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Optimization of KNTU Delta Robot for Pick and Place Application

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***Abstract*—In this paper, the desired configuration for installation of Delta robot is formulated as an optimization problem and has been solved to reach to the highest rate of pick and place operation. The optimization is performed considering the actuators speed and acceleration limitation of the robot within the workspace. Furthermore, energy consumption is considered next as the other optimization objective, and it is shown that the optimal region for the first optimization problem lies within that of the latter one, and therefore, there is no need to propose a multi-objective optimization problem in this case. As a proof of concept, KNTU Delta robot is designed and implemented in practice by using the optimal configuration, and it is observed that the optimal design is very promising in practice.**

***Index Terms*—Delta robot, installation configuration, energy consumption, optimal solution, pick and place application, genetic algorithm, real time implementation.**

I. Introduction

In industries, many robots are used to execute picking and packaging tasks. One of the most celebrating robot for such application is the parallel Delta robot, a 3-DoF light weight robot, which was invented by Reymond Clavel [1]. One of the main property of Delta robot is the maintenance of the end-effector orientation by using parallelograms in its arms. Hence, in production lines, Delta robot is used to execute the fastest possible operations in pick and place application.

Optimization of Delta robot has been studied in several fields, such as kinematics, dynamics, and trajectory planning. [2] discusses a large number of performance criteria dealing with workspace, transmission quality, manipulability, dexterity, and stiffness. [3] proposes a optimization methodology for the dimensional synthesis: workspace, stiffness, kinematic and dynamic performances. Another similar work referrers to [4], in which the optimal dimensional synthesis method of the Delta robot for a prescribed workspace has been studied. To optimize the trajectory, [5] presents algorithms such that the minimum-time trajectory tracking is achieved, when the path is specified and the actuator torque limitations are known. Base on the mentioned methods, [6] and [7] worked on minimum-time manipulator trajectory as well.

Installation configuration is another important issue, that is deprived of the researcher attentions in background literature. Installation configuration for Delta robot refers to the installation parameters such as height and orientation for a specified pick and place operation. Since in applications, the location of the manipulated object is predefined in a bounded area, some configuration optimizations may be executed to gain faster response than the automated predefined task. In most operations, reaching to a more number of pick and places per minute (PPM), is very important, while in some, energy consumption is the issue of consideration, since the speed of production line is constant. To cover both, two scenarios has been analyzed: 1) The number of PPM is optimized, while energy consumption is the secondary objective and, and 2) The number of PPM is given and the energy consumption is optimized. The proposed optimization algorithm has been implemented on KNTU Delta, shown in Fig. 1, which is designed for further research and academic purposes.

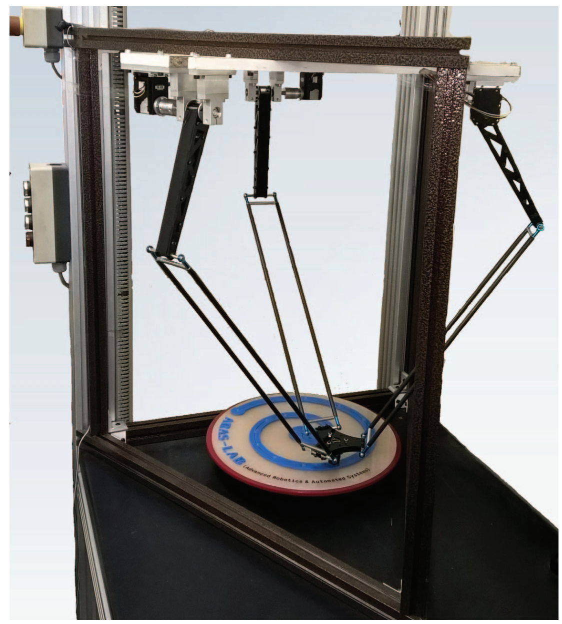

Fig. 1: Real world realization of KNTU Delta robot.

II. Mathematical Analysis

A. Kinematics

Kinematics refers to the study of the robot motion without considering torques and forces that cause it [8]. There are two analysis in kinematics of the robots, namely, inverse kinematics, and forward kinematics. In inverse kinematics it is assumed that the position and orientation of the platform

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is given, and the goal is to find the joint variables. Forward kinematics gives the platform position and orientation by knowing the values of the joint variables. As mentioned in [9] inverse kinematics of Delta robot may be written as

$$q_i = 2\,\tan^{-1}\left(\frac{-\beta + \sqrt{\beta^2 - 4\,\alpha\,\gamma}}{2\,\alpha}\right), \tag{1}$$

where, $i = \{1,\ 2,\ 3\}$ and

$$\begin{aligned} \alpha &:= 2\,a\,\delta_1 + \delta_2, \\ \beta &:= -4\,a\,z_i, \\ \gamma &:= -2\,a\,\delta_1 + \delta_2, \end{aligned} \tag{2}$$

in which, δ_1 and δ_2 are defined as

$$\begin{aligned} \delta_1 &:= R - r + y_i, \\ \delta_2 &:= b^2 - a^2 - \delta_1^2 - x_i^2 - y_i^2, \end{aligned} \tag{3}$$

and $x_i,\ y_i$, and z_i are calculated by using the rotation matrix $R_z(\theta_i)$ as follows

$$(x_i, y_i, z_i)^T = \boldsymbol{R}_z(\theta_i)\,(x_0, y_0, z_0)^T, \tag{4}$$

where, rotations about z-axis are

$$[\theta_1, \theta_2, \theta_3] = \left[0,\ \frac{2}{3}\,\pi,\ \frac{4}{3}\,\pi\right]. \tag{5}$$

The structural parameters has been shown in Fig. (2), in which projected kinematic chain on YZ plane has been illustrated as well to clarify the geometric aspect more precisely. One may obtain the robot workspace by using equation (1). Fig. 3 illustrates the boundary of Delta robot workspace in the ZX and ZY plane.

B. Jacobian

The relation between task space and joint space velocities is defined by the Jacobian matrix. According to [8], the Jacobian matrices in joint ($\boldsymbol{J}_q$) and task ($\boldsymbol{J}_X$) spaces are related as

$$\boldsymbol{J}_x\,\dot{\boldsymbol{X}} = \boldsymbol{J}_q\,\dot{\boldsymbol{q}},\ \ \boldsymbol{J} := \boldsymbol{J}_q^{-1}\,\boldsymbol{J}_x. \tag{6}$$

where, $\boldsymbol{J}$ called the general Jacobian matrix of a parallel robot. For the Delta robot, $\boldsymbol{J}_q$ is written as

$$\boldsymbol{J}_q = \begin{bmatrix} J_{11} & 0 & 0 \\ 0 & J_{22} & 0 \\ 0 & 0 & J_{33} \end{bmatrix}, \tag{7}$$

in which,

$$\begin{aligned} J_{11} &= -l_1\,\sin(\theta_{21})\,\sin(\theta_{31}), \\ J_{22} &= -l_1\,\sin(\theta_{22})\,\sin(\theta_{32}), \\ J_{33} &= -l_1\,\sin(\theta_{23})\,\sin(\theta_{33}), \end{aligned} \tag{8}$$

and, $\boldsymbol{J}_x$ is

$$\boldsymbol{J}_x = \begin{bmatrix} J_{1,x} & J_{1,y} & J_{1,z} \\ J_{2,x} & J_{2,y} & J_{1,z} \\ J_{3,x} & J_{3,y} & J_{1,z} \end{bmatrix}, \tag{9}$$

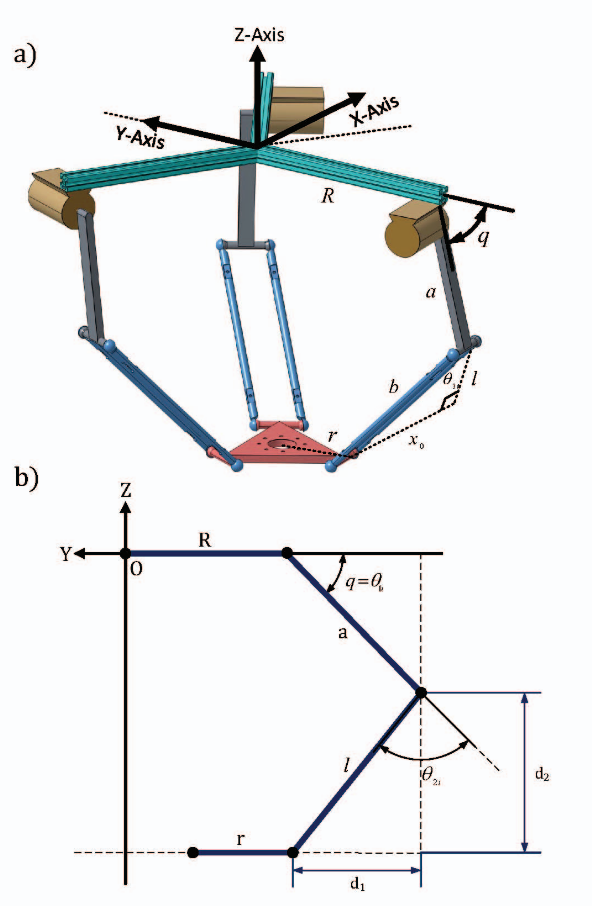

Fig. 2: a) Kinematic parameters of delta robot, b) kinematic chain projected in YZ plane.

in which,

$$\begin{aligned} J_{i,x} &= +\sin(\theta_{3,i})\cos(\theta_{2,i} + \theta_{1,i})\ \cos(\phi_i) + \cos(\theta_{3,i})\ \sin(\phi_i), \\ J_{i,y} &= -\sin(\theta_{3,i})\cos(\theta_{2,i} + \theta_{1,i})\ \sin(\phi_i) + \cos(\theta_{3,i})\ \cos(\phi_i), \\ J_{i,z} &= +\sin(\theta_{3,i})\sin(\theta_{2,i} + \theta_{1,i}). \end{aligned} \tag{10}$$

According to [10], singularity may occur only out of the workspace of the robot.

C. Dynamics

To control, or optimize the robot structure, one should obtain the closed-form dynamic model. By using the Lagrange method the dynamical model of the Delta robot may be determined. As mentioned in [11], the closed-form dynamic model of a parallel robot can be written as

$$\boldsymbol{M}(\boldsymbol{X})\ddot{\boldsymbol{X}} + \boldsymbol{C}(\boldsymbol{X}, \dot{\boldsymbol{X}})\,\dot{\boldsymbol{X}} + \boldsymbol{G}(\boldsymbol{X}) = \boldsymbol{F} \tag{11}$$

in which $\boldsymbol{M}(\boldsymbol{X})$ is the $n \times n$ symmetric positive definite matrix called manipulator inertia matrix, $\boldsymbol{X}$ is the $n \times 1$ array of platform position, $\boldsymbol{C}(\boldsymbol{X}, \dot{\boldsymbol{X}})\dot{\boldsymbol{X}}$ is the $n \times 1$ vector of centrifugal and Coriolis terms, $\boldsymbol{G}(\boldsymbol{X})$ is the $n \times 1$ vector of gravitational effects on the manipulator, and $\boldsymbol{F}$ is the $n \times 1$ array of applied control inputs. Due to space limitations, one

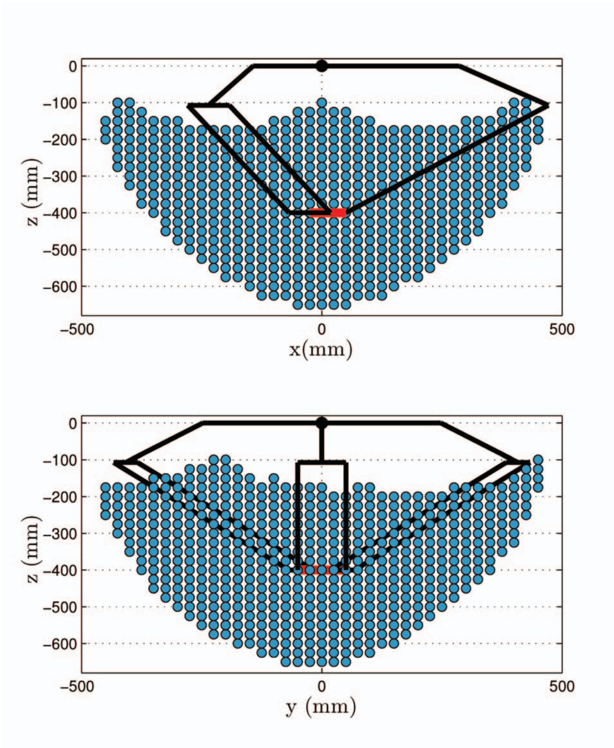

Fig. 3: KNTU Delta workspace boundary in ZX and ZY planes.

may refer to the details of derivation of dynamic matrices in [12].

III. Optimization Problem

Pick and place operation is a frequent task in most production lines. In many applications pick and place operation is designed, considering the fact that the initial and final location of the manipulated object is in a bounded defined region and the end-effector trace lies in a certain subspace. Inspired from this knowledge, in this study, the optimization problem is analyzed in two scenarios; at first the optimal installation configuration, according to a fixed path and actuator restrictions, is calculated to reach the maximum number of PPM in the operation. It means that in this stage pick and place duration is optimized. In many applications, time duration of one pick and place repetition is constant and predefined, due to the constant speed of the production line. Therefore pick and place operation may not be needed to be optimized, while the energy consumption may be analyzed to reach its optimal value in the pick and place task. Thus in the second scenario the energy consumption is considered to be optimized. Before the optimization procedure begins, path and trajectory in the task space should be defined so as to move Delta in a specified trajectory.

A. Path and Trajectory

As a rule, the motions undergone by robotic mechanical systems should be as smooth as possible; i.e., abrupt changes in position, velocity, and acceleration should be avoided. Indeed, abrupt motions require unlimited amounts of power to be implemented, which the motors cannot supply because of their physical limitations [13]. To meet this considerations, boundary conditions has been chosen as

$$\begin{aligned} P(0) &= P_I, \quad \dot{P}(0) = 0, \quad \ddot{P}(0) = 0, \\ P(T) &= P_F, \quad \dot{P}(T) = 0, \quad \ddot{P}(T) = 0, \end{aligned} \tag{12}$$

where, P_I and P_F indicates the initial and final position of the manipulated object. Also, $\dot{P}$ and $\ddot{P}$ denote the velocity and acceleration, respectively. Fig. 4 illustrates the path used in this optimization problem.

Existence of six boundary conditions leads to a 5-th order polynomial interpolation from P_I and P_F as

$$u(s) = a\, s^5 + b\, s^4 + c\, s^3 + d\, s^2 + e\, s + f, \tag{13}$$

in which,

$$0 \leq u(s) \leq 1, \quad 0 \leq s \leq 1, \quad s := \frac{t}{T}. \tag{14}$$

Due to the fact that u and s are normalized, one can write

$$\begin{aligned} P(t) &= P_I + (P_F - P_I)\; u(s), \\ \dot{P}(t) &= T^{-1}\; (P_F - P_I)\; \dot{u}(s), \\ \ddot{P}(t) &= T^{-2}\; (P_F - P_I)\; \ddot{u}(s). \end{aligned} \tag{15}$$

After substitution of (12) in (15), the coefficients of equation (13) may be calculated as follows:

$$u(s) = 6\, s^5 - 15\, s^4 + 10\, s^3. \tag{16}$$

One of the common path used in the pick and place operations is the Adept cycle test [14]. Fig. 4 shows the original and improved cycle test path in YZ plane. Sharp corners cause discontinuity in acceleration. Therefore, the improved version is proposed for pick and place operations. In the smooth-corners path, it has been supposed that $h = 0.5\, H$.

The path formulation along the Z and X axes are given as

$$Z = \begin{cases} Z_I + h\, u(s_1), & 0 \leq t \leq 0.5T \\ Z_F - h\, u(s_2), & 0.5T \leq t \leq T \end{cases}, \tag{17}$$

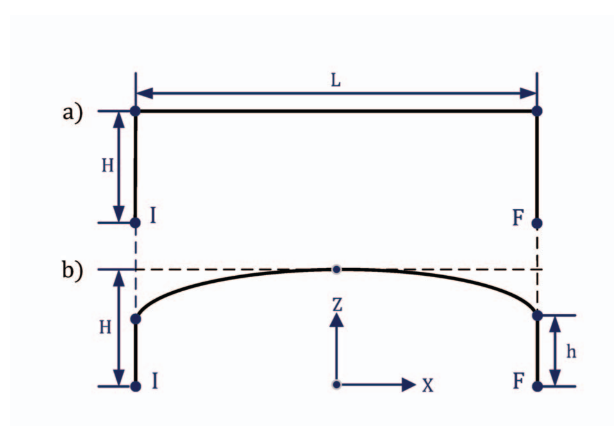

Fig. 4: Path Planning: a) sharp corners, b) smooth corners.

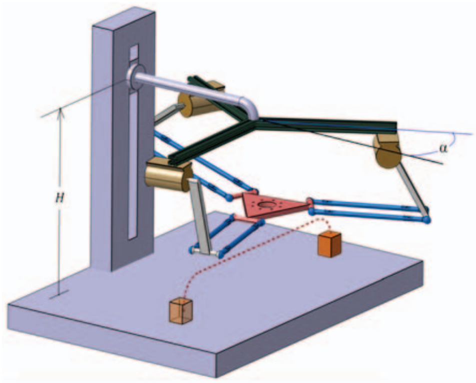

Fig. 5: Installation configuration parameters.

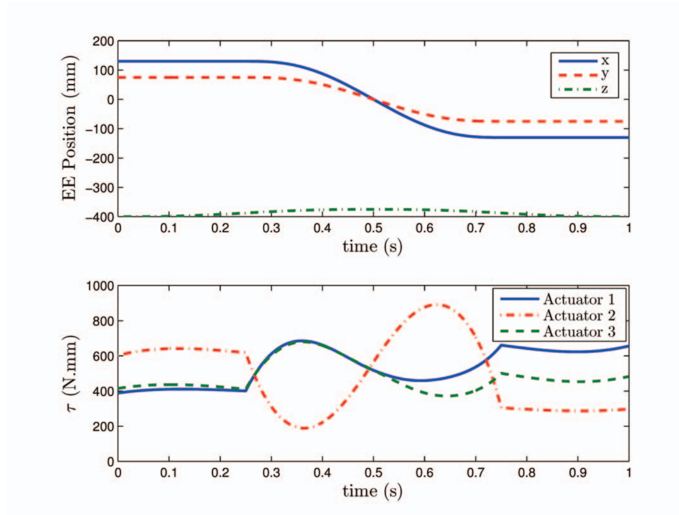

Fig. 6: Desired trajectories in the task space, and equivalent actuator torques.

and,

$$X = \begin{cases} X_I, & 0 \leq t \leq 0.25T \\ X_I + w\,u(s_3), & 0.25T \leq t \leq 0.75T \\ X_F, & 0.75T \leq t \leq T \end{cases}, \tag{18}$$

where,

$$\begin{aligned} s_1 &= 2T^{-1}\,t, \\ s_2 &= 2T^{-1}\,(t - 0.5T), \\ s_3 &= 2T^{-1}\,(t - 0.25T). \end{aligned} \tag{19}$$

B. Pick and Place Optimization

One of the major challenges is to increase pick and place repetition in a specified time, or increase PPM. As PPM increases, the time period of each pick and place task decreases, which means an increase in velocity and acceleration of the actuators. Since acceleration is in relation with actuator torque, one can expect increase in PPM is bounded with actuators torque and speed restrictions. As depicted in Fig. 5, there are two optimization parameters, namely H and α, to change the robot installation configuration. Variations in H and α change the task space coordinate system which produces new actuator trajectories and subsequently new actuator torque profiles. In this work, the objective is to obtain optimal boundaries for H and α, so as for a given trajectory, the actuator profile lies in the OAB torque-speed limiting triangle, shown in Fig. 7, and simultaneously has the maximum distance from the line AB. To clarify the problem, suppose the trajectories in Fig. 6, with $H = 400\,mm$, $\alpha = 30\,deg$, and $PPM = 30$ ($T = 1\,s$). For this configuration, the $\tau - \omega$ profiles of the actuator are shown in Fig. 7. It should be noted from this figure that the critical distance to the actuator restriction is d, which creates the optimization index in this research. The goal is to increase d, which means T can be made smaller. To do so, genetic algorithm (GA) has been used [15]. However, the goal is to find a subspace not a point, since the robot is utilized in a region not a specified point in its workspace. Therefore, all possible values for H and α has been considered to find the optimal configuration boundary.

C. Energy Optimization

To optimize energy, the same mentioned procedure may be performed, although in here the energy consumption along the given PPM is the goal of the optimization problem. The following equation shows the total consumed energy along the specified path.

$$E(\alpha, H) := \int_0^T \left(\sum_{i=1}^{3} \dot{q}_i\,\tau_i \right) dt. \tag{20}$$

To obtain the optimal energy, GA has been used, due to similar reason, mentioned in previous section, all possible values for H and α has been considered to find the optimal configuration boundary.

IV. Optimization Results

By using the Optimization Toolbox of MATLAB [15], a genetic algorithm was used to solve (21) and (23). The optimization results has been presented in Table I.

TABLE I: Optimization Results

Scenario	Iteration	H^* (mm)	α^* (deg)	$\lvert CostFunction \rvert$
PPM	23	-572.3	121.5	34.17
Energy	14	-589.6	104.2	1044.8

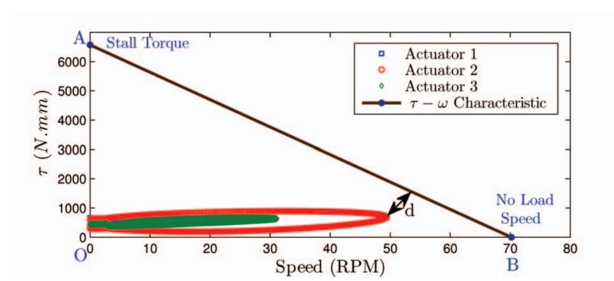

Fig. 7: DC motor τ-ω characteristics for $50\,PPM$.

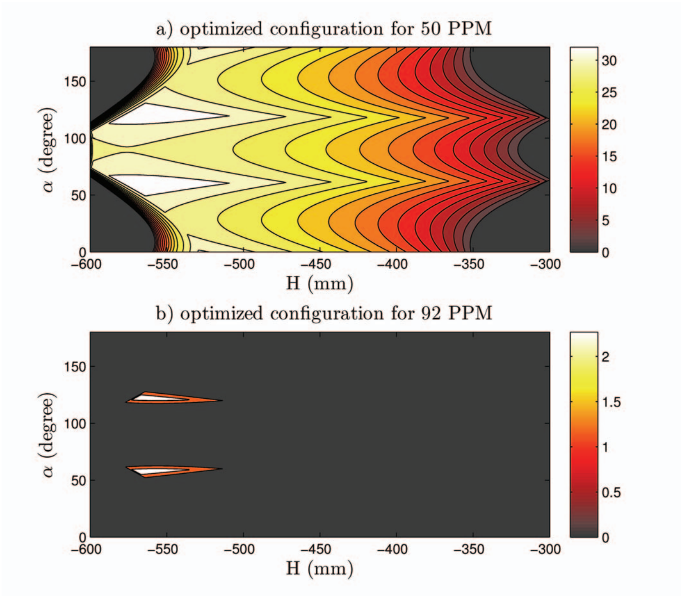

Fig. 8: Visual graphs for non-optimized and optimized PPM

As shown Fig. 8a, the optimal point is in the white region of the spectrum. Due to the fact that the objective is to find a optimal subspace, the areas with approximate white spectrum, which includes the optimal point, indicate the optimal configurations. It is obvious from Fig. 8a that in this area the 2-norm of d is greater than 30. This figure, which is for $50\,PPM$, indicates that it is possible to increase PPM by optimizing d, which means using the most possible capability of the actuators. According to the Fig. 8a, there are two bounded areas for optimal configurations in which PPM can be increased. Note that, darker areas indicated worse cases. In Fig. 8a, the dark gray spectrum with zero index indicates the workspace limitation, while in Fig. 8b it means the inability to reach PPM greater than 92. After the optimization, the number of pick and place increases to $92\,PPM$. Fig. 9 shows that for this PPM, actuator restriction has not been offended.

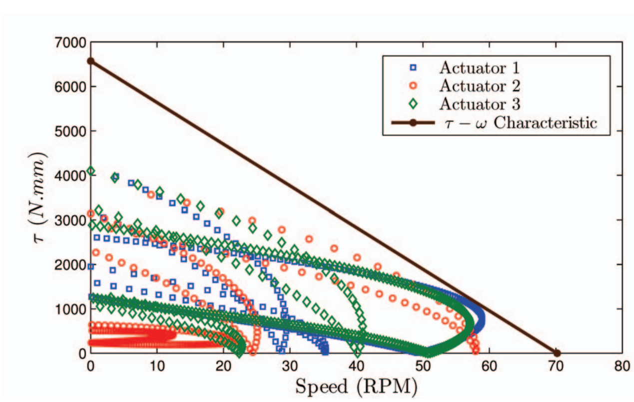

Fig. 9: DC motor τ-ω characteristics for $92\,PPM$

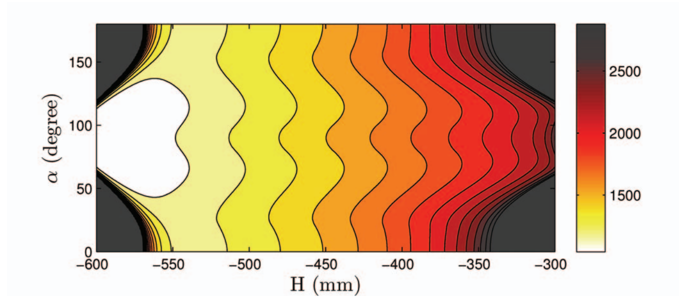

Fig. 10: Visual graphs for energy optimization

To show the validity of the result, suppose a non-optimized configuration with $H = -330\,mm$ and $\alpha = 90\,deg$. In this situation, d-index for $PPM = 50$ equal to 5.13, which means d can be changed to be zero by increasing PPM. Thus, In this manner the maximum number of PPM is about 54, while in the optimal configuration this quantity is 92, which precisely indicates the difference.

PPM optimization problem formulated as follows

$$\min_{over\, X=[\alpha,H]} \{ -d(\alpha, H) \}, \tag{21}$$

where,

$$0 < \alpha < \pi, \quad -600 < H < -300. \tag{22}$$

With GA, energy consumption optimization leads to the results in Table. I. The optimal point, which is in the heart-shape region, indicates the minimum energy consumed in the operation. This region is the desired configuration, in the sense of energy consumption.

Energy optimization problem can be formulated as

$$\min_{over\, X=[\alpha,H]} \{ E(\alpha, H) \}, \tag{23}$$

where,

$$0 < \alpha < \pi, \quad -600 < H < -300. \tag{24}$$

It should be noted from Fig. 8 and Fig. 10 that the optimized regions obtained in the both optimization scenarios are approximately the same and optimized region in energy optimization cover optimized regions obtained in PPM. This means that with the optimized configuration in the sense of PPM, one may obtained optimal energy consumption as well. This results paves the path to obtain an optimal configuration to accommodate both objectives, without need to have a weighted optimal criteria or multi-objective optimization routines.

As the height of installation is increased, the limbs of the branches are stretched outwards. According to the physical sense it is perceived that the actuator effort for moving the end effector at this configuration is less compared with that of low height installation of robot. This is because the gravity forces of the limbs exerts lower torques on the actuators. This justification may result the optimum area to be located in higher heights in energy optimization.

V. Implementation

The optimization result is mainly being adopted for optimal design and implementation of a prototype robot in the Lab. KNTU Delta robot shown in Fig. 1 has been made with lightweight material in Advanced Robotics and Automated System, Robotics lab for further research and academic purposes, from the outcome of this optimization. The detail design, and controller design and implementation of this robot is not intended to be elaborated in this paper, and only some real time experiments is reported in here, to show the proof of concept defined and solved in this paper. This robot is controlled with Real-Time Windows Target in Simulink/Matlab. A specified trajectory proposed in III-A has been implemented to show the performance of robot, and to verify the speeds that can be practically achieved. Initial and final points are set as $P_I = [-150, 0, -550]$ and $P_F = [150, 0, -550]$, respectively. By considering 60 PPM a half pick and place operation in half second is shown in Fig. 11. The performance of tracking especially in picking and placing indicate the accuracy of robot. In practice higher PPM near to the designed 92 PPM has been achieved in practice, however, the tracking performance in transient needs further improvement. This is the current stage of research on the design and implementation of more robust controllers in practice,

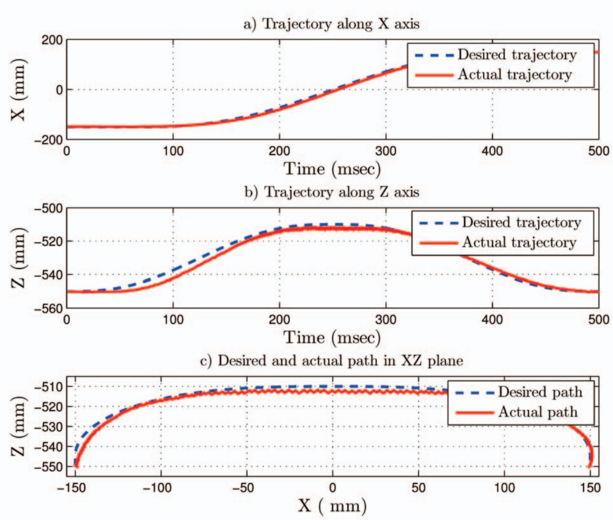

Fig. 11: Performance of KNTU Delta robot in 60-PPM operation cycle.

VI. Conclusions

In this work a new optimization problem is formulated and being studied in detail for the design and implementation of a Delta robot. Installation configuration has been analyzed to obtain the desired pick and place trajectory within the optimal workspace of the robot. By considering the actuator limitation, two bounded area in installation is found that may cause the highest rate of pick and place operation. The importance of optimizing the robot installation configuration in order to reach highest rate of PPM, is clarified by comparing the significant difference of optimized configuration PPM versus the non-optimized one. Furthermore, the problem of optimal energy consumption for a preset value of PPM is considered next. It is shown that the optimal region for the first optimization region lies within the latter one, and therefore, there is no need to use multi-objective optimization routines to solve both problems simultaneously. As a proof of concept, the optimal solution is used to design and implement KNTU Delta robot in practice, and it is observed that it is possible to reach to the theoretical PPM in practice. Future research is continued on the improvement of the controller design of the KNTU Delta robot, in order to reach to the highest possible PPM with desirable transient performance in trajectory tracking.

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