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Type Synthesis of 2R-T Parallel Mechanisms Based on the Screw Theory for Haptic Applications

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Abstract— **Recently haptic devices are increasingly used in industry and research. As their applications become widespread, their design is needed to be more efficient. At design stage, determinant features of haptic devices such as rigidity, force bandwidth, accuracy etc. must be considered and improved. Structurally, parallel mechanisms (PMs) are appropriate candidates for haptic devices. Due to multi legged structure of PMs and their grounded motors, inertia and stiffness feautures of them are desirable and it also made them popular for applications that require high mechanical transparency. Spherical kinematics (two rotational and one translational motion, 2R-T) is a very common type of motion in haptic devices that is also capable of general rendering. In this paper, several 3-DOF 2R-T PMs are synthesized for haptic applications by means of the screw theory. All of these mechanisms have center of motion (CM) which is a key property in variety of applications such as surgery. These mechanisms are compared qualitatively and their applications as haptic devices are discussed.**

I. INTRODUCTION

Haptic devices permit force reflections between users and virtual or remote environments by which they improve the full immersion and interactivity with an operator [1]. They are applied in many applications such as virtual reality, medical robotics, game industry and teleoperations [2]. For the past decades, various haptic devices have been developed and successfully utilized [3].

Performance of haptic devices mostly relies on their mechanical structures and properties. Several measures are used for evaluating their performance. One important measure of these force-reflecting interfaces is the force bandwidth that shows the range of frequency which the device can generate the desired force [4]. Another feature of a haptic device is transparency. This means that transmitted impedance of environment to the user should be without any distortions [5]. The significant factor which disturbs the transmitted impedance is the dynamics of the haptic device. Therefore, to improve the transparency, haptic mechanism should be designed such that it's inertia and mass is eliminated or reduced. On the other hand, one more critical capability of haptic device is backdrivability, that the actuators should be able to follow the users hand motion rapidly [6]. Altogether, haptic devices are mechanisms with strict and special considerations.

Recently, many researches has been devoted to using parallel mechanisms as haptic devices [7],[8],[9],[10]. A generalized parallel robot is a mechanism with closed loop kinematic chains, whose moving platform is attached to the base by several independent kinematic chains [11]. Because of their multi-legged configurations, they have high structural stiffness, force capability and accuracy. Moreover, installing the actuators near the fixed base causes their mass and inertia greatly reduced. Therefore, they are good in the sense of force bandwidth, mechanical transparency and backdrivability which make them suitable options for haptic mechanisms [12].

Generally, required DOF of a haptic device is fewer than six that makes their structure simple and reduces their cost. Types of their DOF have been selected such that they can do their specified tasks. A notable task for general haptic devices is global rendering that means it can reach all the points in 3D space [13]. In variety of tasks two rotational motions and one translational motion (2R-T) are required for haptic devices that also is capabale of global rendering. For instance, in applications such as minimal invasive surgery, game and simulation, 2R-T haptic devices are adequate. Furtheremore, A structural feature of haptic mechanisms in the mentioned applications is center of motion (CM) [14]. The CM is a point of mechanism which the moving platform rotations pivoted at that point. For instance, in minimaly invasive surgery, medical tool inserted at pivot point and rotates around it with yaw and pitch rotations.

In this paper, several 2R-T PMs that have center of motion are synthesized for haptic applications. Different approaches in the literature are devoted to type synthesis of PMs such as screw theory, displacement group theory and single opened chain units [15],[16],[17]. Recently, the screw theory is successfully utilized for synthesizing special mechanisms [18],[19]. A systhematic method for synthesizing PMs based on the screw theory and virtual chain is introduced in [20]. Kong et al also present type synthesis of several PMs such as 3T, 3R-T, S etc. , as case studies. However, type synthesis of many of PMs including the 2R-T are not addressed in their work. Their systhematic stepwise algorithm is employed in this paper for type synthesis of 2R-T PMs that also have center of motion. The remainder of this paper is organized as follows. In section II, important results from screw theory are introduced. In section III, 2R-T PMs based on the screw theory are synthesized. Finally, the synthesized mechanisms are compared and their application as haptic devices is discussed.

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II. SCREW THEORY

Screw coordinates are 6×1 coordinates system that can be used for describing general motion of a rigid body [21]. The general motion is decomposed to a rotation around an axis and the translation along of that axis , which is called the screw axis . An unit screw coordinate ,$\hat{\$}$, is defined as follows:

$$\hat{\$} = \begin{bmatrix} \hat{s} \\ s_0 \times \hat{s} + \lambda \hat{s} \end{bmatrix} = \begin{bmatrix} \$_1 \\ \$_2 \\ \$_3 \\ \$_4 \\ \$_5 \\ \$_6 \end{bmatrix} \tag{1}$$

in which, $\hat{s}$ is the unit vector that aligned with the screw axis , and s_0 is the distance of the origin of the fixed frame A to the screw axis , as shown in Fig. 1. λ, is the ratio of the translation to the rotation.

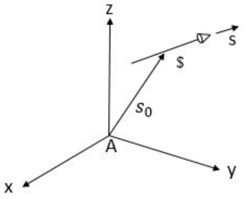

Fig. 1. Screw representation

A general mechanism consist of variety of joints which can be decomposed into a mechanism including only prismatic (P) and revolute (R) joints. For a revolute joint, the ratio of the translation to the rotation i.e. $\lambda = 0$, therefore the normalized screw of the joint is expressed by:

$$\hat{\$} = \begin{bmatrix} \hat{s} \\ s_0 \times \hat{s} \end{bmatrix} \tag{2}$$

For a prismatic joint, $\lambda = \infty$ and the unit vector $\hat{\$}$ of the joint is as follows:

$$\hat{\$} = \begin{bmatrix} 0 \\ \hat{s} \end{bmatrix} \tag{3}$$

For convenience, usually $\$_0$ and $\$_\infty$ are used to represent the screw of a revolute joint and the screw of a prismatic joint, respectively.

Type of joints (P or R), determines the relative motion and constraints between each two links which can be analyzed using screw theory. Twist system represents the instantaneous relative motion and wrench system represents constraints between each two links caused by their joint. Wrench system is reciprocal of twist system based on virtual work principle [20]. Briefly, ξ and ζ represent twist system and wrench system respectively, that their subscript may be 0 or ∞ that is related to revolute and prismatic joints. The twist system of R and P joint are respectively $1-\xi_0$ and $1-\xi_\infty$ and the wrench system which is resiprocal to the twist system are $3-\zeta_0-2-\zeta_\infty$ and $2-\zeta_0-3-\zeta_\infty$, respectively.

Type synthesis of a mechanism actually is determination of the constraints on the moving platform which is applied with its legs and joints. In type synthesis of mechanisms not only number of DOFs is significant, but also type of DOFs that is called motion pattern is determinant. In order to illustrate the motion pattern of a mechanism, virtual chain (VC) concept is introduced. A VC associated with a motion pattern is a serial or parallel kinematic chain (KC) whose moving platform has the given prescribed motion pattern. Therefore, type of DOFs of a PM can be described by its related VC. It shoud be noted that for determining related VC for any type of DOFs, there are several serial or parallel kinematic KCs that have the prescribed motion pattern, that the simplest KC is better to be selected as the VC.

Regarding that PMs consist of several legs, type synthesis of a PM is divided into type synthesis of each leg which determines constraints that each leg exerted on the moving platform. The key for synthesis of each leg is motion constraint that the leg can apply to the end-effector i.e. wrench system of the leg. Therefore, after detemining the wrench system of the desired PM, the wrench system of its legs called leg-wrench systems shall be obtained.

Conceptually for a parallel structure, final motion constraint that applied to the end-effector is union of constraints of all it's legs whereas for a serial KC final motion constraint would be intersection of constraint of all it's joints. Therefore the wrench system of a PM is linear combination of the wrench system of each leg and also the wrench system of each leg which is a serial KC is intersection of the wrench system of all it's joints.

For type synthesis of a leg that have specfied wrench system and belongs to a PM, a single loop which contain the VC and the leg must be constituted. As we know a single loop is a serial KC which the first and last joint of it is attached together. Then for synthesis of a single loop , the number of joints should be determined with regard to the following formula:

$$f = F + (6 - c) \tag{4}$$

in which, c, f and F are the order of wrench system of the leg, total number of 1-DOF joints and mobility of the single loop KC, respectively.

After determination of number of joints in a single loop, type of joints i.e. P or R must be assigned such that the considered wrench system of the single-loop/leg can be obtained. To do so, geometric conditions on joints and their axes must be found based on reciprocity condition of screws [20]. Accordingly, general algorithm for synthesis of PMs is as follows:

- *Step 1.* Firstly, leg wrench system must be found.
  Since the wrench system of the PMs is the linear combination of legs wrench system , the wrench system of each leg must be a subset of specified wrench system of the PM. Therefore, specified wrench system of the PM must be decomposed to obtain possible wrench system of each leg.
- *Step 2.* All possible legs must be synthesized.
  For synthesis of each leg, which has specified DOF

and wrench system (that obtained from previous step), each leg of the PKC must be such that with the VC, constitutes an F-DOF single loop KC. Conceptually, by adding VC to the leg for single-loop formation, this condition which the base and the end-effector (first and last joint of leg) must have a specified relative motion is applied. Furthermore, wrench system of leg ensured that legs apply required constraints to the end-effector. Therefore, each possible leg for the specified PM may be obtained from synthesis of the single loop with the considered DOFs and leg wrench system.

- *Step 3*. Structure of the PM must be determined.
  Final structure of the desired PM may be generated by selecting m legs that obtained from *Step 2* such that the linear combination of the wrench system of the legs is the same as that of the desired PM. In this paper, the non-redundant PMs have been designed i.e. $m = F$.
- *Step 4*. Actuated joints must be assigned.
  After assembly of legs and determination of the structure of PM, active joints must be selected. They should be selected such that when actuated joint are fixed, whole PM will be locked [20]. In the selection of the actuated joints, it is better that the number of actuated joints in the legs be equal and due to inertia consideration it is recommended that actuated joints be close to the base.

Above description is summarized as a flowchart in Fig. 2.

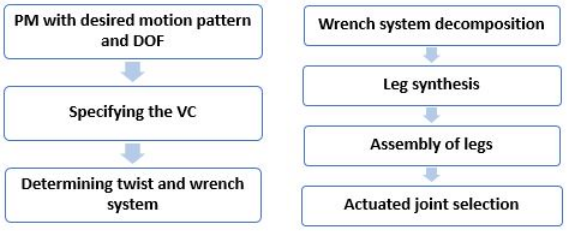

Fig. 2. Type synthesis of PMs procedure

III. SYNTHESIS OF 2R-T PARALLEL MECHANISMS

In this section, several 2R-T PMs which have CM are synthesized. The 3-DOF 2R-T PMs are a class of PMs with reduced DOFs. These mechanisms have two rotational DOF (pitch and yaw) and one radial motion, i.e. spherical kinematics, which is capable of global rendering and suitable for haptic devices. This type of motion have used numerously in surgery, game and simulation applications. Using the general approach reviewed in the previous section, type synthesis of 2R-T PMs is presented as follows.

The mentioned motion pattern, in the simplest form, can be achieved by a U joint and a P joint, then the VC of the considered mechanisms is UP as shown in Fig. 3. Noting that the mechanism have CM, the axis of P joint and U joint must be concurrent at the pivot point.

The twist system of a 2R-T PM or its VC is a 2-ξ_0 -1- ξ_∞ system in which the axes of screws intersect at one point on the base or somewhere else. The twist system is a 3-order system and then its reciprocal is also 3-order system. Then the wrench system of this PM, based on its reciprocal is a 2-ζ_0 -1-ζ_∞ system whose axes pass through the center of the U joint and ζ_0 axes are perpendicular to the P joint within the VC as shown in Fig. 3. It should be noted that final PM must be synthesized such that each leg of it and related VC form a 3-DOf single-loop and also the wrench system of the mechanism in any general configuration must be a 2-ζ_0 -1-ζ_∞ system.

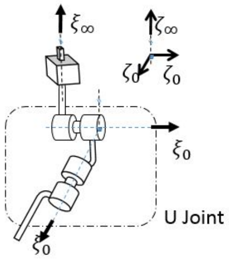

Fig. 3. Virtual chain of the desired 2R-T parallel mechanisms

TABLE I
ALL POSSIBLE WRENCH SYSTEM FOR LEGS OF THE PM

1-order	2-order	3-order
$1-\xi_0$	$2-\xi_0$	$2-\xi_0-1-\xi_\infty$
$1-\xi_\infty$	$1-\xi_0-1-\xi_\infty$	—

A. Synthesis of legs

As the first step of synthesis procedure, all possible legs for this mechanism must be obtained. For leg synthesis, leg-wrench systems must be determined. To do so, the wrench system of the 2R-T PMs is decomposed. Any leg-wrench system in the PMs is a sub-system of its wrench system. All the leg-wrench systems for the UP PMs are shown in Table. 1.

In the other word, any leg that belongs to the desired 2R-T PM must have one of the wrench systems represented in the Table. 1. As shown in this table, the possible order of wrench system of legs is 1, 2 and 3. When the order of wrench system of a leg increases, the number of constraints that exerted by that leg increases too and causes decreasing the number of joints in that leg which makes the mechanism simpler. Therefore, to obtain mechanisms that are rather simpler, structurally and kinematically, the possible legs with high order wrench system is more suitable to be selected .

After leg-wrench system detemination, the second step related to type synthesis of each leg. Each leg must be synthesized such that its combination with VC yields a 3-DOF single-loop with specified leg-wrench system. Then for each case in Table. 1. possible legs are synthesized.

1) Cases with a 2-ζ_0 -1-ζ_∞ -system : The legs with the wrench system of 2-ζ_0 -1-ζ_∞ must be obtained from type synthesis of 3 DOF single-loop that involve an UP virtual chain. According to equation (4), order of wrench system of

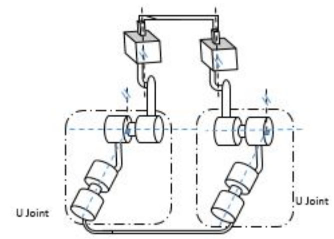

Fig. 4. Single-loop with 2-ζ_0 -1-ζ_∞ -system, UP or $\hat{R}\hat{R}P$

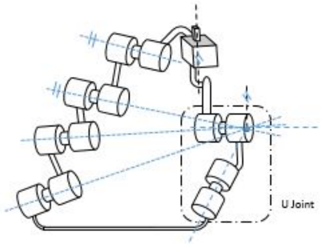

Fig. 5. Single-loop with 2-ζ_0 -system, $\hat{R}\hat{R}\hat{R}R_L$

single loop is three (c=3) and its number of DOF is three (F=3) then number of joints in the single loop will be six (f=6) and also number of joints excluding virtual chain's joints is three (number of leg's joints). In this case, both the VC and the single loop have the wrench system of 2-ζ_0 -1-ζ_∞, if the leg has the same wrench system, then the required conditions will be satisfied. Therefore, the type synthesis procedure yields RRP or UP leg. The related single-loop is shown in Fig. 4.

2) Cases with a 2-ζ_0 -system : In this case, order of the wrench system is two (c=2), according to equation (4) number of joints of single-loop will be seven (f=7) and number of leg joints is four. Such a single-loop that involves an UP virtual chain and has a 2-ζ_0-system and also has CM, is formed by inserting (a) three concurrent revolute joints and another revolute joint which is coaxial with the last one ($\hat{R}\hat{R}\hat{R}R_L$) (b) three concurrent revolute joints and a prismatic joint which is codirectional with the P joint of the UP ($\hat{R}\hat{R}\hat{R}P_L$) (c) two concurrent revolute joints and two parallel axis revolute joints whose associated plane is prependicular to the P joint of the UP ($\hat{R}\hat{R}(RR)_E$). $\hat{R}$ denotes R joints whose axis intersect at the center of the U joint of the UP. Therefore, three different legs are synthesized with 2-ζ_0 -system as shown in Fig. 5. to Fig. 7.

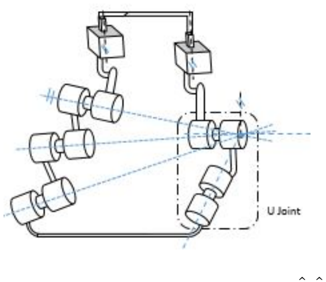

Fig. 6. Single-loop with 2-ζ_0 -system,$\hat{R}\hat{R}\hat{R}P_L$

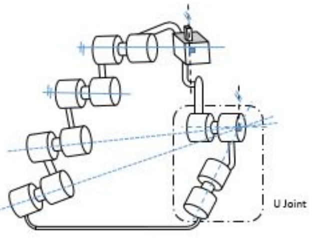

Fig. 7. Single-loop with 2-ζ_0 -system,$\hat{R}\hat{R}(RR)_E$

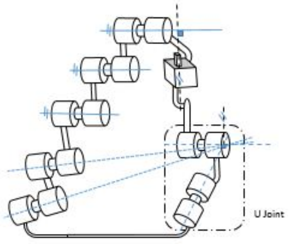

Fig. 8. Single-loop with 1-ζ_0 -system,$\hat{R}\hat{R}(RRR)_E$

3) Cases with a 1-ζ_0 -system : In this case, order of the wrench system is one (c=1), according to equation (4) number of joints of single loop will be eight (f=8) and number of leg joints is five. Synthesizing of this case, leads to legs with two concurrent revolute joints and three parallel axis revolute joints whose associated plane is prependicular to the P joint of the UP ($\hat{R}\hat{R}(RRR)_E$). Single loop associated with this case is shown in Fig. 8.

4) Cases with a 1-ζ_0 -1-ζ_∞ system : In this case, order of the wrench system is two (c=2), according to equation (4) number of joints of single loop will be seven (f=7) and number of leg joints is four. This wrench system can be obtained usually by inserting one coaxial compositional unit into one single-loop KC composed of one planar compositional unit [20]. But because of significant constriant that applied to the mechanism i.e. center of motion, this arrangement need some modification. This modification leads to legs with one revolute joint along one of U joint axes and three parallel axis revolute joints whose associated plane is perpendicular to the P joint of the UP ($\hat{R}(RRR)_E$). Single loop associated with this case is shown in Fig. 9.

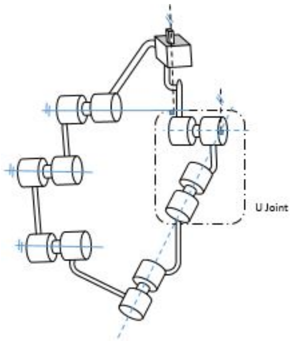

Fig. 9. Single-loop with 1-ζ_0 -1-ζ_∞ -system,$\hat{R}(RRR)_E$

5) Cases with a 1-ζ_∞ -system : In this case, order of the wrench system is one (c=1), according to equation (4) number of joints of single loop will be eight (f=8) and number of leg joints is five. Regarding that wrench system should be 1-ζ_∞, the joint arrangement must be such that axes of prismatic joint be perpendicular to all revolute joints. This arrangement is obtained with revolute joint along one of U joint axes and four parallel axis revolute joints whose associated plane is perpendicular to the P joint of the UP ($\hat{R}(RRRR)_E$). Single loop associated with this case is shown in Fig. 10.

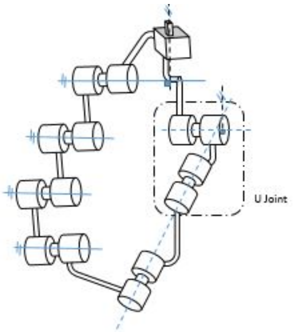

Fig. 10. Single-loop with 1-ζ_∞ -system,$\hat{R}(RRRR)_E$

It should be noted that it is possible to have some other arangements leading to the same wrench system, especially arrangement with more than two passive prismatic joints. Those arrangements did not investigated becasue of their infeasibility in practice.

B. Assembly of Legs

In the previous part, all possible legs that may be a leg of 2R-T PMs are obtained. The final PM can be obtained from assembly of different synthesized legs. Each three legs which linear combination of their wrench systems yeilds to a 2-ζ_0 -1-ζ_∞ wrench system, may synthesize a 3-DoF 2R-T PM. Therefore in this step all three-member subset of synthesized leg shall be selected considering the final wrench system of mechanism. In the Table. 2. all three-member combinations of legs that can together constitute the desired PM are introduced.

It should be noted that these combinations are practically feasible while their geometry and axes must be set such that linear combination of their wrench yeilds 2-ζ_0 -1-ζ_∞ wrench system. For example $\zeta_0, \zeta_0, \zeta_\infty$ combination yields a 2-ζ_0 -1-ζ_∞ wrench system if the two ζ_0 is not linearly dependent. Here, for instance three PM are synthesized using combination of different wrench systems. In Fig. 11. $1-\zeta_0-1-\zeta_\infty, 1-\zeta_0-1-\zeta_\infty, 1-\zeta_0-1-\zeta_\infty$ combination selected for PM synthesis. In Fig. 12. $1-\zeta_0-1-\zeta_\infty, 1-\zeta_0, 1-\zeta_0$ combination selected for PM synthesis and also CAD model of this PM is shown in Fig. 13. In Fig. 14. $1-\zeta_0-1-\zeta_\infty, 2-\zeta_0, 2-\zeta_0$ combination selected for PM synthesis.

Actuated Joints : Actuated joint selection and validity detection of actuated joints for synthesized mechanisms not detailed in this paper because of lack of space. However, this procedure can be simply pursued as in [20]. Usually for a mechanism several set of actuated joints may be assigned but for haptic application the inertia is a very critical property and therefore the actuated joints must be selected near the base. It should be noted that actuated joints are usually one DoF joints; therefore, in the chains that U joint is used, this joint can not be actuated. This fact causes that actuated joints take some distance from the base in some of synthesized mechanisms , that is needed to be reconsidered.

TABLE II

ALL POSSIBLE THREE-MEMBER SUBSET OF LEG-WRENCH SYSTEMS FOR PM SYNTHESIS

$2\xi_0$,$2\xi_0$,$1\xi_\infty$ $2\xi_0$,$2\xi_0$,$1\xi_0-1\xi_\infty$ $2\xi_0$,$2\xi_0$,$2\xi_0-1\xi_\infty$
$2\xi_0$,$1\xi_0$,$1\xi_\infty$ $2\xi_0$,$1\xi_0$,$1\xi_0-1\xi_\infty$ $2\xi_0$,$1\xi_0$,$2\xi_0-1\xi_\infty$
$2\xi_0$,$1\xi_\infty$,$1\xi_\infty$ $2\xi_0$,$1\xi_\infty$,$1\xi_0-1\xi_\infty$ $2\xi_0$,$1\xi_\infty$,$2\xi_0-1\xi_\infty$
$2\xi_0$,$1\xi_0-1\xi_\infty$,$1\xi_0-1\xi_\infty$ $2\xi_0$,$1\xi_0-1\xi_\infty$,$2\xi_0-1\xi_\infty$
$1\xi_0$,$1\xi_0$,$1\xi_\infty$ $1\xi_0$,$1\xi_0$,$1\xi_0-1\xi_\infty$ $1\xi_0$,$1\xi_0$,$1\xi_0-1\xi_\infty$
$1\xi_0$,$1\xi_\infty$,$1\xi_0-1\xi_\infty$ $1\xi_0$,$1\xi_\infty$,$2\xi_0-1\xi_\infty$
$1\xi_0$,$1\xi_0-1\xi_\infty$,$1\xi_0-1\xi_\infty$ $1\xi_0$,$1\xi_0-1\xi_\infty$,$2\xi_0-1\xi_\infty$
$1\xi_0$,$2\xi_0-1\xi_\infty$,$2\xi_0-1\xi_\infty$
$2\xi_\infty$,$1\xi_\infty$,$2\xi_0-1\xi_\infty$
$1\xi_\infty$,$1\xi_0-1\xi_\infty$,$1\xi_0-1\xi_\infty$ $1\xi_\infty$,$1\xi_0-1\xi_\infty$,$2\xi_0-1\xi_\infty$
$1\xi_\infty$,$2\xi_0-1\xi_\infty$,$2\xi_0-1\xi_\infty$
$1\xi_0-1\xi_\infty$,$1\xi_0-1\xi_\infty$,$1\xi_0-1\xi_\infty$ $1\xi_0-1\xi_\infty$,$1\xi_0-1\xi_\infty$,$2\xi_0-1\xi_\infty$
$1\xi_0-1\xi_\infty$,$2\xi_0-1\xi_\infty$,$2\xi_0-1\xi_\infty$
$2\xi_0-1\xi_\infty$,$2\xi_0-1\xi_\infty$,$2\xi_0-1\xi_\infty$

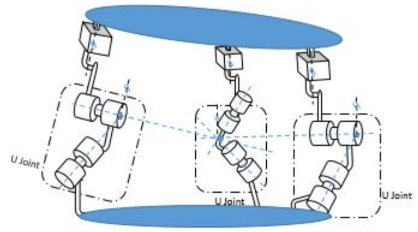

Fig. 11. $3-UP$ synthesized PM

IV. DISCUSSIONS

Considering special criteria that haptic devices must fulfill, a number of final synthesized mechanisms may not be appropriate for this application. In addition to mass, inertia, force producibility and stiffness properties of mechanisms that tried to satisfied by using PM, some more items need to meet. Friction, kinematic solvability, controllability and implementation consideration must also be noticed. Kinematic solvability is an important item in selecting appropriate mechanisms for haptic application. Kinematic calculations

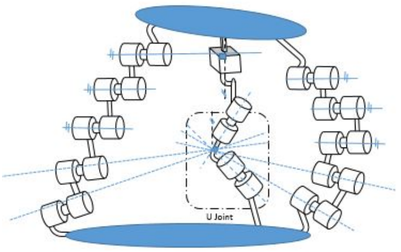

Fig. 12. $UP-2\hat{R}\hat{R}(RRR)_E$ synthesized PM

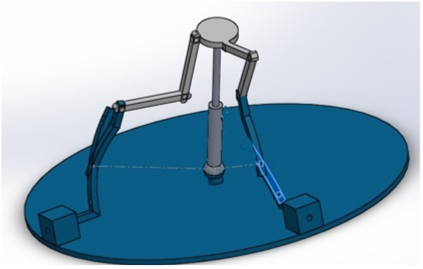

Fig. 13. Cad model of the $UP-2\hat{R}\hat{R}(RRR)_E$ synthesized PM

should be performed real time. Kinematic computing and its simplicity mostly depends on the number and type of joints. Also similarity of legs simplify kinematic calculations.

Friction often reduce haptic performance. Friction considerations usually are related to implementation but in mechanism selection also play a significant role. As the number of joints increases, friction and energy losses increase too. On the other hand, passive prismatic joint impose notable friction to system. For this reason, all mechanisms with passive prismatic joint must be ignored.

Actuator location, gravity compensation capability, scalablity, compactness are also important factors that must be considered in mechanisms selection carefully.

V. CONCLUSIONS

Parallel mechanisms are suitable structures as haptic devices, because of their mechanical properties. In this research using screw theory and virtual chain approach , several 3-DOF 2R-T Parallel manipulators are synthesized. A stepwise and conceptual procedure pursued. Virtual chain and wrench system of mechanisms and then leg-wrench systems determined. After that, all possible legs for these mechanisms synthesized. Then by assembly of legs final structure of mechanisms obtained. All of obtained mechanisms have center of motion that make them suitable for variety of applications such as surgery and game. Finally usability of synthesized mechanisms as haptic devices are discussed considering inertia properties, kinematical complexity etc.. Regards to all before-mentioned consideration the $UP-2\hat{R}\hat{R}(RRR)_E$ and also $UP-\hat{R}\hat{R}(RRR)_E-\hat{R}\hat{R}(RR)_E$ may be suitable for desired application.

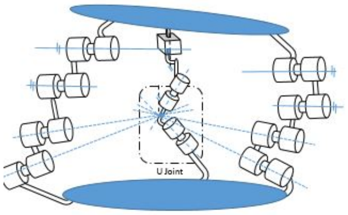

Fig. 14. $UP-2\hat{R}\hat{R}(RR)_E$ synthesized PM

REFERENCES

[1] Janez Podobnik et al. *Haptics for virtual reality and teleoperation*, volume 64. Springer Science & Business Media, 2012.

[2] Vincent Hayward, Oliver R Astley, Manuel Cruz-Hernandez, Danny Grant, and Gabriel Robles-De-La-Torre. Haptic interfaces and devices. *Sensor Review*, 24(1):16–29, 2004.

[3] Vincent Hayward and Oliver R Astley. Performance measures for haptic interfaces. In *Robotics research*, pages 195–206. Springer, 1996.

[4] Seiichiro Katsura, Yuichi Matsumoto, and Kouhei Ohnishi. Analysis and experimental validation of force bandwidth for force control. *IEEE Transactions on Industrial Electronics*, 53(3):922–928, 2006.

[5] Paul G Griffiths. *Design and analysis of haptic interface and teleoperator feedback systems*. ProQuest, 2008.

[6] Uroš Mali and Marko Munih. Hife-haptic interface for finger exercise. *IEEE/ASME Transactions on Mechatronics*, 11(1):93–102, 2006.

[7] Jumpei Arata, Hiroyuki Kondo, Norio Ikedo, and Hideo Fujimoto. Haptic device using a newly developed redundant parallel mechanism. *Robotics, IEEE Transactions on*, 27(2):201–214, 2011.

[8] Force dimension. `http://www.forcedimension.com/`.

[9] Roger Baumann, Willy Maeder, Dominique Glauser, and Reymond Clavel. The pantoscope: A spherical remote-center-of-motion parallel manipulator for force reflection. In *Robotics and Automation, 1997. Proceedings., 1997 IEEE International Conference on*, volume 1, pages 718–723. IEEE, 1997.

[10] Jungwon Yoon and Jeha Ryu. Design, fabrication, and evaluation of a new haptic device using a parallel mechanism. *IEEE/ASME Transactions on mechatronics*, 6(3):221–233, 2001.

[11] Jean-Pierre Merlet. *Parallel robots*, volume 74. Springer Science & Business Media, 2012.

[12] Jumpei Arata, Norio Ikedo, and Hideo Fujimoto. Force producibility improvement of redundant parallel mechanism for haptic applications. In *Intelligent Robots and Systems (IROS), 2011 IEEE/RSJ International Conference on*, pages 2145–2150. IEEE, 2011.

[13] Reuben Brewer, Adam Leeper, and J Kenneth Salisbury. A friction differential and cable transmission design for a 3-dof haptic device with spherical kinematics. In *Intelligent Robots and Systems (IROS), 2011 IEEE/RSJ International Conference on*, pages 2570–2577. IEEE, 2011.

[14] Roger Baumann and Reymond Clavel. Haptic interface for virtual reality based minimally invasive surgery simulation. In *Robotics and Automation, 1998. Proceedings. 1998 IEEE International Conference on*, volume 1, pages 381–386. IEEE, 1998.

[15] Jorge Angeles. The qualitative synthesis of parallel manipulators. *Journal of Mechanical Design*, 126(4):617–624, 2004.

[16] Marco Carricato and Vincenzo Parenti-Castelli. A family of 3-dof translational parallel manipulators. *Journal of mechanical design*, 125(2):302–307, 2003.

[17] Qiong Jin and Ting-Li Yang. Synthesis and analysis of a group of 3-degree-of-freedom partially decoupled parallel manipulators. *Journal of Mechanical Design*, 126(2):301–306, 2004.

[18] Marco Carricato. Fully isotropic four-degrees-of-freedom parallel mechanisms for schoenflies motion. *The International Journal of Robotics Research*, 24(5):397–414, 2005.

[19] Xianwen Kong and Clement M Gosselin. Type synthesis of 3-dof translational parallel manipulators based on screw theory. *Journal of mechanical design*, 126(1):83–92, 2004.

[20] Xianwen Kong and Clément Gosselin. *Type Synthesis of Parallel Mechanisms*, volume 70. 2004.

[21] Hamid D Taghirad. *Parallel robots: mechanics and control.* CRC press, 2013.