrium points are selected.

$$x_0 = f(x_0, u_0) \qquad (2)$$

After selection equilibrium point, x_i by using RMPC algorithm the appropriate control law about each equilibrium point is designed. This procedure is demonstrated in the following.

By using Taylor Series, system (1) about the equilibrium point x_i can be expressed as,

$$x(k+1) = f(x_i, u_i) + A_i(x - x_i) + B_i(u - u_i) + \cdots \quad (3)$$

Where $A_i = \frac{\partial f}{\partial x}(x_i, u_i)$, $B_i = \frac{\partial f}{\partial u}(x_i, u_i)$. By defining $\tilde{x}_i(k) = x(k) - x_i$, $\tilde{u}_i(k) = u(k) - u_i$ then the dynamic system (3) can be formulated as

$$\tilde{x}_i(k+1) = A_i\tilde{x}_i(k) + B_i\tilde{u}_i(k) + \tilde{f}_i(\tilde{x}_i(k), \tilde{u}_i(k)) \quad (4)$$

where,

$$\tilde{f}_i(\tilde{x}_i(k), \tilde{u}_i(k)) = f\big(x(k), u(k)\big) - (A_i\tilde{x}_i(k) + B_i\tilde{u}_i(k)) \quad (5)$$

And $\tilde{f}(.,.)$ is a Lipschitz non-linearity. This term is bounded by:

$$\tilde{f}_i(x,u)^T \tilde{f}_i(x,u) \le \left[\tilde{x}_i^{\,T} \;\; \tilde{u}_i^{\,T}\right] W_i^{\,T} W_i \begin{bmatrix}\tilde{x}_i \\ \tilde{u}_i\end{bmatrix}$$

The state and control variables are required to satisfy the following constraints

$$\tilde{x}_i(k+n|k) \in \bar{X}, \quad \tilde{u}_i(k+n|k) \in \bar{U}, \;\; i \ge 0 \quad (6)$$

Where $\bar{X}$ and $\bar{U}$ are compact subsets of R^n and R^m , respectively, both containing the origin as an interior point. The RMPC algorithm's object is minimizing the maximum value of cost function.

$$\min_{\tilde{u}_i(k+n|k)} \quad \max_{\tilde{x}_i(k+n|k)\in\bar{X},\; \tilde{u}_i(k+n|k)\in\bar{U},\; i\ge 0} \quad J_i(k) \quad (7)$$

where,

$$J_i(k) = \sum_{n=0}^{\infty} \tilde{x}_i(k+n|k)^T Q \tilde{x}_i(k+n|k) + \tilde{u}_i(k+n|k)^T R \tilde{u}_i(k+n|k) \quad (8)$$

in which, Q_c and R_c are positive definite weighting matrices. Let introduce a quadratic function $V_i(\tilde{x}) = \tilde{x}_i^{\,T} P_i \tilde{x}_i$, $P_i > 0$ of the state $\tilde{x}_i(k|k)$ of the system (4), with $V_i(0) = 0$. At sampling time k, suppose the following inequality is given:

$$V_i(k+n+1|k) - V_i(k+n|k) \le -\big(\tilde{x}_i(k+n|k)^T Q \tilde{x}_i(k+n|k) + \tilde{u}_i(k+n|k)^T R \tilde{u}_i(k+n|k)\big) \quad (9)$$

Summation of (9) from $n = 0$ to $n = \infty$, results in

$$\tilde{x}_i(\infty|k)^T P_i \tilde{x}_i(\infty|k) - \tilde{x}_i(k|k)^T P_i \tilde{x}_i(k|k) \le -J_i$$

If the resulting closed loop system for (4) is stable, $\tilde{x}(\infty|k)$ must be zero. Hence,

$$J_i \le \tilde{x}_i(k|k)^T P_i \tilde{x}_i(k|k) \le \gamma_i \quad (10)$$

where γ_i is a positive scalar and is regarded as an upper bound of the objective in (8).

III. CONTROL SYSTEM DESIGN

In this section, we discuss the MPC problem formulation for non-linear system and then, we incorporate input and state constraints.

A. State-feedback MPC

In this section, a convex optimization method to solve the model predictive control problem is presented. In this algorithm, instead of minimizing J_i in (8) an upper bound of J_i is minimized. Primarily this upper bound is minimized with a state-feedback control law $u(k+n|k) = u_i + F_i(k)\tilde{x}_i(k+n|k)$ $(n \ge 0)$ for system (4), and then a representation of model predictive control law to linear matrix inequalities is given. The following theorem is devoted to constructing the state-feedback matrix, F_i.

***Lemma* 1:** ([5]) Consider the discrete-time system (4) at each time k and let $\tilde{x}_i(k|k)$ be the measured state, $\tilde{x}_i(k)$. Then, the state-feedback matrix F_i in the control law that minimize the upper bound $V_i(\tilde{x}_i(k|k))$ of objective function at instant k is given by $F_i = Y_i X_i^{-1}$, where $X_i > 0$ and Y_i are obtained from the solution of the following optimization problem with variables $\gamma_i, \xi_i, X_i, Y_i$ and $Z_i = [X_i \;\; ; Y_i]$:

$$\min_{\gamma_i, \xi_i, X_i, Y_i} \gamma_i$$

Subject to

$$\begin{bmatrix} -I & * \\ \tilde{x}_i(k) & -X_i \end{bmatrix} \le 0 \quad (11)$$

and

$$\begin{bmatrix} -X_i & * & * & * & * \\ \sqrt{(1+\varepsilon)}(A_iX_i + B_iY_i) & -X_i & * & * & * \\ \sqrt{(1+\frac{1}{\varepsilon})}W_iZ_i & 0 & -\xi_i I & * & * \\ Q_i^{1/2}X_i & 0 & 0 & -\gamma_i I & * \\ R_i^{1/2}Y_i & 0 & 0 & 0 & -\gamma_i I \end{bmatrix} \le 0 \quad (12)$$

Where $X_i = \gamma_i P_i^{-1}$, $X_i > 0, Y_i = F_i X_i$ and $\xi_i = \gamma_i \mu_i^{-1}$ which $p_i \le \lambda_{i,max} I \le \mu_i I$

Proof: See [5]

Remark: In this algorithm, the stability domain about each equilibrium point is defined by an ellipsoidal invariant set as

$$\mathcal{E}_i = \left\{\tilde{x}_i(k|k) \middle| \tilde{x}_i^{\,T}(k|k)\, X_i^{-1}\, \tilde{x}_i(k|k) \le 1\right\} \quad (13)$$

B. State Constraints

Inherent physical limitations in the process imposed hard constraint on the manipulated variables. Imposing two-norm input constraint in the problem discussed in Lemma 1 is proposed in [5]. In this paper two-norm state constraint is developed a routine to incorporating in optimization solution. Consider a state two-norm constraint in the form of

$$\tilde{x}_i(k+n|k)\tilde{x}_i(k+n|k)^T \le W_{\tilde{x}_i max,2} \quad (14)$$

where $\tilde{x}_i{}^T = [\tilde{x}_{i,1}{}^T, \tilde{x}_{i,2}{}^T, \dots, \tilde{x}_{i,n}{}^T]$ and $W_{\tilde{x}_i} = diag(\tilde{x}_{i,1}, \tilde{x}_{i,2}, \dots, \tilde{x}_{i,n})$. The inequality (14) is rewritten as

$$X_i \le W_{\tilde{x}_i max,2} \Rightarrow X_i - W_{\tilde{x}_i max,2} \le 0 \quad (15)$$

This inequality in the form of linear matrix inequality and can be combined with problem (8).

IV. CONTROLLER SYNTHESIS

In section III-A, designing RMPC algorithm about various equilibrium points for system (1) is detailed. In this section, we discuss how to incorporate controllers in various regions to both convergence the system to desired equilibrium point and preserving overall stability of closed loop system.

A. Solution Procedure

The first step is to consider the origin as desired equilibrium point for the system. According to equality (13) the stability domain about origin is estimated. Next, the second equilibrium point is selected such that both equality (2) is satisfied and must be put in the region of convergence about origin, the prior equilibrium point. This method is repeated for other equilibrium points.

Definition 1: [8] Given a system $\dot{x} = f(x,\omega)$ with $\omega(t) \in \mathcal{W} \subseteq R^l$ for all $t \ge 0$ and a compact subset $\Omega \subset R^n$, define $\|x\|_\Omega \doteq \inf\{\|x-y\|, y \in \Omega\}$ the system is *robustly, uniformly, asymptotically stable over* χ with respect to Ω, or RUAS(χ,Ω). We call the set χ a *region of stability* (RS) for the system.

Definition 2: [8] A level set of a proper positive definite function $V(x)$ at origin, $(0) = 0$, is defined by real numbers $c_2 > c_1 \ge 0$ via $\Omega \doteq V^{-1}[c_1, c_2] = \{x \in R^n | c_1 \le V(x) \le c_2\}$. The numbers c_1 and c_2 are called the lower and upper level values, respectively.

Definition 2 can be extended to region of convergence about each equilibrium point, x_i.

$$\Omega^i \doteq V_i{}^{-1}[c_1^i, c_2^i] = \{\tilde{x} \in R^n | c_1^i \le V(\tilde{x}) \le c_2^i\}$$

To enlarge the region of convergence following complete solution procedure is proposed.

1) Design a Lyapunov function about desired equilibrium point x_0 *,origin.*

a) Select a quadratic Lyapunov function $V_0(\tilde{x})$.

b) Determine sets $\Omega_1^0 \subset \Omega_2^0$ such that the nonlinear system is RUAS(Ω_2^0, Ω_1^0).

2) Design Lyapunov functions about other equilibrium points of the system. At each iteration i do the following.

a) Find an equilibrium point of the system, x_i contained in the portion of the state space already covered by level sets, i.e., $x_i \in int(\Omega_2^j)$ for some $j < i$.

b) Select a quadratic Lyapunov function $V_i(\tilde{x})$ about x_i.

c) Determine sets $\Omega_1^i \subset \Omega_2^i$ such that the nonlinear system is RUAS(Ω_2^i, Ω_1^i) and such that $\Omega_1^i \subseteq \Omega_2^j$. If no such sets can be found, go back to step 2a) using a different equilibrium point. (The trim point is selected in an iterative method until the condition $\Omega_1^i \subseteq \Omega_2^j$ can be achieved.)

d) Repeat until the region $\chi \doteq \bigcup_{j=0}^{i} \Omega_2^j$ covers a desired portion surface.

3) Implement the control law. If the state is at a point $x \in \chi$*, do the following.*

a) Select one of the level sets in which the point x lies.

b) Apply the feedback control law by using Lemma 1.

B. Switching

For implementing the control law, it is necessary to find the index i with the smallest value of $V_i(\tilde{x})$, which satisfies the condition $V_i(\tilde{x}) \le c_2^i$. This index gives us the correct level set on which to base the control input computation. Then it is an easy matter to compute a control input. For example by the state -feedback control law.

$$\tilde{u}_i(x) = k_i(\tilde{x}) = \begin{cases} F_i\tilde{x}_i, & c_1^i \le V_i(\tilde{x}) \le c_2^i \\ 0, & otherwise \end{cases} \quad (16)$$

However, at each switching point, where the state trajectory enters a new level set, linear interpolation on the control law can be used. This interpolation requires additional analysis to ensure that the interpolated control law still guarantees stability. Specifically, if $x(k_1) \in \Omega_2^i \cap \Omega_2^j$, Then $T > 0$ and an interpolation function $\alpha \in C^0([0,T] \to [0,1])$ exist, increasing on $[0,T]$, with $\alpha(0) = 0$ and $\alpha(T) = 1$, such that $x(k) \in \Omega_2^i \cap \Omega_2^j$, for all $k \in [k_1, k_1 + T]$, under the control law

$$\tilde{u}(x,k) \doteq \alpha(k-k_1)\tilde{u}_j(x) + [1-\alpha(k-k_1)]\tilde{u}_i(x) \quad (17)$$

Subject to (17), control law in the intersections between level sets is continuous. In next section by using this control law overall close loop stability is analyzed.

C. *Stability Analysis*

Theorem 1: Consider the discrete time system (4). While this system has various equilibrium points, desired equilibrium point is origin. If by using RMPC algorithm, introduced in Lemma1, appropriate controllers about each region of convergence are designed and in common regions a linear interpolation on the control law, subject to (17), is used then overall closed-loop stability of system is guaranteed.

Proof: Let us introduce the following definition.

Definition 3: [9] Given a system $\dot{x} = f(x, \omega)$ with $\omega(t) \in \mathcal{W} \subseteq R^l$ for all $t \geq 0$, a positively invariant set $\chi \subseteq R^n$, and a compact subset $\Omega \subset \chi$, the system is *robustly, uniformly, asymptotically stable over* χ with respect to Ω, or RUAS(χ, Ω), if it is RUAS-Ω whenever $x_0 \in \chi$. We call the set χ a *region of stability* (RS) for the system.

1.) Uniform Stability: A $\mathcal{K}_\infty$ function δ exists such that for any $\epsilon > 0$ $\|x(t)\|_\Omega < \epsilon$ for all $t \geq 0$ whenever $\|x(0)\|_\Omega < \delta(\epsilon)$, for all $\omega(t) \in \mathcal{W}$. (See [10] for the definition of function of the class $\mathcal{K}_\infty$.)

2.) Uniform Attraction: For any $r, \epsilon > 0$, $T > 0$ exists such that for every $\omega(t) \in \mathcal{W}, \|x(t)\|_\Omega < \epsilon$ whenever $\|x(0)\|_\Omega < r$ and $t \geq T$.

Indeed, subject to Definition 3, if the algorithm presented in *Theorem 1*, satisfies all conditions of upper definition then the system is asymptotically stable over χ with respect to Ω whenever $x_0 \in \chi$. Suppose at time t, $x(t) \in \Omega_2^0$, a control law $\tilde{u}_0(x, k)$ exists that makes $V_i(k + i + 1|k) - V_i(k + i|k) \leq 0$ for the closed-loop system when $c_1^0 \leq V_0(\tilde{x}) \leq c_2^0$. Therefore, from condition 1 the system has uniform stability. Now suppose $i \neq 0$, and select $j \neq i$ satisfying condition 2. Because $x(k) \in \Omega_2^i$, and $c_1^i > 0$, a finite $\Delta k > 0$ exists such that $x(\tau) \in V_i^{-1}[0, c_2^i]$ for all $\tau \in [k, k + \Delta k]$ and $x(k + \Delta k) \in \Omega_1^i \subset \Omega_2^j$. Because $V_j(\tilde{x}) < V_i(\tilde{x})$ at switching point, therefore, the function $V_i(\tilde{x})$ decreases each time the control switches between level sets. Because the number of level sets is finite and the Δt required to transition from one level set to the next is always finite, the trajectory reaches the set Ω_2^0 in finite time whenever $x(0) \in \bigcup_{j=0}^{i} \Omega_2^j = \chi$. Hence, one may conclude asymptotic stability of the closed-loop system (1) over χ. From this proof, it is also deduced that χ is positively invariant set. Because $x(0)$ always puts in one of the level sets Ω_2^i. Therefore, the algorithm proposed in this paper enlarges the region of convergence.

D. *Modifications on control law continuity*

In this section modifications on control law continuity, $\alpha(k)$, is performed. The purpose of this discussion is to enhance the performance and to reach to a maximum rate of convergence to desired equilibrium point for the system.

Suppose the state is in the level set corresponding to $V_1(\tilde{x})$ and it has just entered the level set corresponding to $V_0(\tilde{x})$. At some time k_0, $x(k_0) \in \Omega_2^1 \backslash \Omega_2^0$. Assume the equilibrium points and level sets have been chosen so as $[0, c_1^1 + \Delta c_1^1] \subseteq [0, c_2^0 - \Delta c_2^0]$ for some $\Delta c_1^1, \Delta c_2^0 > 0$. Then, $k_1 > k_0$ exists such that, for some $\Delta c_1^0 > 0$

$$x(k_1) \in V_1^{-1}[c_1^1 + \Delta c_1^1, c_2^1] \cap V_0^{-1}[c_1^0 + \Delta c_1^0, c_2^0 - \Delta c_2^0]$$

Given the control laws $\tilde{u}_0(x)$ and $\tilde{u}_1(x)$, this paper is interested in derive a time interval $T > 0$ and an interpolation function $\alpha(k)$ such that $x(k) \in \Omega_1^0 \cap \Omega_1^1$, for all $k \in [k_1, k_1 + T]$, under the control law

$$\tilde{u}(x, k) \doteq \alpha(k)\tilde{u}_0(x) + (1 - \alpha(k))\tilde{u}_1(x) \tag{18}$$

By using comparison principle and the requirement that $V_1(\tilde{x}(k)) \leq c_2^1$, for all $k \in [k_1, k_1 + T]$, to constrain the function α as follows.

$$\max_{x \in V_1^{-1}[c_1^1, c_2^1], u \in \tilde{U}} \quad \sup \frac{V_1(x(k+1)) - V_1(x(k))}{V_1(x)} \leq b_1 + b_2\alpha \tag{19}$$

Suppose $b_1 < 0$ and $b_2 > 0$. Because α is a monotonically increasing function, we know $V_1(\tilde{x}(k)) \leq c_2^1$, for all $k \in [k_1, k_1 + T]$, if

$$\sum_{k=0}^{T-1} b_1 +, b_2\alpha\left(\frac{k}{T}\right) \leq 0 \Rightarrow b_1(T) + b_2 \sum_{k=0}^{T-1} \alpha\left(\frac{k}{T}\right) \leq 0 \tag{20}$$

Moreover, it can be written

$$T.\frac{1}{T}\sum_{k=0}^{T-1} \alpha\left(\frac{k}{T}\right) \leq T\int_0^1 \alpha(t)dt < T\alpha(1) \tag{21}$$

Therefore inequality (20) can be rewritten as

$$\int_0^1 \alpha(t)dt \leq -\frac{b_1}{b_2} \tag{22}$$

If $b_2 \leq 0$, then no constraint is on the integral of α. Any interpolation function satisfying this condition and the other constraints listed above may be used as a suitable routine. The method for computation of b_1 and b_2 is detailed in [11].

Theorem 2: The maximum allowable interpolation interval can be determined by

$$T \leq \frac{-1}{\left(b_3 + b_4 \int_0^1 \alpha(t)dt\right)}\left(1 - \frac{c_1^1 + \Delta c_1^1}{c_2^1}\right)$$

Proof: see [11].

Corollary: hard switching, $\int_0^1 \alpha(t)dt = 1$, minimizes the time interval, T. Therefore it increases the rate of convergence of system to the target values. Because of *Theorem 2*, the relation between T and $\int_0^1 \alpha(t)dt$ is inverse. Therefore, the maximum value of $\int_0^1 \alpha(t)dt$ results to the minimum value of T.

V. CASE STUDY: TWO-TANK SYSTEM

Consider a two-tank system modeled by [12]:

$$\rho S_1 \dot{h}_1 = -\rho A_1 \sqrt{2gh_1} + u$$
$$\rho S_2 \dot{h}_2 = \rho A_1 \sqrt{2gh_1} - \rho A_2 \sqrt{2gh_2}$$
$$y = h_2$$

where $= 0.001 \frac{kg}{cm^3}$, $g = 980 \frac{cm}{s^2}, S_1 = 2500\ cm^2, A_1 = 9\ cm^2,\ S_2 = 1600 cm^2, A_2 = 4\ cm^2, h_1 \leq 50 cm, h_2 \leq 120 cm, 0 \leq u \leq 2.5 \frac{kg}{s}$ representing density, acceleration of gravity, first tank area, first valve area, second tank area, second valve area, first tank height, second tank height, inlet flow of system, respectively. The second state is considered as the output. The sampling time is one second. Let $Q = diag(0,1), R = 0.01, W_i = 0.001 diag(1,1,0)$ and $\varepsilon_i = .01$. Consider the regulation from an initial state $h(0) = \begin{bmatrix} 14 \\ 72 \end{bmatrix}$ to the target equilibrium point (h_0, u_0)=($\begin{bmatrix} 19.753 \\ 100 \end{bmatrix}$, 1.7710). RMPC algorithm introduced in Lemma 1 is implemented to this system with one equilibrium point. Fig. 1 shows that initial point is beyond of stability region of (h_0, u_0). Therefore, the method of expansion region of convergence proposed in this paper may be easily used. For this purpose, besides target equilibrium point (h_0, u_0), two equilibrium points are selected. Fig. 2 shows three regions of stability for three equilibrium points. Two equilibrium points are (h_1, u_1)=$\left(\begin{bmatrix} 17.77 \\ 90 \end{bmatrix}, 1.68\right)$, (h_2, u_2)=($\begin{bmatrix} 15.8 \\ 80 \end{bmatrix}$, 1.37). There are three state-feedback controls corresponding to these three equilibrium points and the control law is discontinuous at each switching points. Fig. 2 shows the transitions between three control laws.

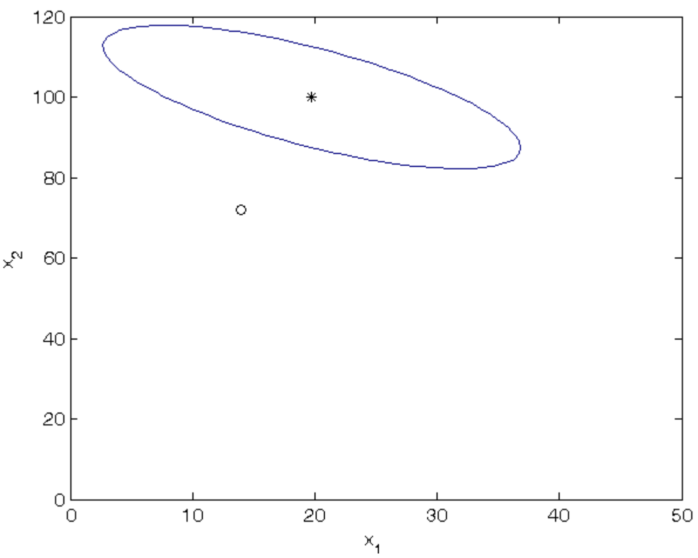

Fig.1. Region of convergence of constrained system. Solid line: region of convergence, *: target equilibrium point, o: initial point.

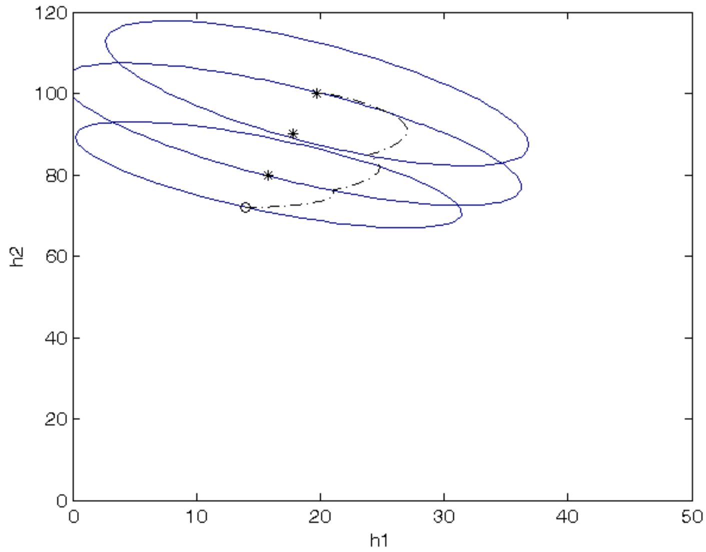

Fig.2. Region of convergence. Solid line: region of convergence about each equilibrium point, dashed line: state trajectory, *: equilibrium points, o - initial point.

Fig. 3 shows the time responses. Close-up views of the responses of the states and control input. Moreover, it shows after almost t=400s states of system reaches to target values.

Now at each switching point, the interpolated control law, $\alpha(k - k_1) = \frac{k - k_1}{T}$ is used. Where k_1 is the time that state trajectory enters a new level set and T is the time interval that state trajectory in the intersections between level sets. Fig. 4 shows the time responses of the states and control input.

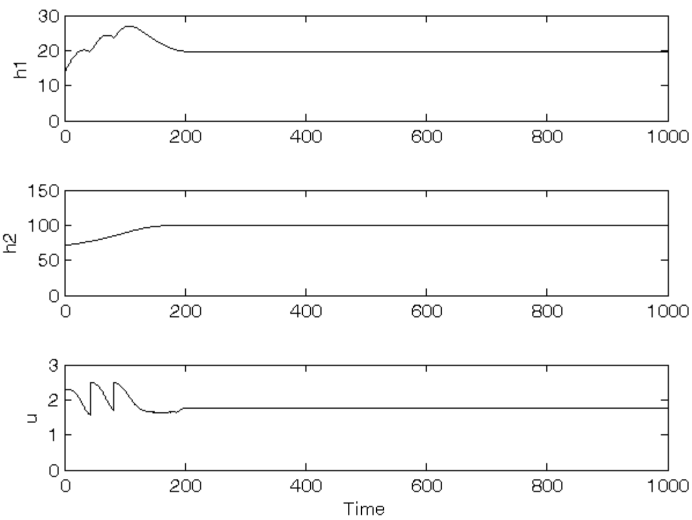

Fig 3. Time responses of the regulations from $h(0) = (14,72)$ to the equilibrium (19.753,100)

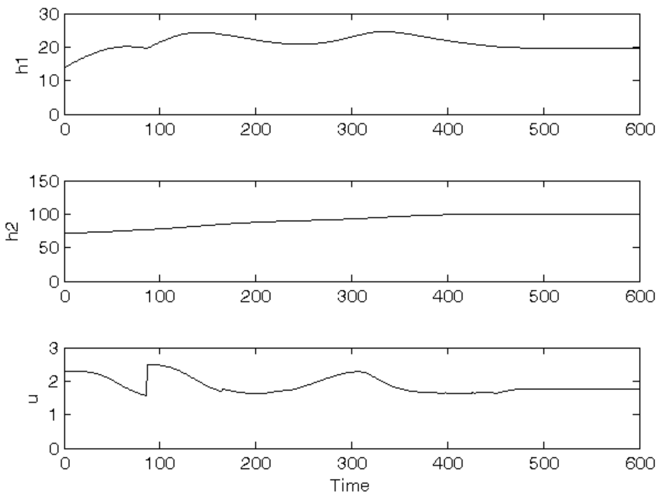

Fig 4. Time responses of the system with interpolated control law at switching points.

Fig. 4 shows after t=500s states and control input reaches to target values. Therefore this result is clearly verified Theorem 2. The fact that hard switching enhances the rate of convergence of system.

VI. CONCLUSIONS

The most serious hindrance to progress on the nonlinear control problem is the online computational complexity. Therefore, the methods which change the optimization problem to a quadratic convex problem, are required. In this paper, we have proposed robust MPC for nonlinear discrete-time systems. Because of small region of convergence at equilibrium points, gain scheduling method is used. Therefore, the region of stability of system is expanded by using several equilibrium points. Although gain scheduling typically doesn't provide guarantees on the stability, this procedure preserve closed-loop stability of the system based on control Lyapunov functions. Subject to analysis which is performed in the design of the control law continuity, this result is yield that hard switching increase the rate of convergence of system to the target value. Furthermore, this algorithm is implemented to a two-tank process, and the desired results are discussed.

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