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Loop Closure Detection by Compressed Sensing for Exploration of Mobile Robots in Outdoor Environments

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***Abstract*—In the problem of simultaneously localization and mapping (SLAM) for a mobile robot, it is required to detect previously visited locations so the estimation error shall be reduced. Sensor observations are compared by a similarity metric to detect loops. In long term navigation or exploration, the number of observations increases and so the complexity of the loop closure detection. Several techniques are proposed in order to reduce the complexity of loop closure detection. Few algorithms have considered the loop closure detection from a subset of sensor observations. In this paper, the compressed sensing approach is exploited to detect loops from few sensor measurements. In the basic compressed sensing it is assumed that a signal has a sparse representation is a basis which means that only a few elements of the signal are non-zero. Based on the compressed sensing approach a sparse signal can be recovered from few linear noisy projections by l_1 minimization. The difference matrix which is widely used for loop detection has a sparse structure, where similar observations are shown by zero distance and different locations are indicated by ones. Based on the multiple measurement vector technique which is an extension of the basic compressed sensing, the loop closure detection is performed by comparison of few sensor observations. The applicability of the proposed algorithm is investigated in some outdoor environments through some publicly available data sets. It has been shown by some experiments that the proposed method can detect loops effectively.**

I. Introduction

Various applications for mobile robots are considered from military, scientific or industrial prospects [1]. Exploration and navigation in an unknown environment is a vital task for a mobile robot which provides more advanced functionalities after solving the SLAM problem. In order to have a consistent map from the surrounding environment, it is required to reduce the estimation error of SLAM algorithm by closing loops and keeping the localization uncertainty bounded [2]. Various sensors are used for perception of an unknown environment such as Lidar, laser scanner, stereo camera or RGB-D sensors like Microsoft Kinect [3], [4].

Raw measurements acquired from sensors can hardly be used for detection of similar locations, due to the difficulty of classification and high computational complexity. Therefore, state-of-the-art techniques are either based on the extraction of features from sensor observations [5], learning from raw measurements [6] or by extraction of low-dimensional model [7]. Almost every loop closure detection technique is aimed to construct a difference matrix which represents similarity of sensor observations [8]. Considering a normalized difference matrix, the similarity of sensor observations are shown by a number between zero and one where the similar measurements have difference of near to zero and the difference of dissimilar places is indicated by a number near to one.

The authors in [9] have modeled the loop closure detection problem as a node clustering in a graph. Features are represented as nodes and are connected to each other when they are located closed. A probabilistic model of the environment is incrementally updated.

In the FAB-Map [5] and FAB-MAP 3D [10] a dictionary of visual words is used for loop closure detection over large trajectories. Extraction of a Bag of Words (BoW) is considered in several feature based loop closure detection algorithms. In [11] a bag of binary words is extracted from camera images in order to detect similar places by measuring a similarity metric and testing geometrical constraint.

The authors in [12] have presented a global describing method based on point clouds. The proposed method is applied to the appearance-based loop closure detection problem. The normal distributions transform are employed as a global descriptor for acquired point clouds.

The curse of dimentionality is considered in [7] where the sensor observations are represented in a lower dimensional by sparse modeling. A parametric dictionary is constructed from a Gaussian mother function and a sparse model is extracted from sensor observations. A normalized similarity metric from information theory is employed for comparison of sensor measurements. The difference matrix can be generated from either camera image or range image observations.

Most of the conventional methods require to compare a newly acquired observation to all previously captured ones in order to compute the difference matrix. During the long term exploration or navigation of a mobile robot, the number of sensor measurements increases and the computation of difference matrix for loop closure detection requires more and more computational resources. In order to remedy this problem, in this paper a loop closure detection method is proposed based on the compressed sensing. The loop closure detection is modelled as a matrix completion problem, where the difference matrix is constructed from few samples. By

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performing some experiments, it has been shown that based on a few sensor observations, the difference matrix can be recovered.

The structure of the paper is as follows. A brief review of some related works is presented in the next section. In Section II some required theories are provided. The next section is devoted to the proposed method for detection of loop closure from a sub set of sensor measurements. Finally, Section IV is dedicated to the experimental results, which is followed by the concluding remarks.

II. Preliminaries

In this section some required theories are briefly reviewed. The normalized compression distance from information theory is used for similarity measurement of sensor observations. The difference matrix can be constructed when the similarity measurements are readily available where the loop closure can be performed very simple.

A brief review of compressed sensing technique and especially the problem of multiple measurement vectors is also presented here. Based on the compressed sensing approach a loop closure detection is developed in Section III which provides the loop closure detection from a few sensor observations.

A. Algorithmic Information Theory

In order to estimate the amount of information, some tools are provided by information theory. While the Shannon offers entropy as an estimate of information, in algorithmic information theory the upper bound of information is estimated by a compression algorithm. The algorithmic information theory is widely used in various fields such as visual perception [13], image similarity measurement [14], feature extraction [15] and loop closure detection [7].

In algorithmic information theory, it is assumed that each object can be represented by a binary sequence which is expressed by X. Furthermore, a universal computer $\mathcal{C}$ with a program $\mathcal{P}$ are considered too. The universal computer $\mathcal{C}$ is employed for execution of program $\mathcal{P}$ which generates the data X and halts. Then, the Kolmogorov complexity of an object is defined as the smallest program $\mathcal{P}$ which is executed by the universal computer $\mathcal{C}$ can generate the data sequence X. In the algorithmic information theory, the entropy of object X is substituted by the Kolmogorov complexity $K(X)$.

In addition to the Kolmogorov complexity of an object X, the conditional Kolmogorov complexity is defined in order to measure the complexity of that object, when the object Y is also given. The conditional Kolmogorov complexity is represented by $K(X|Y)$. The computation of an upper bound for the Kolmogorov complexity is accomplished by a compression program as the smallest program that can regenerate the representation of object X from a compressed presentation. The usage of a compression program implies that the Kolmogorov complexity is not computable and an upper bound is achieved which is denoted by $C(X)$. In order to compute $C(X)$, a file is constructed by storing the binary sequence representation of an object and then a conventional compression algorithm is applied. The length of the compressed file in bytes is considered as the Kolmogorov complexity of that object.

In order to measure the amount of shared information between representations of two objects, a normalized metric is defined in algorithmic information theory [16]. The normalized compression distance is defined as

$$\mathrm{NCD}(X,Y) = \frac{C(XY) - \min\{C(X), C(Y)\}}{\max\{C(X), C(Y)\}} \tag{1}$$

where $C(XY)$ is achieved by computing the size of compressed file with X and Y as content.

The NCD metric satisfies the following condition

$$0 \leq NCD \leq 1$$

where the zero is reported in the case of comparing two completely similar representations and one is achieved in the case of comparing two completely dissimilar sequences.

In the next section the proposed method is presented. The complexity based representation of sensor measurements as either range or camera images is derived. Then the object classification is accomplished by comparing these representations using NCD.

B. Compressed Sensing

The emerging field of compressed sensing has received great amount of attraction from various communities such as image processing [17], video processing [18] and medical imaging [19].

While conventional sampling methods are based on the Shannon theory, in the compressed sensing it has been shown that few samples are sufficient for reconstruction of original signal, when the signal has a sparse representation in a basis [20]. Some practical implementations are available, such as basis pursuit [21], CoSaMP [22] and Orthogonal Matching Pursuit(OMP) [23] just to mention a few.

In the CS theory, a signal x is called *k-sparse* if it has at most k non-zero coefficients.

$$\Sigma_k = \{x \in \mathbb{R}^N : \| x_0 \| \leq k\} \tag{2}$$

A measurement matrix is used for taking m linear observation by

$$y = Ax, \quad A \in \mathbb{R}^{m \times N} \tag{3}$$

When the measurement matrix A satisfies Restricted Isometry Property (RIP), the signal reconstruction from noiseless measurements will be possible. The problem of recovering the k-sparse signal is defined as

$$\underset{x}{minimize} \; ||x||_1 \text{ subject to } Ax = b \tag{4}$$

In the case of noisy observations we have

$$y = Ax + n, \quad A \in \mathbb{R}^{m \times N} \tag{5}$$

and the problem is modeled as

$$\underset{x}{minimize} \; ||x||_1 \text{ subject to } ||Ax - b||_2 \leq \sigma \tag{6}$$

where σ^2 is noise variance.

If the measurement matrix A satisfies the RIP property with constant δ_k then

$$(1-\delta_k) \parallel x \parallel_2^2 \leq \parallel Ax \parallel \leq (1+\delta_k) \parallel x \parallel_2^2 \tag{7}$$

we have

$$||\hat{x}-x||_2^2 \leq ck\sigma^2 logN \tag{8}$$

where c is a constant.

A special class of compressed sensing that has received a great amount of attention is the multiple measurement vectors (MMVs) [24]. In MMVs problem we are considered in recovering a set of sparse signals jointly with the constraint of having a common support. The MMVs problem can be solved by SPGL1 algorithm [25]. The sparse signals are gathered in a matrix and the MMVs problem is defined as

$$\underset{X}{minimize} \; ||X||_{1,2} \text{ subject to } ||AX-B||_{2,2} \leq \sigma \tag{9}$$

where we have

$$||X||_{1,2} = \sum_i ||row_i X^T||_2 \tag{10}$$

$$||X||_{2,2} = (\sum_i ||row_i X^T||_2^2)^{1/2} \tag{11}$$

and $row_i X$ is the ith row of matrix X.

Equipped with the normalized similarity measurement NCD and the compressed sensing technique, a loop closure detection method based on few sensor observations is developed in the next section.

III. Proposed Method

In this section the proposed method is explained in detail. It is assumed that a mobile robot is traversing an unknown environment while perceiving the surrounding environment by a sensor providing either camera or range images. During the exploration or navigation a SLAM algorithm is used for estimation of the position of the robot in the map. In order to keep the localization error bounded, it is required to close loops which is made possible by detecting previously visited places.

Following the proposed method in [7], after acquisition of sensor observations, a parametric dictionary is constructed which is the base of generating sparse representations from sensor observations. A Gaussian function is used for sparse modeling of camera or range images.

$$g(x,y) = \frac{1}{k} e^{-(x^2+y^2)} \tag{12}$$

where x, y are pixel coordinates of image and k is a normalization factor. The orthogonal matching pursuit (OMP) [28] is widely used for sparse modeling. The camera or range image is fed into this algorithm and a sparse model is achieved after some fixed iterations.

In order to compare sparse models, a complexity based representation is constructed by mapping the numeric values of the sparse model into the complexity space. This mapping is accomplished by replacing each numeric value with a relevant pesudo random binary sequence (PRBS) [29]. The PRBS sequence is made by a shift register having the mentioned numeric values as the initial value. At the end of this process, a representation of complexity as a PRBS sequence is achieved from sensor measurements.

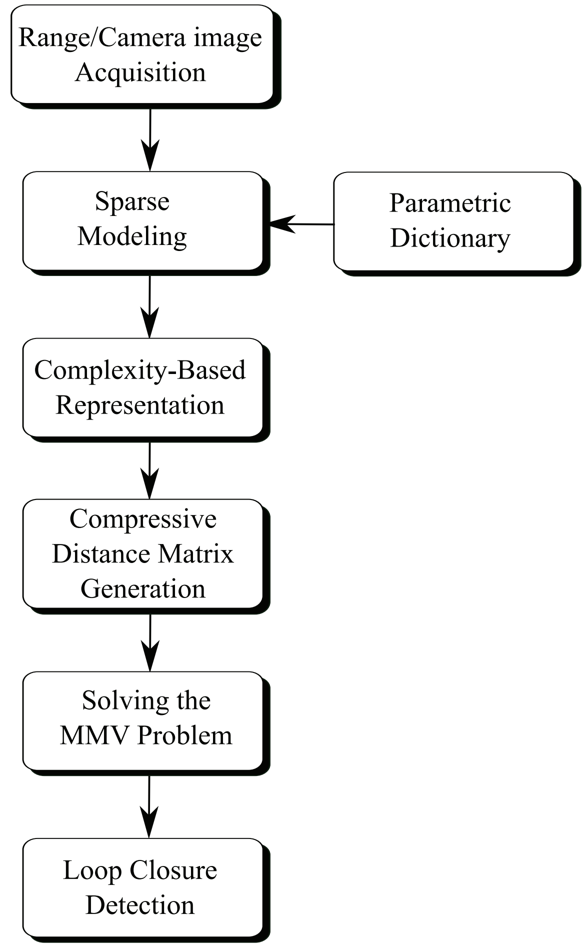

Fig. 1. The overall process of the proposed method.

The standard difference matrix indicates the similarity of sensor measurements in the form of a matrix. A value of zero is related to two similar observations while two dissimilar measurements are represented by a distance value of one. A similarity matrix can also be used for loop closure detection instead of a difference matrix. In similarity matrix a value of zero indicates two dissimilar sensor measurements and vice versa.

Here we define a custom similarity matrix which is not symmetric and the main diagonal is set to zero. This matrix is very sparse and only the sensor observations related to loops have values of one. Here the standard similarity matrix (13) and the modified one (14) are shown in a toy example.

TABLE I
DATASETS

Data Set	# Scans	Traveled Dist.(m)	Scene Type	Sensor Type	Image Size
KITTI - Sequence(00) [26]	4541	3721.73	Urban - Dynamic	Stereo camera	1241×374
Lip6 - Outdoor [27]	531	1300	Outdoor- Dynamic	Monocular camera	240×192

$$S = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

$$S^{'} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{14}$$

In this example, a set of 5 observations is considered where the observations 1 and 3 are similar. Also the observation 2 is also similar to 4. In the modified version the lower triangle of the matrix is filled with zero and a sparse matrix is achieved. This modification results in a sparse matrix which is actually a set of jointly sparse vectors.

In order to solve the loop closure detection by MMVs problem, the following process is accomplished. From a set of N sensor observations, a set of $M \ll N$ are selected randomly and compared together by the NCD metric. Therefore, the measurement matrix A is a random binary matrix. Since the similarity measurement achieved by NCD can not be considered as an ideal measure, the noise is also considered. Therefore, the MMVs problem is constructed by

$$\underset{S^{'}}{minimize} \, ||S^{'}||_{1,2} \text{ subject to } ||AS^{'} - B||_{2,2} \leq \sigma \tag{15}$$

Since the number of similar sensor observations is very lower than the total number of sensor measurements, the assumption of sparsity for the modified similarity matrix is true. In order to select the number of required sensor observations for solving the MMVs problem, an upper bound of the number of loops is required. Based on the localization estimation, it is possible to provide an upper bound for number of loops and therefore the sparsity of the MMVs problem. After construction of the compressive measurements, the SPGL1 algorithm is employed for solving the MMVs problem (15).

After reconstruction of the modified similarity matrix from compressive data, the loops can be detected by values of one in the similarity matrix. In the next section, some experiments are provided to prove the applicability and efficiency of the proposed method in loop closure detection in outdoor environments.

IV. EXPERIMENTAL RESULTS

In this section the proposed method is applied to the problem of loop closure detection in two challenging outdoor environments. The results are also compared to that of our previously published technique [7]. The datasets used in this section are denoted in Table I. The KITTI-Sequence (00) dataset is acquired in a dynamic urban outdoor environment using a stereo camera which is shown in Fig. 2. The Lip6 outdoor dataset is grabbed in a dynamic outdoor environment by a monocular camera.

The experimental result of the KITTI-Sequence (00) is shown in Fig. 3. The figures are the modified difference matrix where black lines indicates detected loops. The ground truth of the modified difference matrix is shown in Fig. 3(a) while the result of the proposed method is depicted in Fig. 3(b). The MMVs problem is constructed from only 8% of total observations. As it can be seen, the compressed sensing based method is capable of detecting loops efficiently.

In the next experiment, at first 8% and then 20% of the total observation are used for construction of MMVs problem and deriving the modified difference matrix. The results of this experiment for the Lip6 - Outdoor environment is shown in Fig. 4. The ground truth of the modified difference matrix is shown in Fig. 4(a). As it can be seen, there are two loops in this environment. The results of our previously published method is shown in Fig. 4(b) which is based on processing all of the acquired sensor observations. In Fig. 4(c) and Fig. 4(d) only 8% and 20% of total sensor observations are used for loop closure detection.

Fig. 2. The KITTI Sequence(00) dataset. The green lines indicate the travelled path.

Based on these experiments, it can be concluded that the proposed method can efficiently detect loops in challenging unknown outdoor environments. The proposed method required no prior information about the surrounding environment or training. Using a few sensor observations for loop closure detection makes the proposed method suitable for long term navigation and exploration of mobile robots.

V. Conclusions

In this paper we have proposed a method for detection of loops in an unknown environment for navigation or exploration of mobile robots. Loop closure detection is an essential part of the SLAM problem so the localization error can be kept bounded. While most of the state-of-the-art techniques use all sensor observations for construction of a difference matrix and computation of similarities, the proposed method detects loops based on a few sensor observations. Exploiting special properties of compressed sensing, few sensor observations are compared and the difference matrix is recovered from them.

The multiple measurement vectors problem is solved by SPGL1 algorithm where a set of sparse signals are recovered jointly. Stacking these sparse signals, the difference matrix is constructed where the loops can be derived readily. In order to indicate the applicability of the proposed method, some experiments on publicly available data sets are performed. The results show that the developed method can efficiently detect loops from few camera image or range image observations in various outdoor environments.

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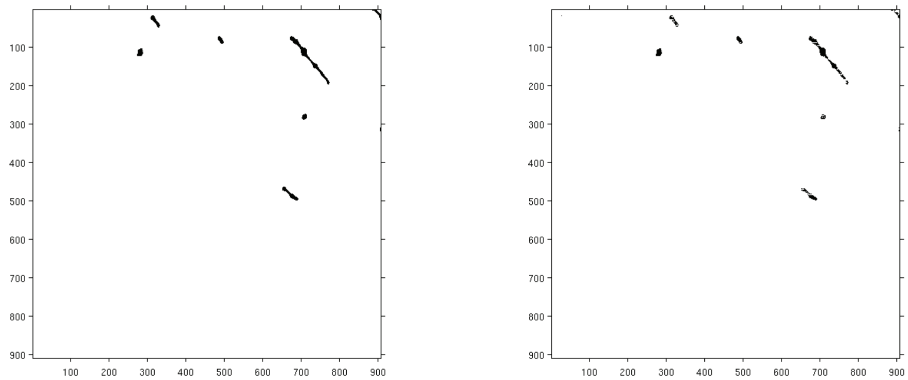
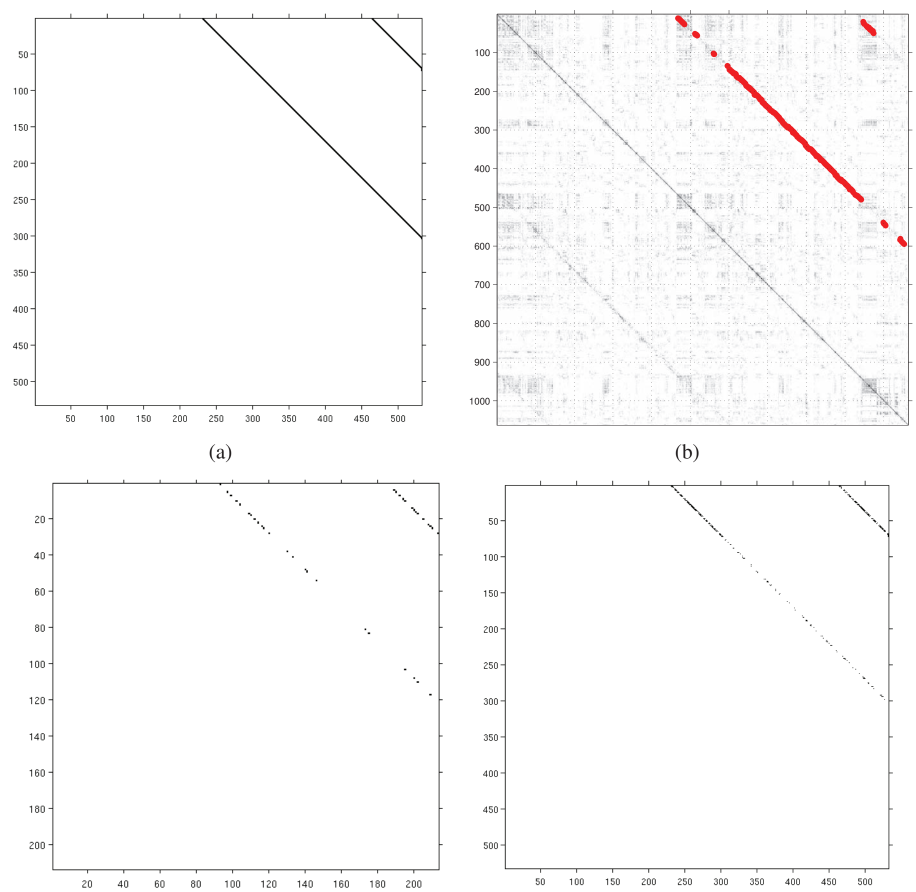


(a) (b)

Fig. 3. Experimental results of KITTI - Sequence(00) [26]. (a) The ground truth of modified difference matrix. (b) The sparse reconstruction result by 8% of sensor observations.



(a) (b) (c) (d)

Fig. 4. Experimental results of Lip6 - Outdoor [26]. (a) The ground truth of modified difference matrix. (b) The experimental results of [7]. (c) The sparse reconstruction result by 8% of sensor observations. (d) The sparse reconstruction result by 20% of sensor observations.